Stochastic cracking of composites with micromechanical formulation of crack bridges

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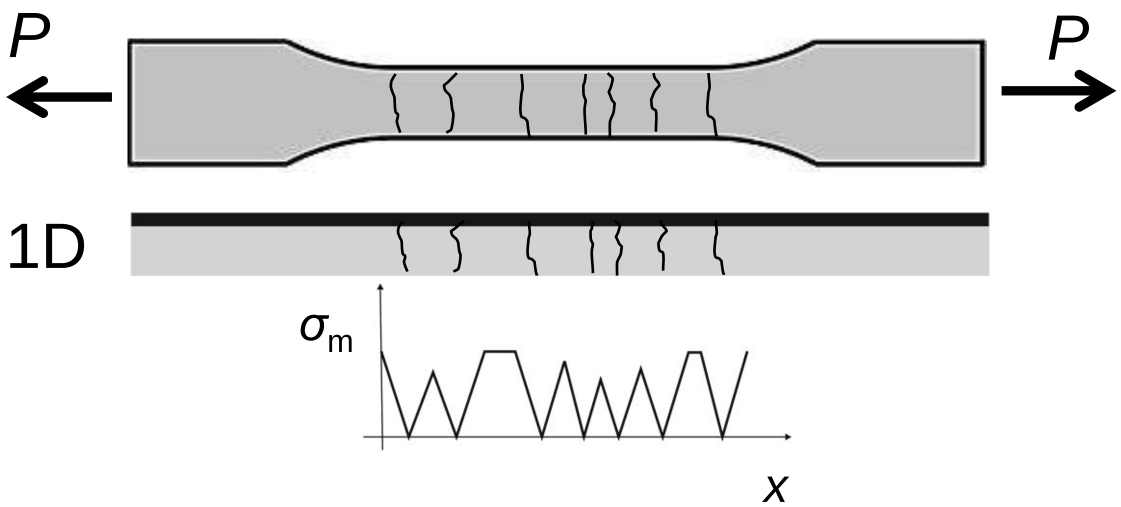
GERMANY

CZECH REP.

motivation



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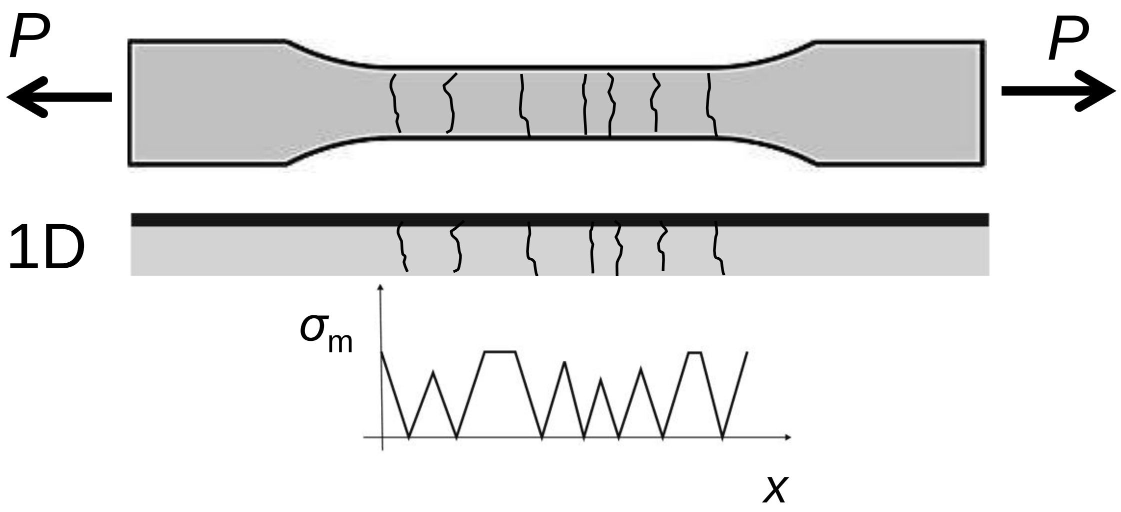
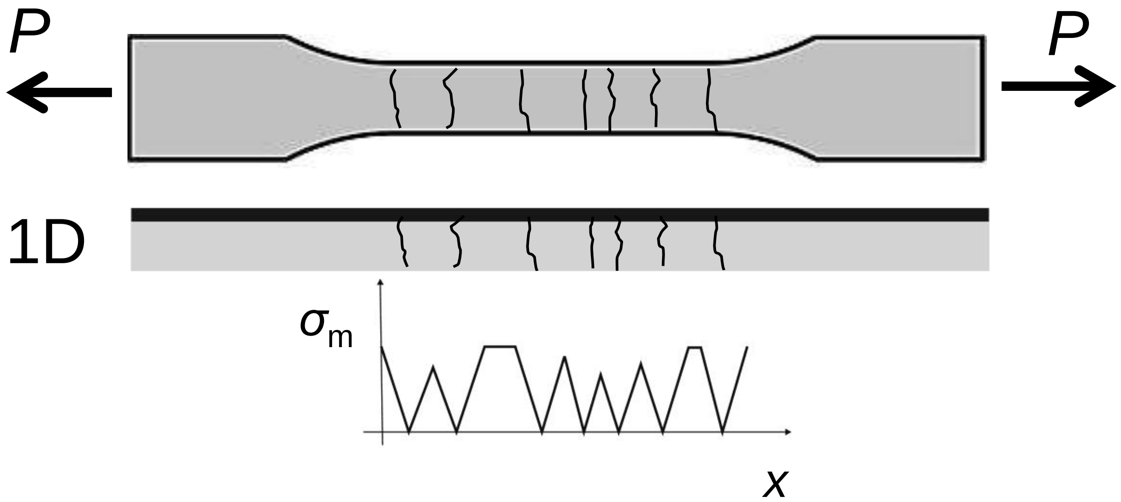
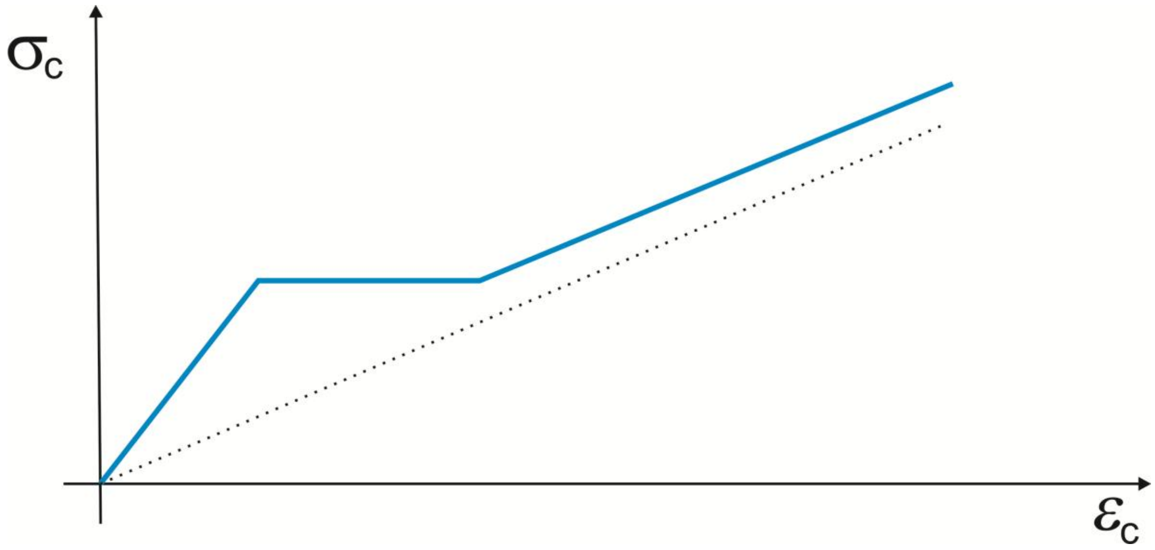


ACK modell

σ_{mu} = const.

motivation





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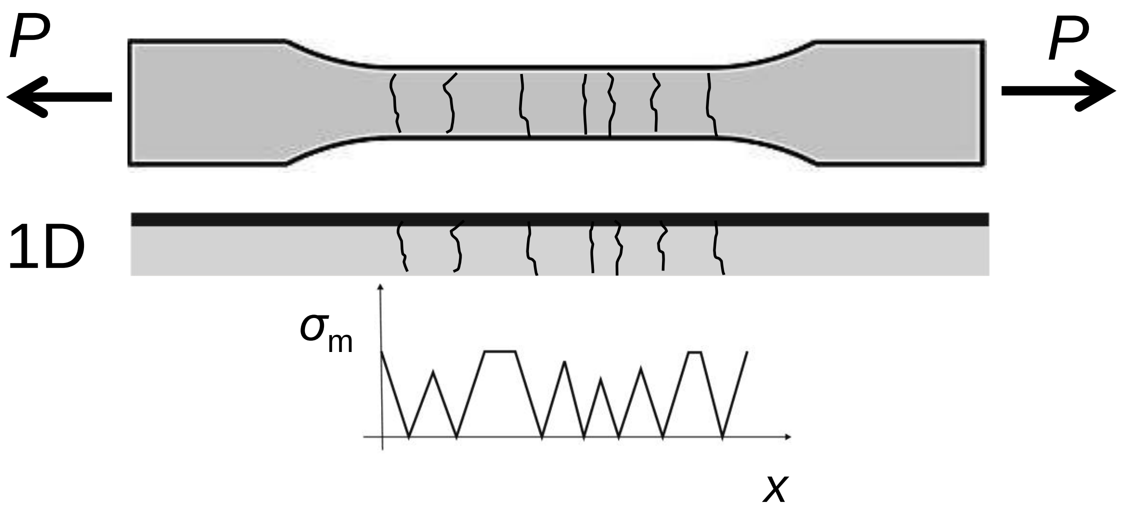

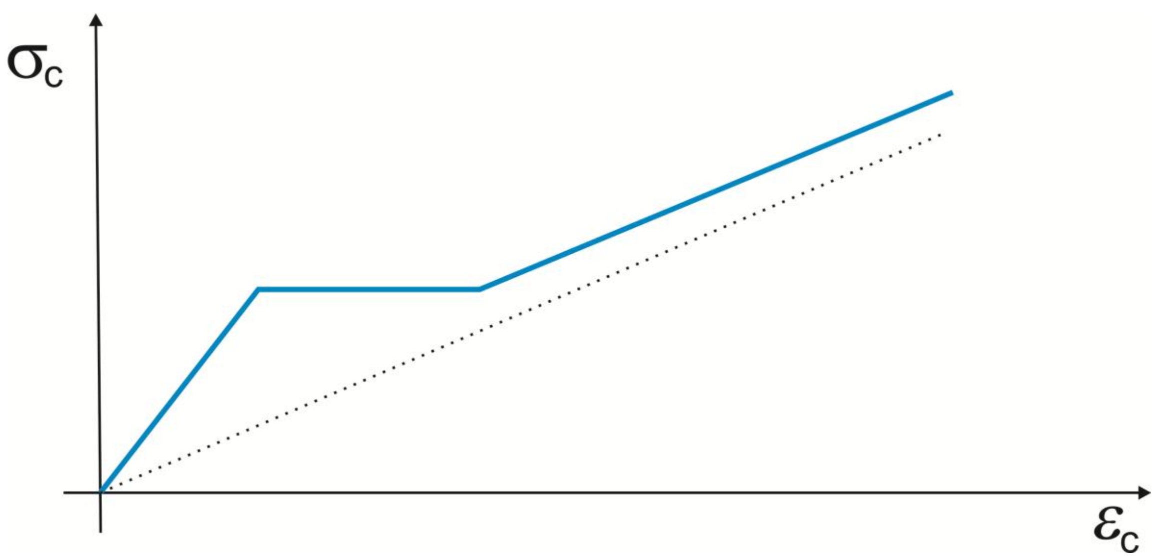

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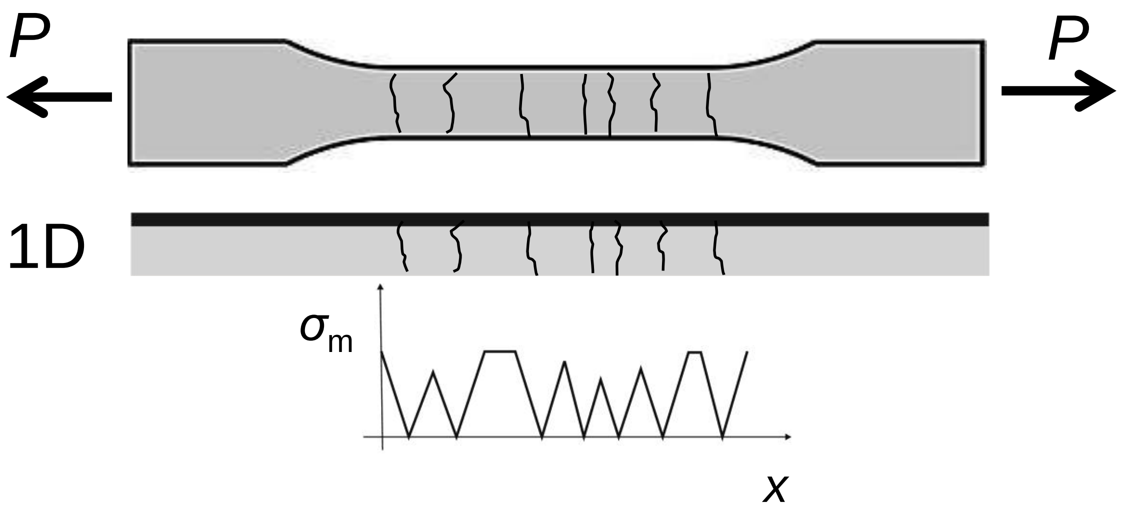
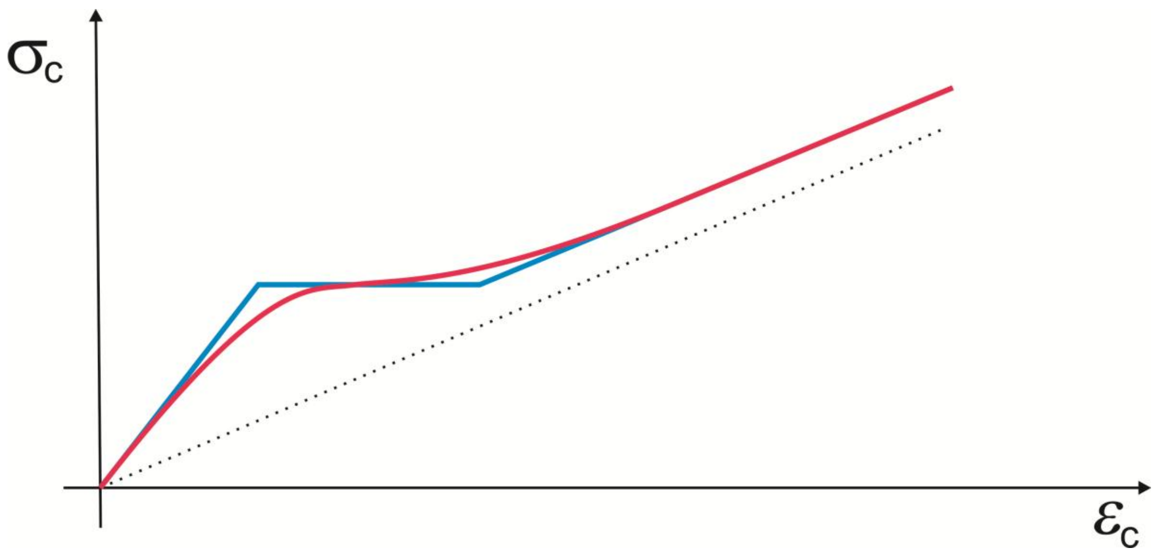
Curtin modell

σ_{mu} = random

(Weibull distributed)





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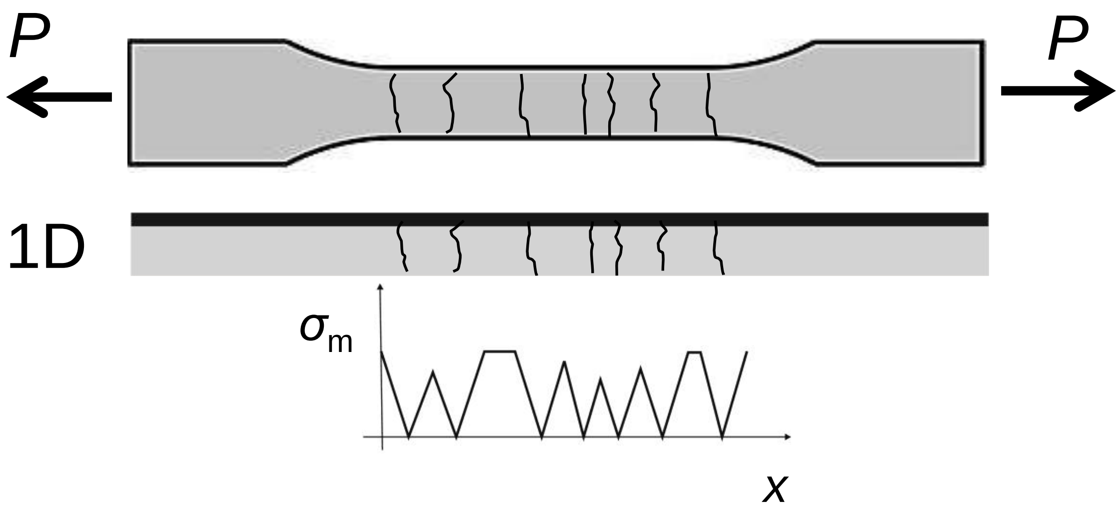

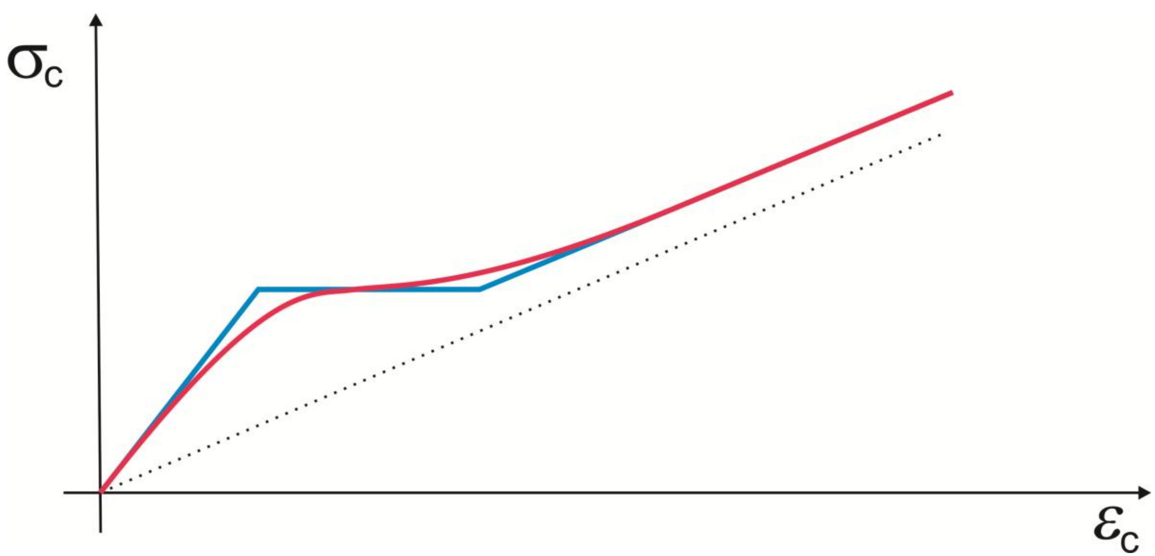

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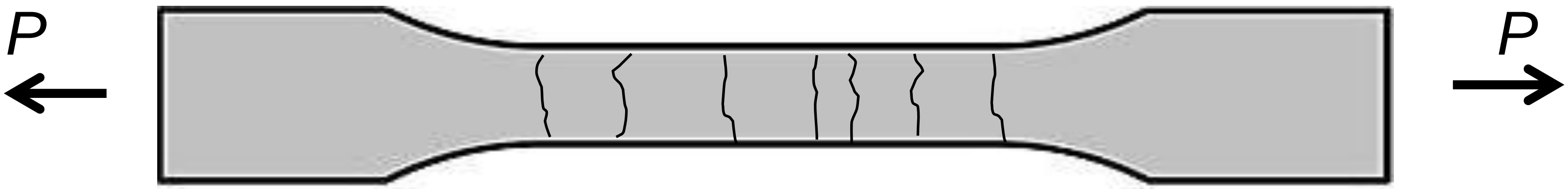
assumptions:

- const. shear flow at the interface

- final crack spacing can be evaluated

motivation

heterogeneous reinforcement (yarns, short fibers, combined reinf.)



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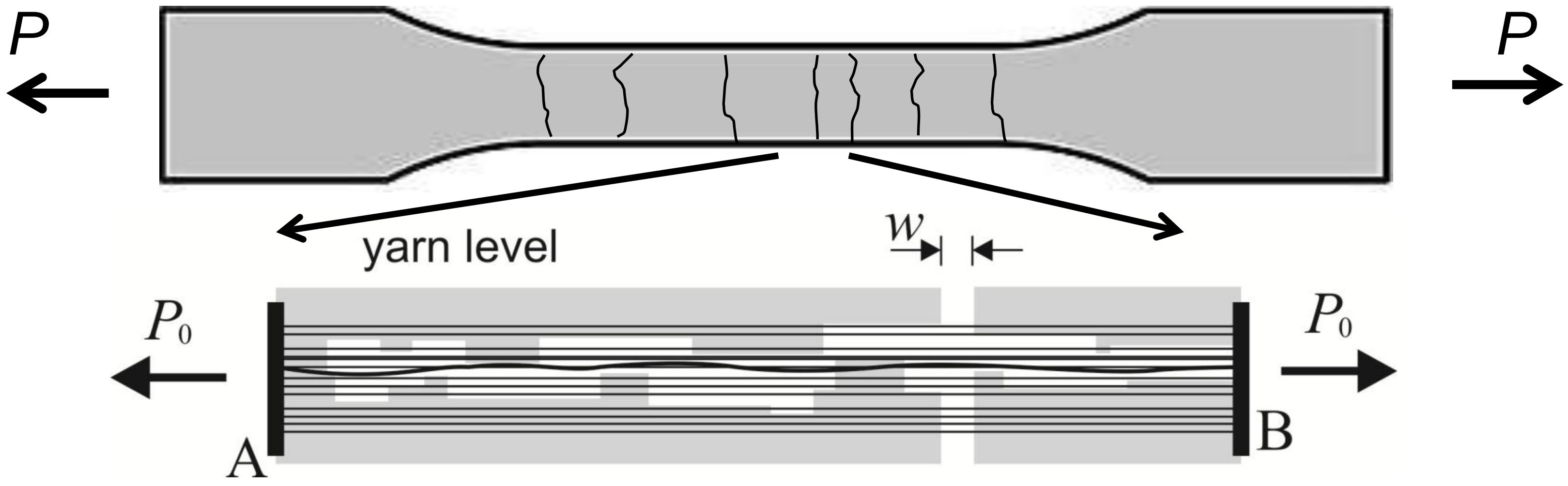

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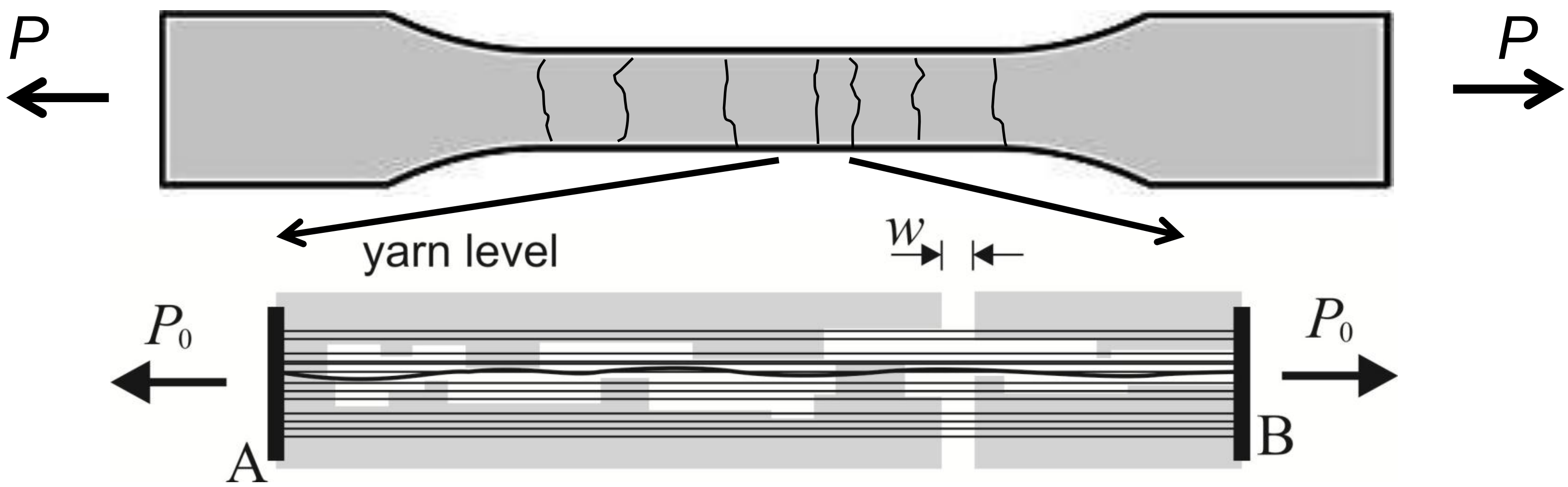
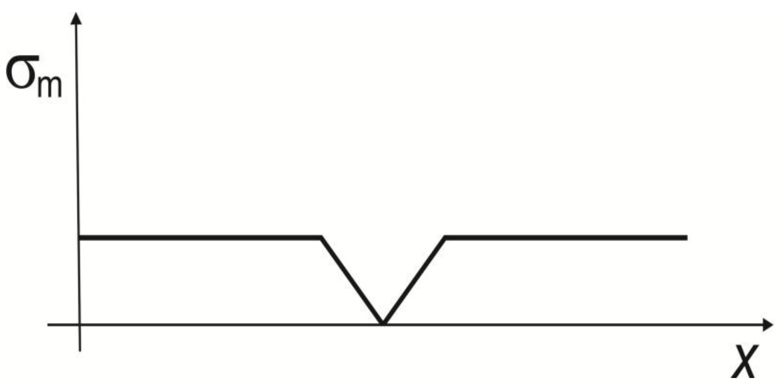
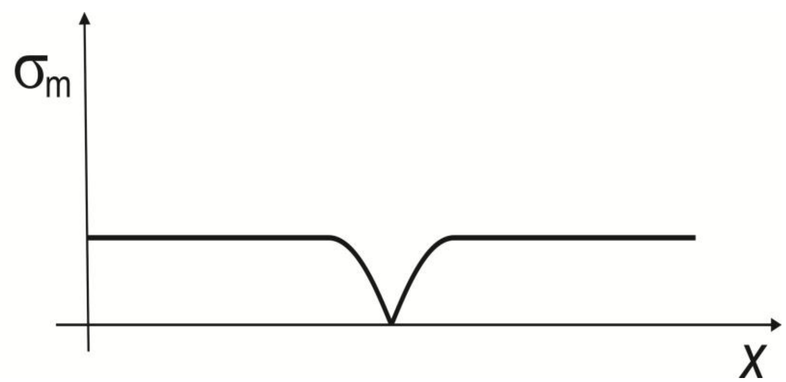




homogeneous interface



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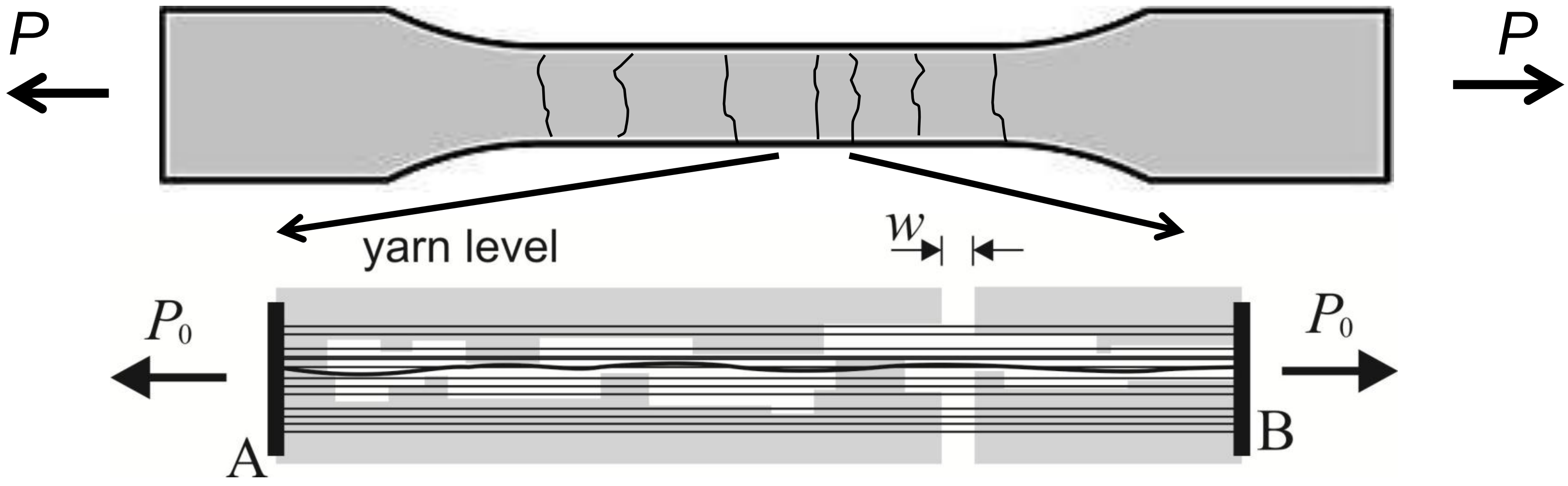

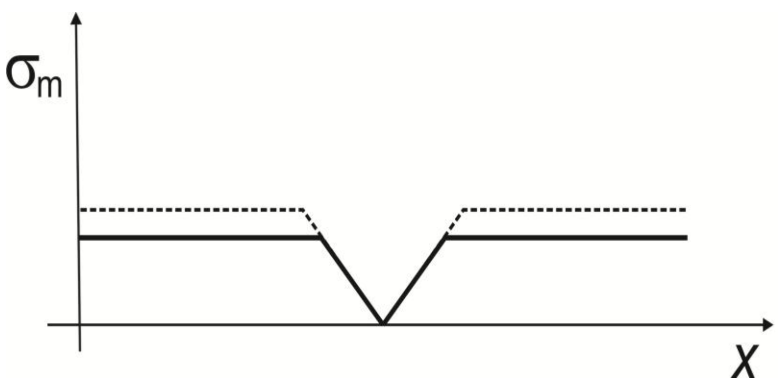

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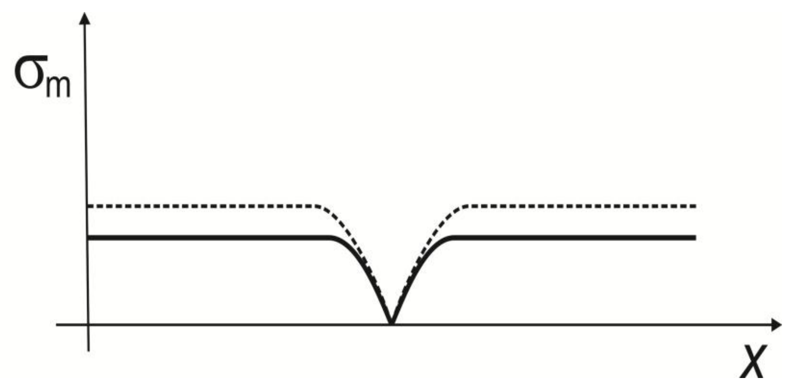

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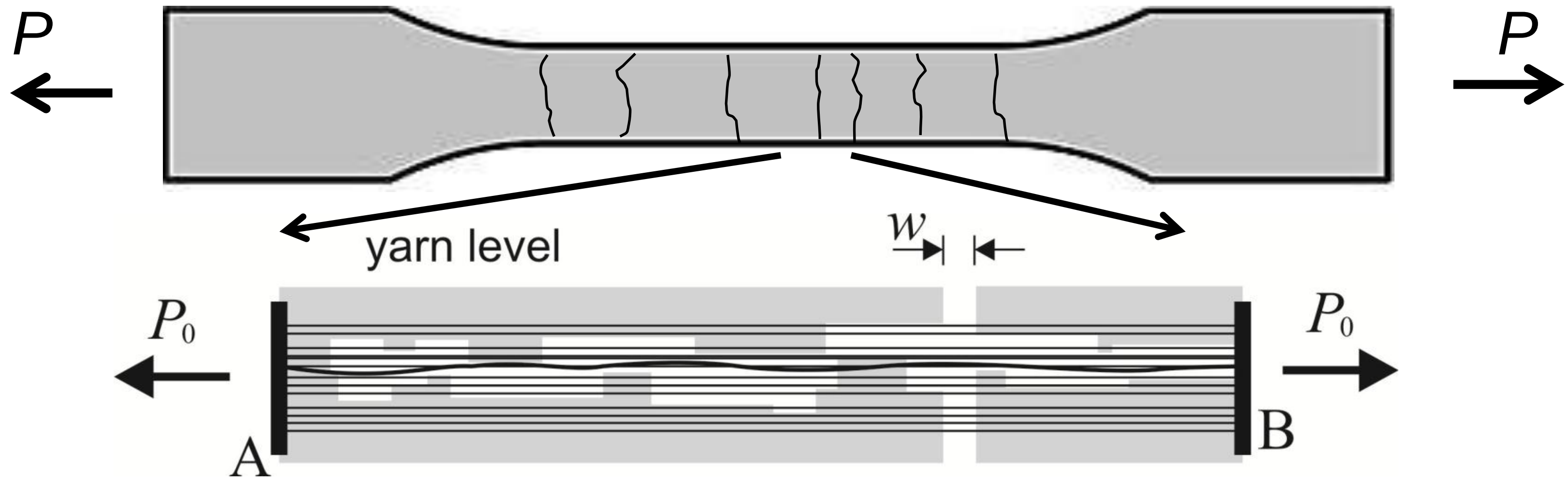

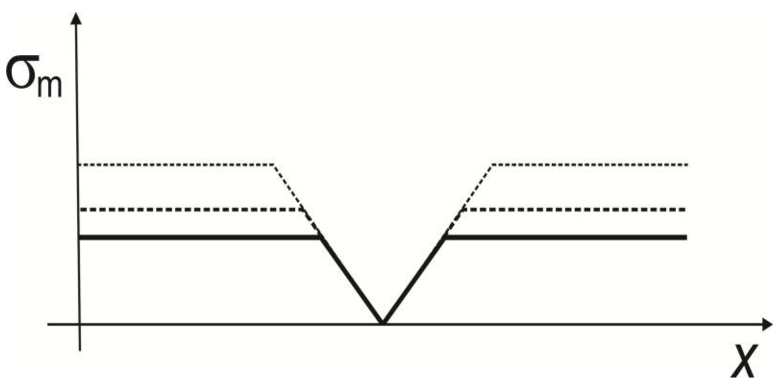

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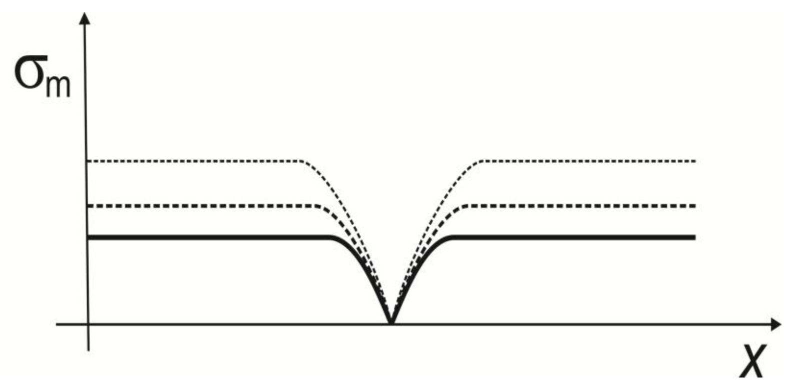
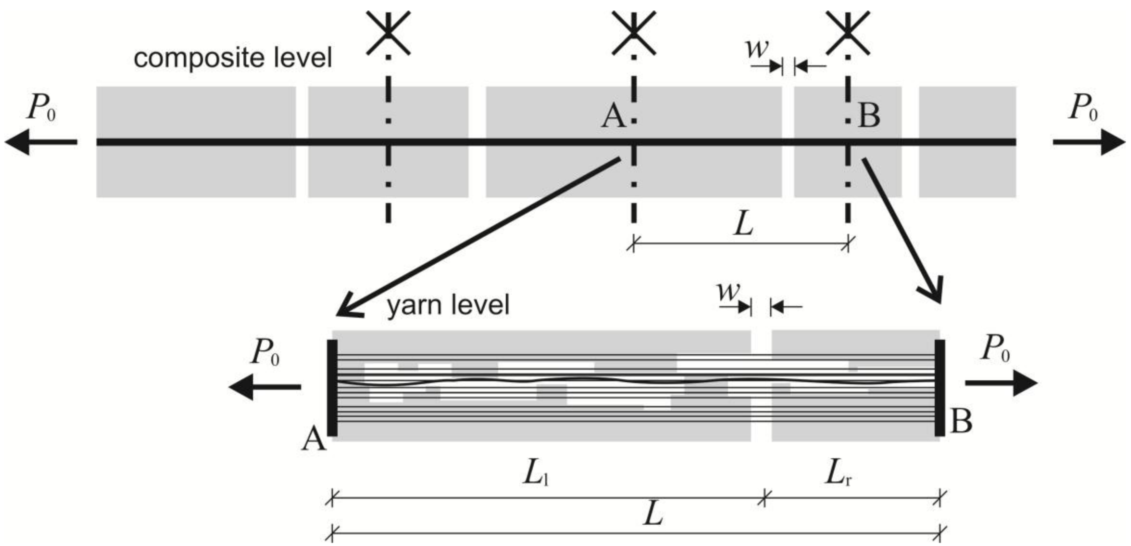


heterogeneous interface

micromechanical formulation of a crack



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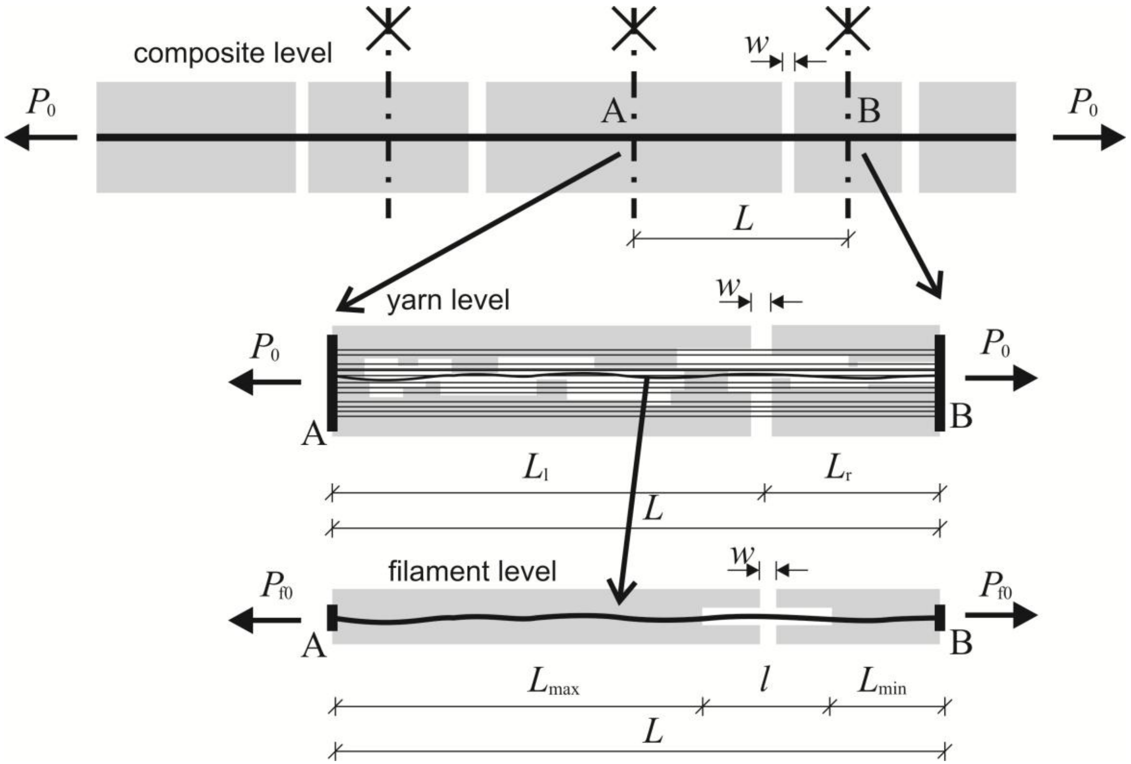

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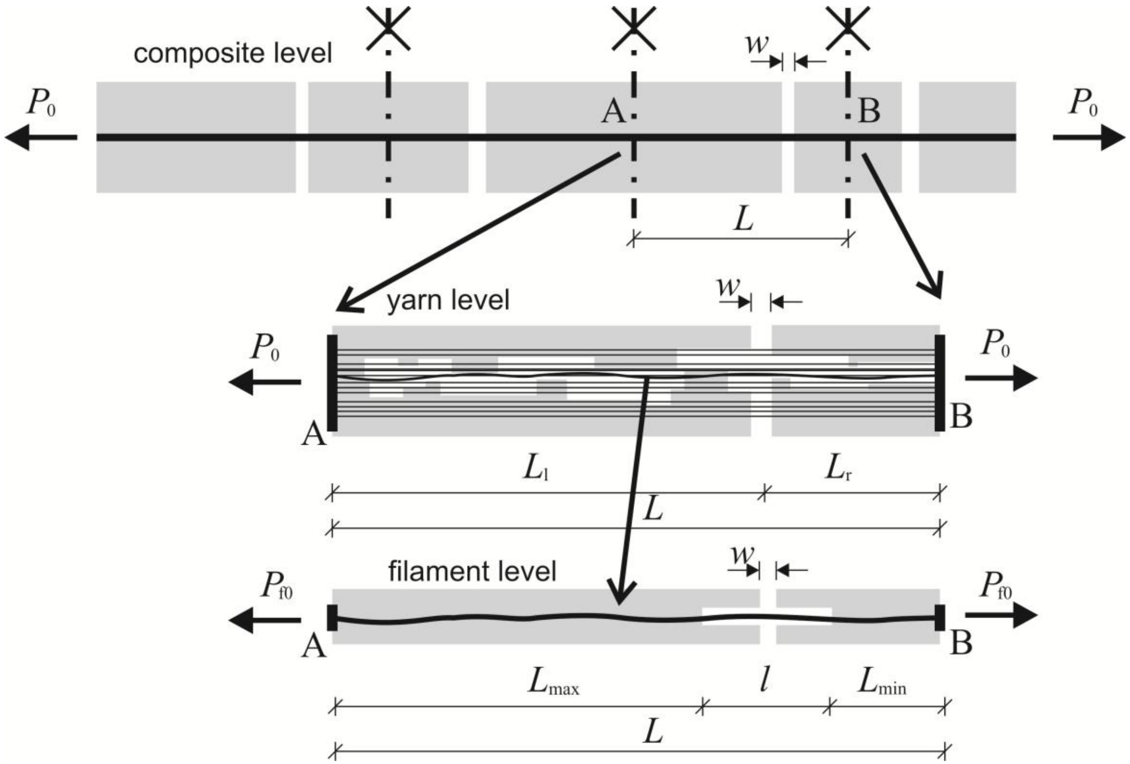

homogenization

1) single filament response

$$\varepsilon_{\mathrm{f}}(w, x, L_l, L_r, \theta)$$

θ vector of random variables

$$\theta = (\tau, l, A_{\mathrm{f}}, E_{\mathrm{f}} \ldots)$$

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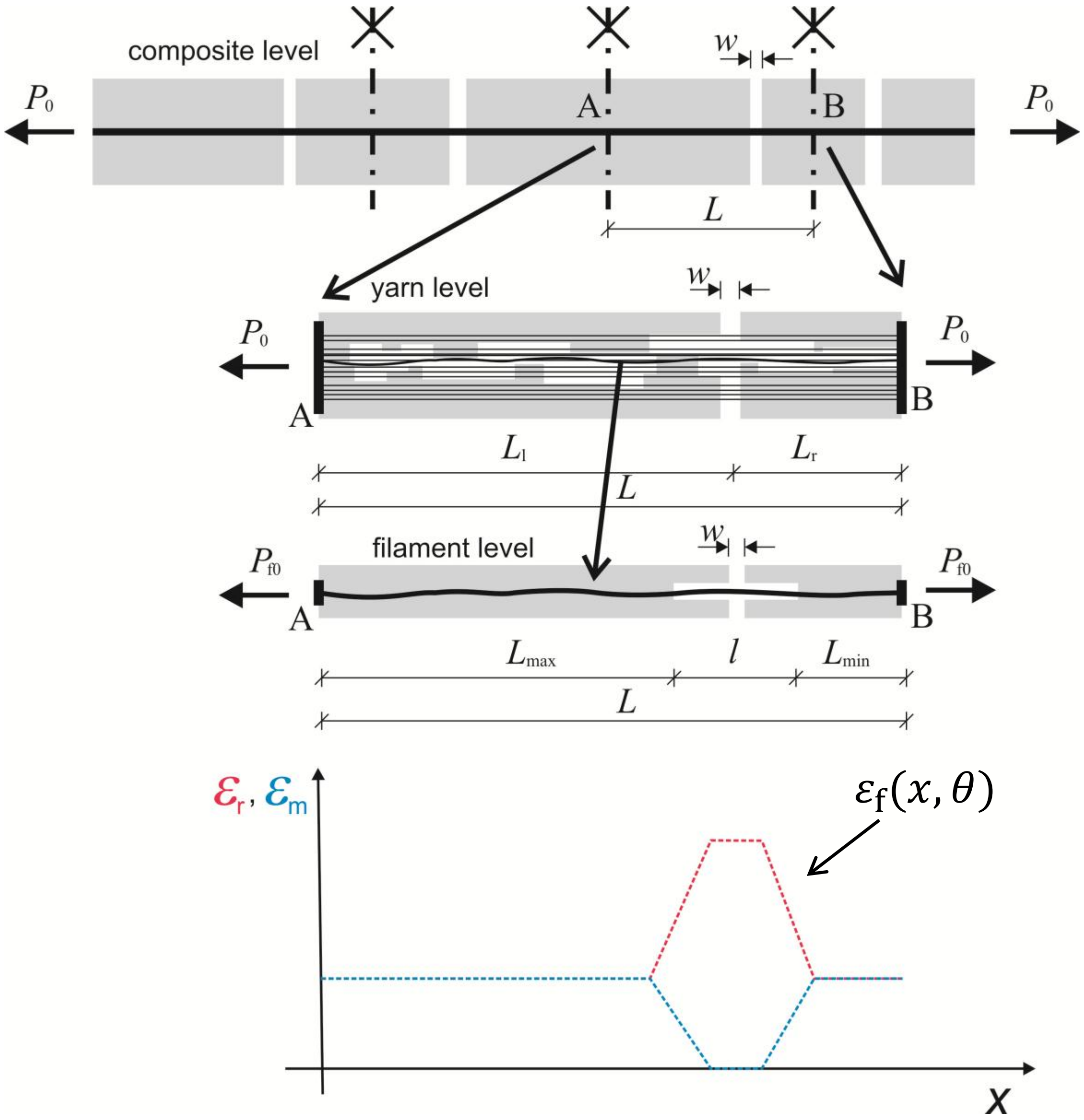

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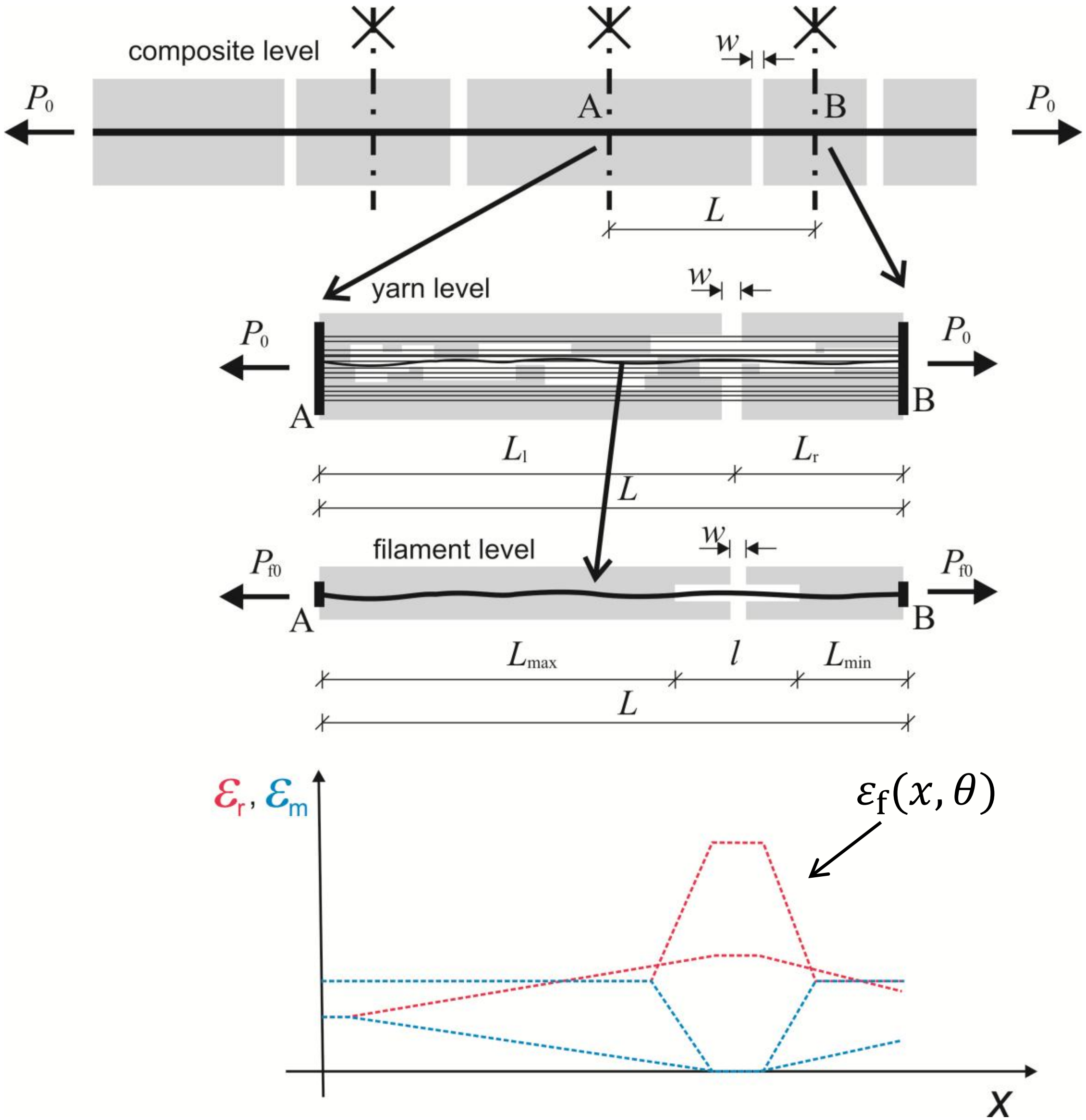

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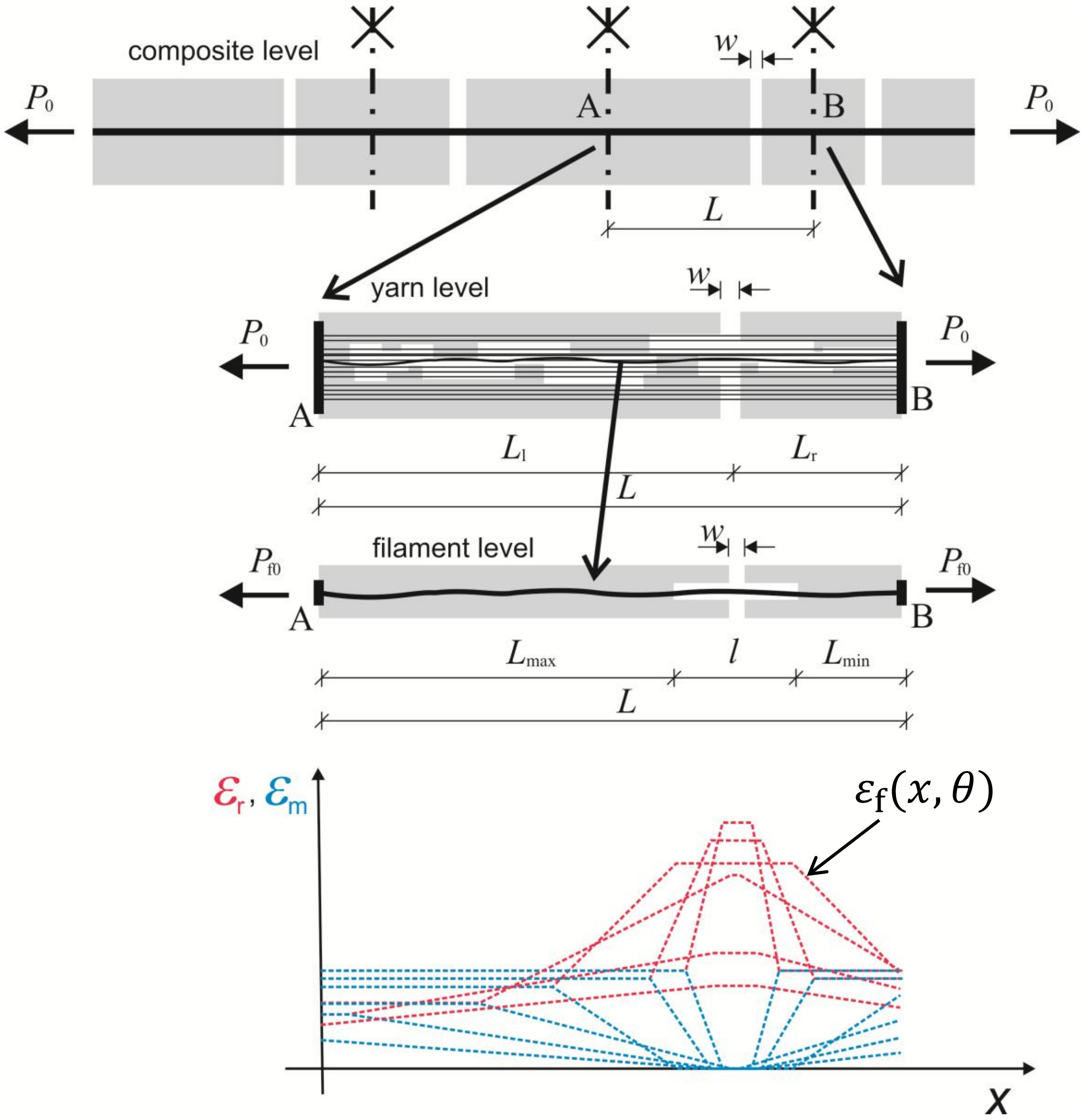

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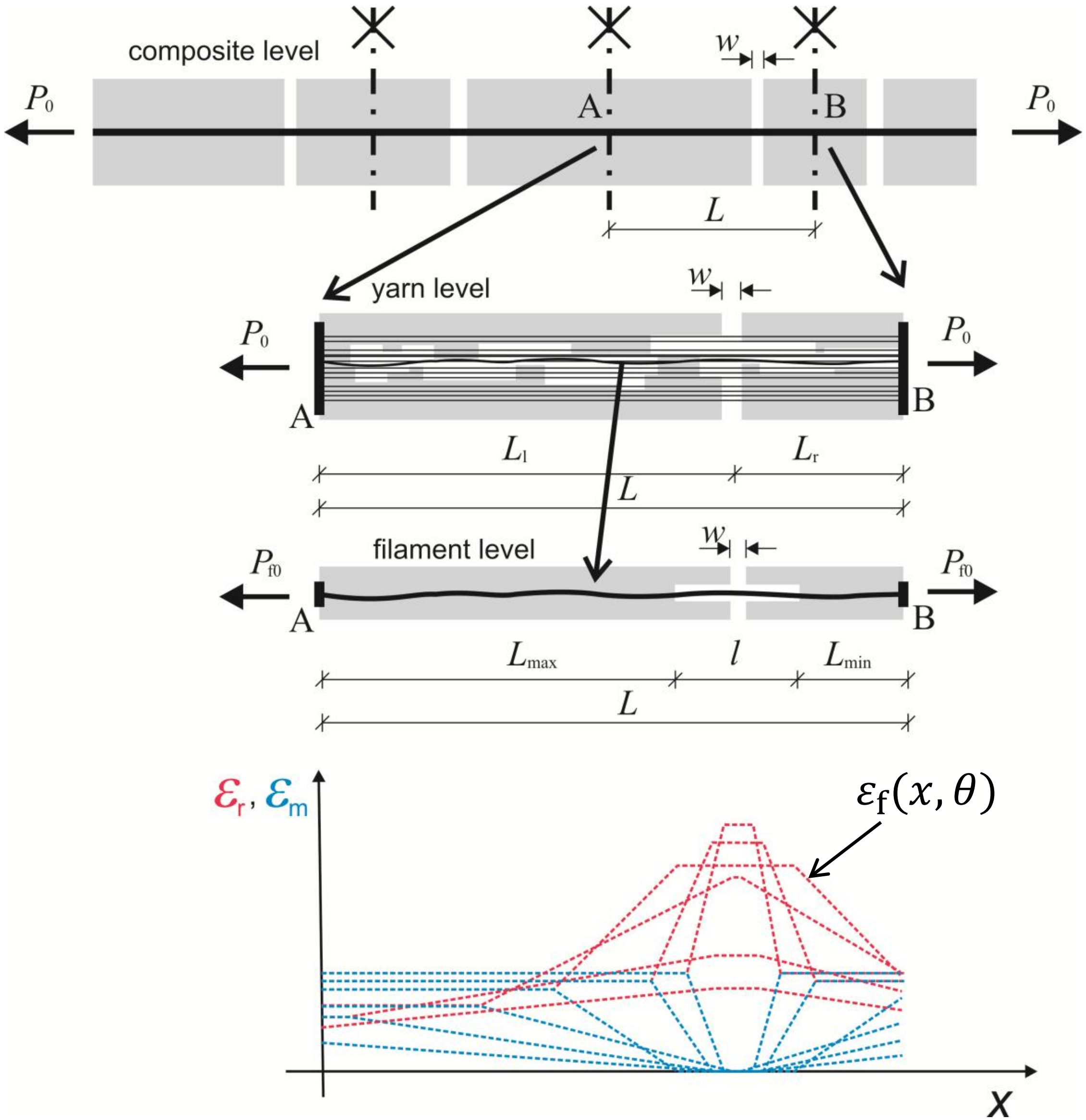

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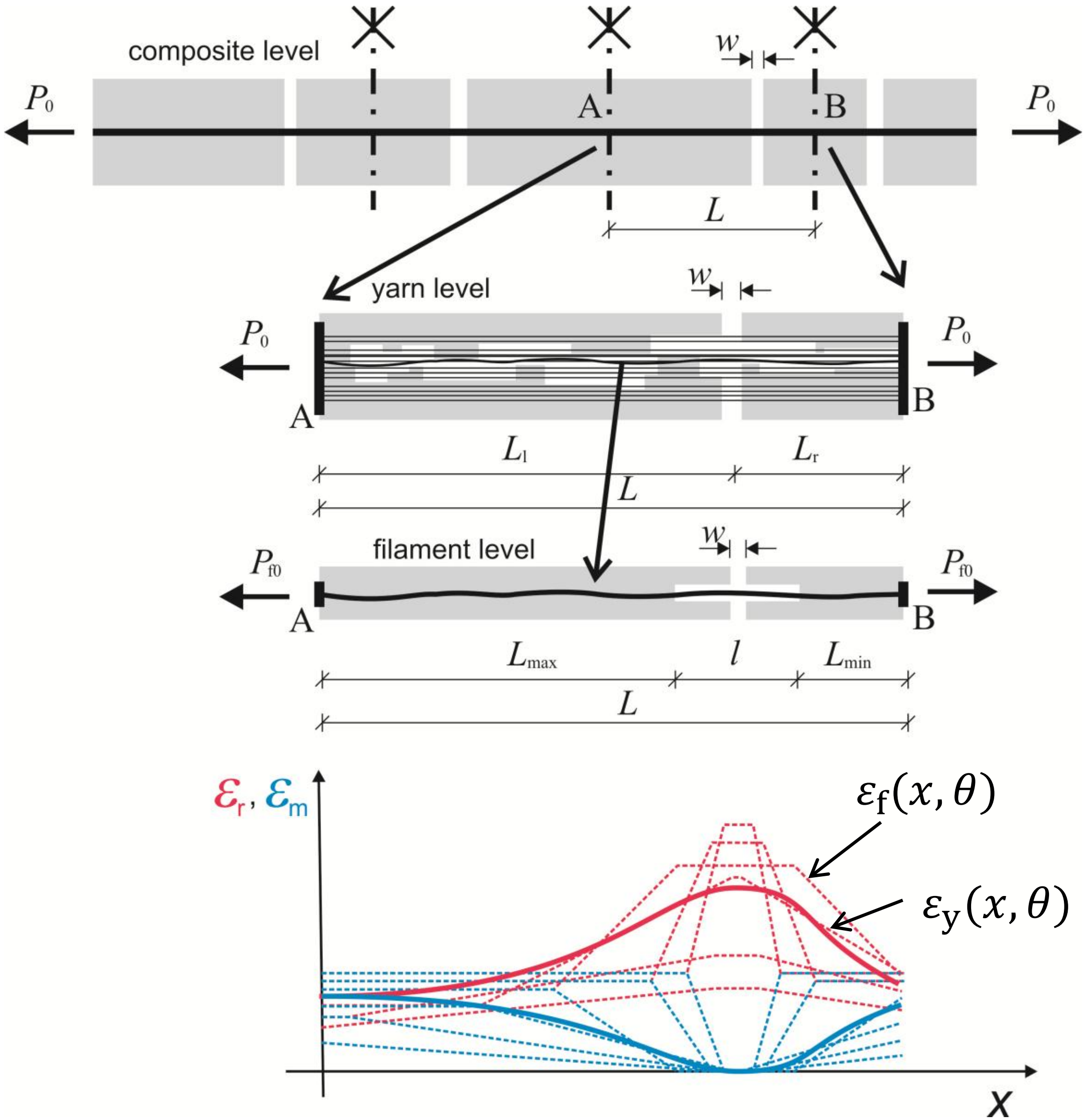
= average filament response

$$\varepsilon_y = \int_\theta \varepsilon_f(w, x, L_l, L_r, \theta) g(\theta) d\theta$$

$g(\theta)$ joint PDF for θ

micromechanical formulation of a crack



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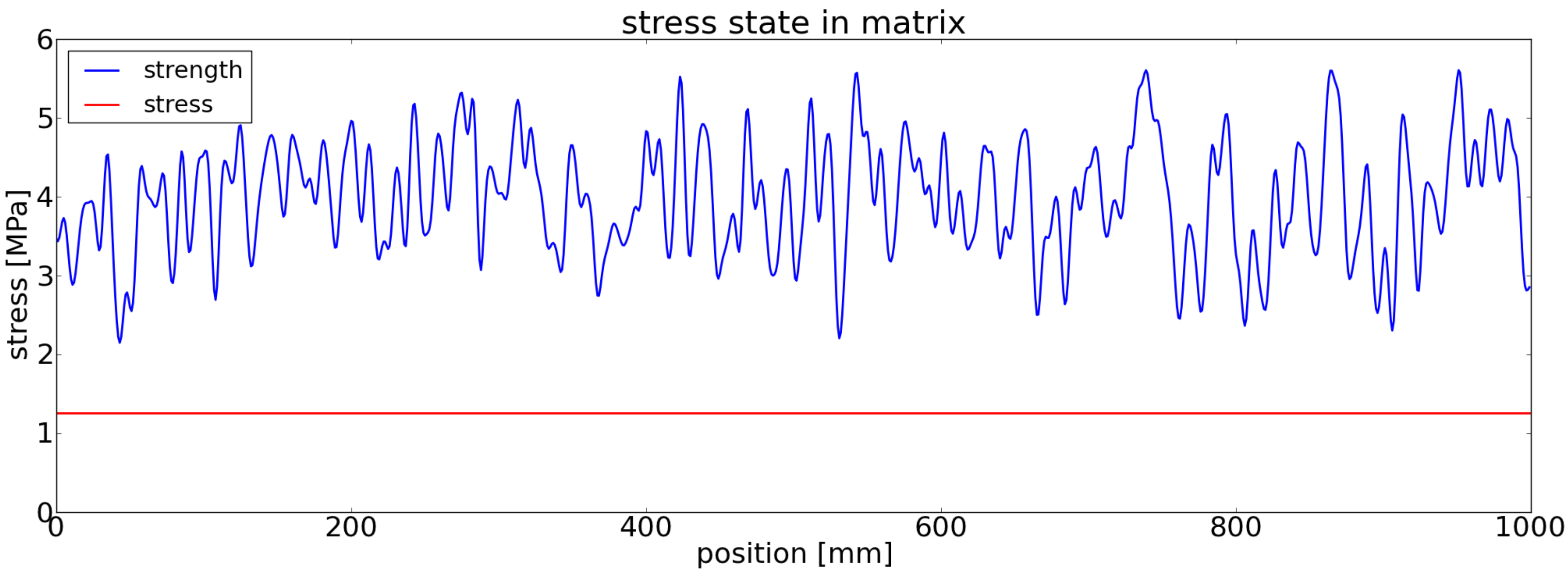
crack developement

matrix stress **>** matrix strength



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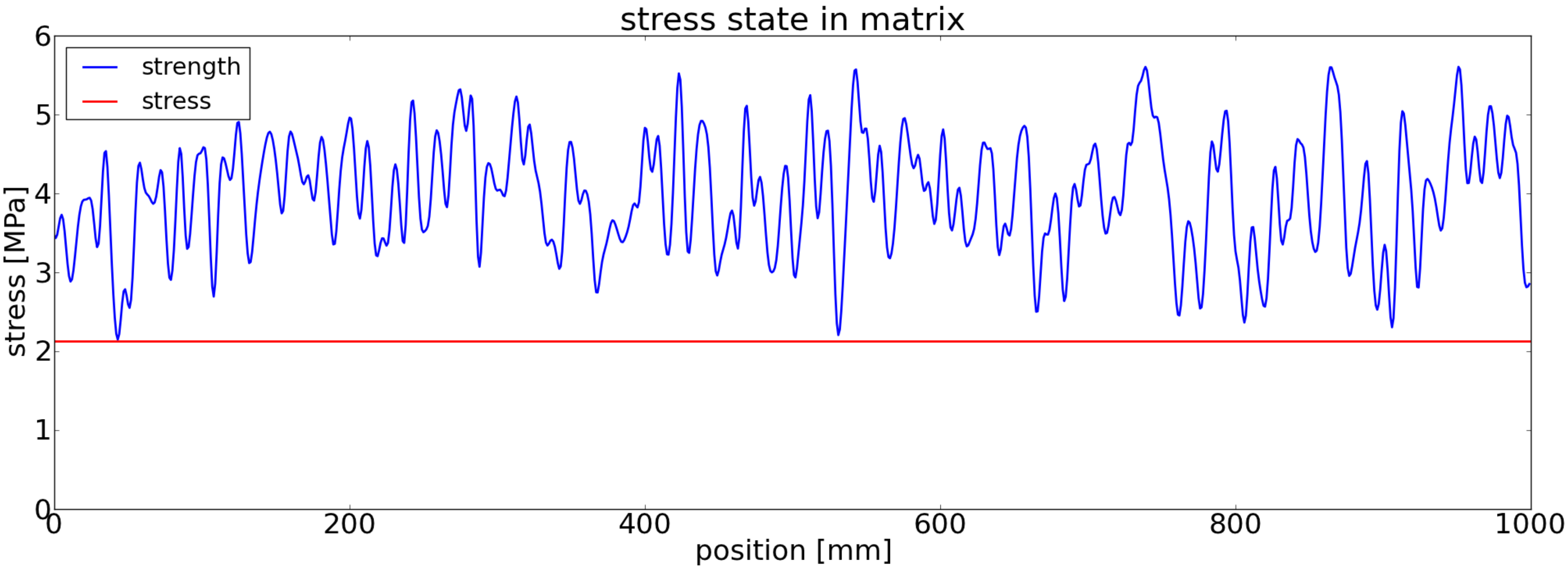

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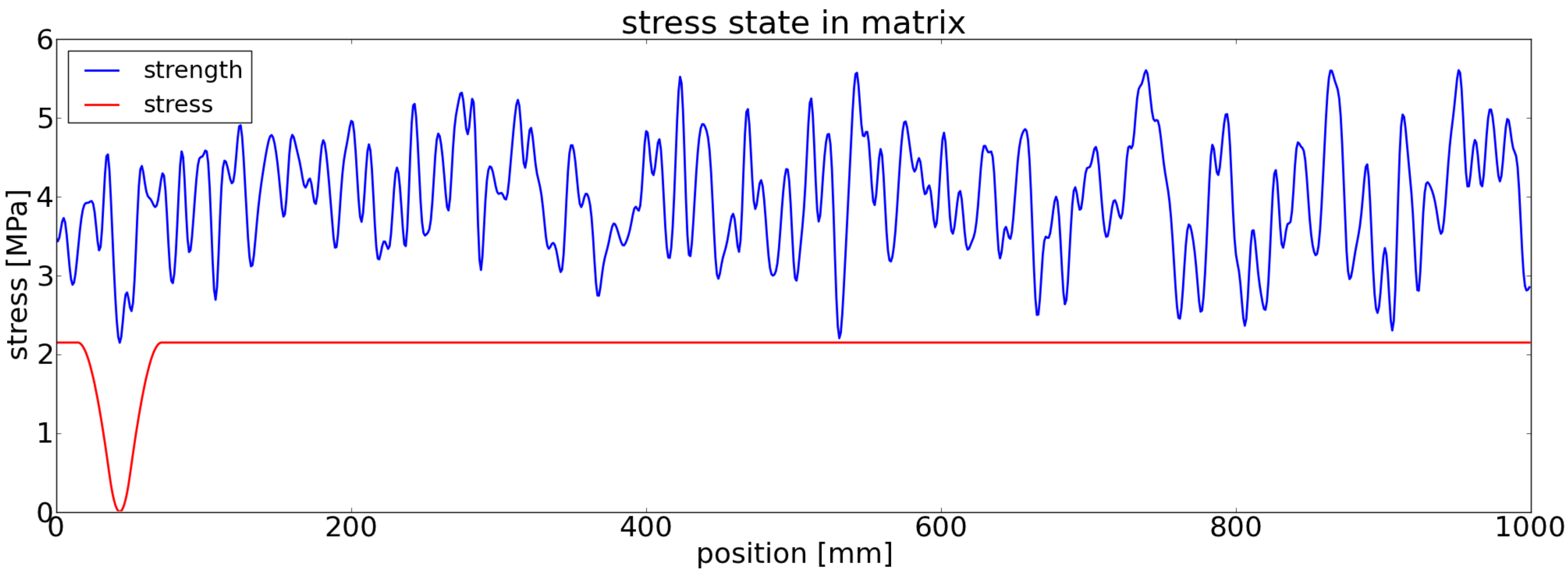

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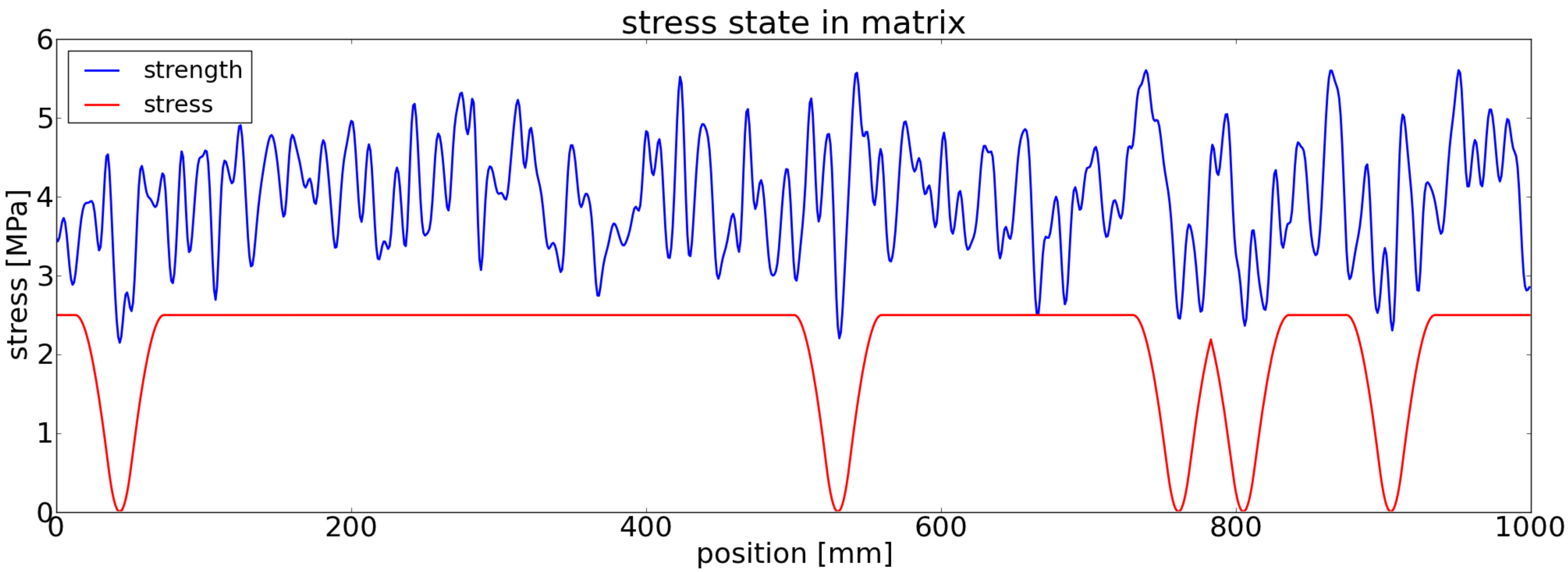

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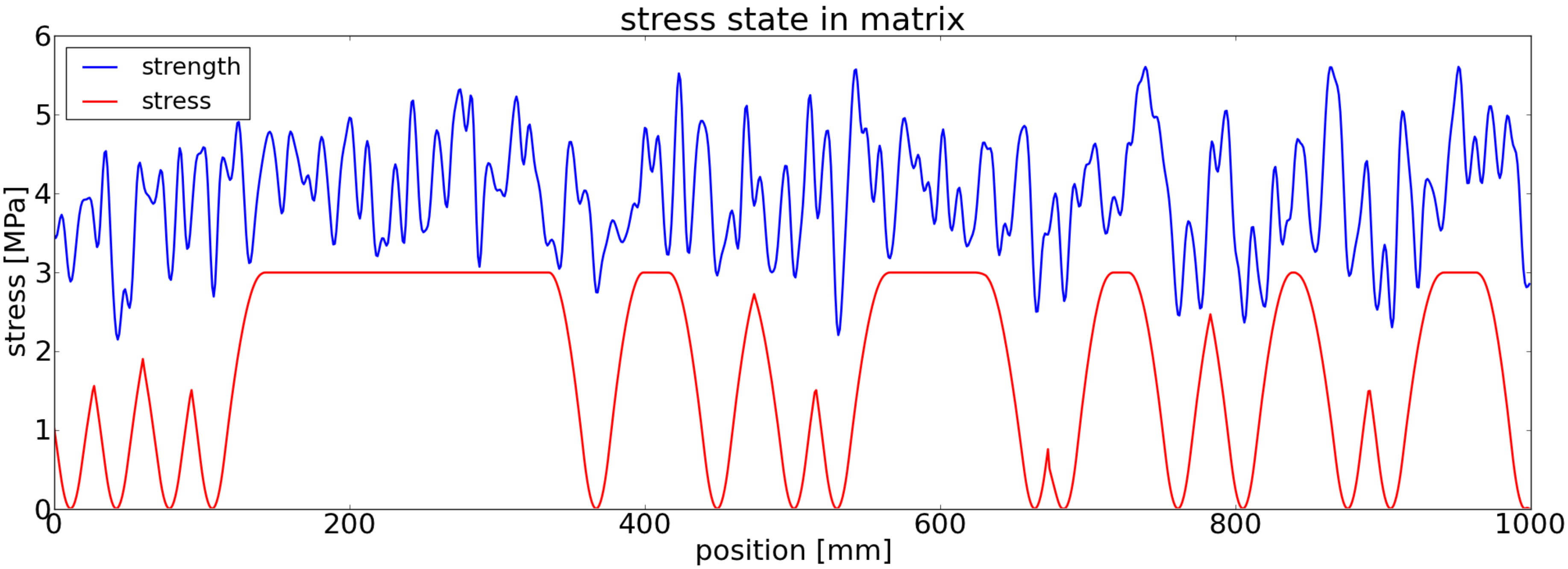

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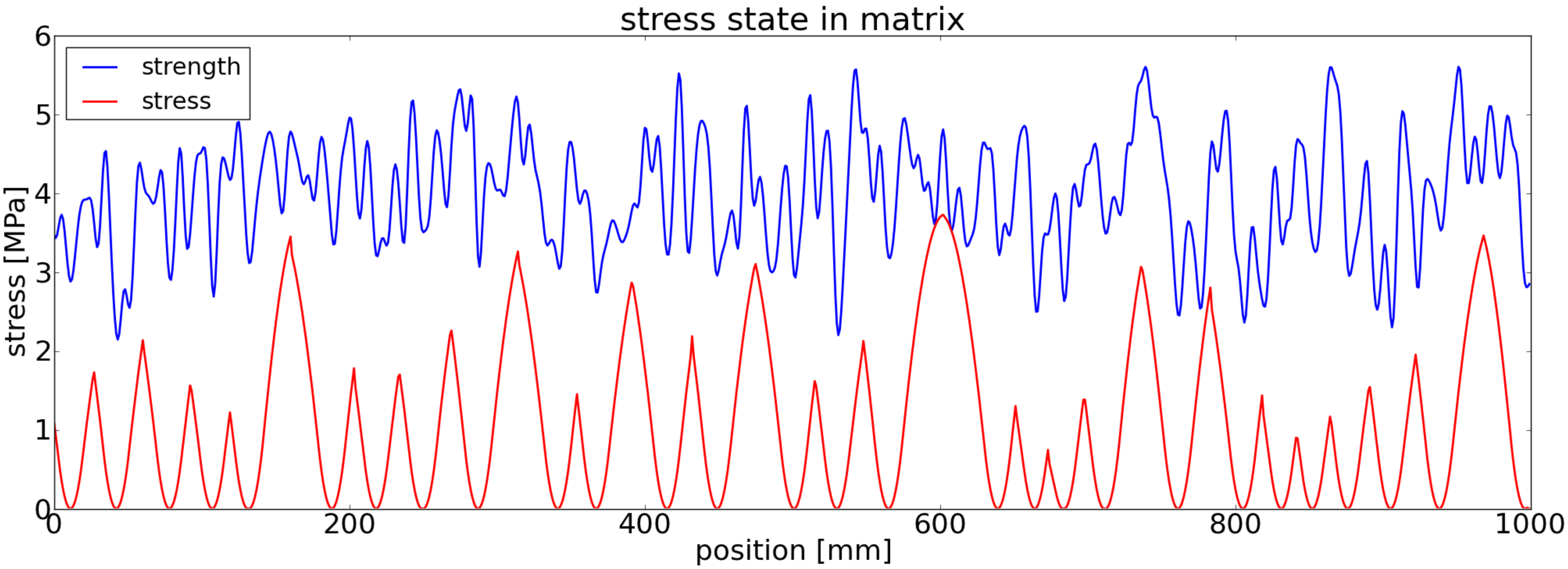

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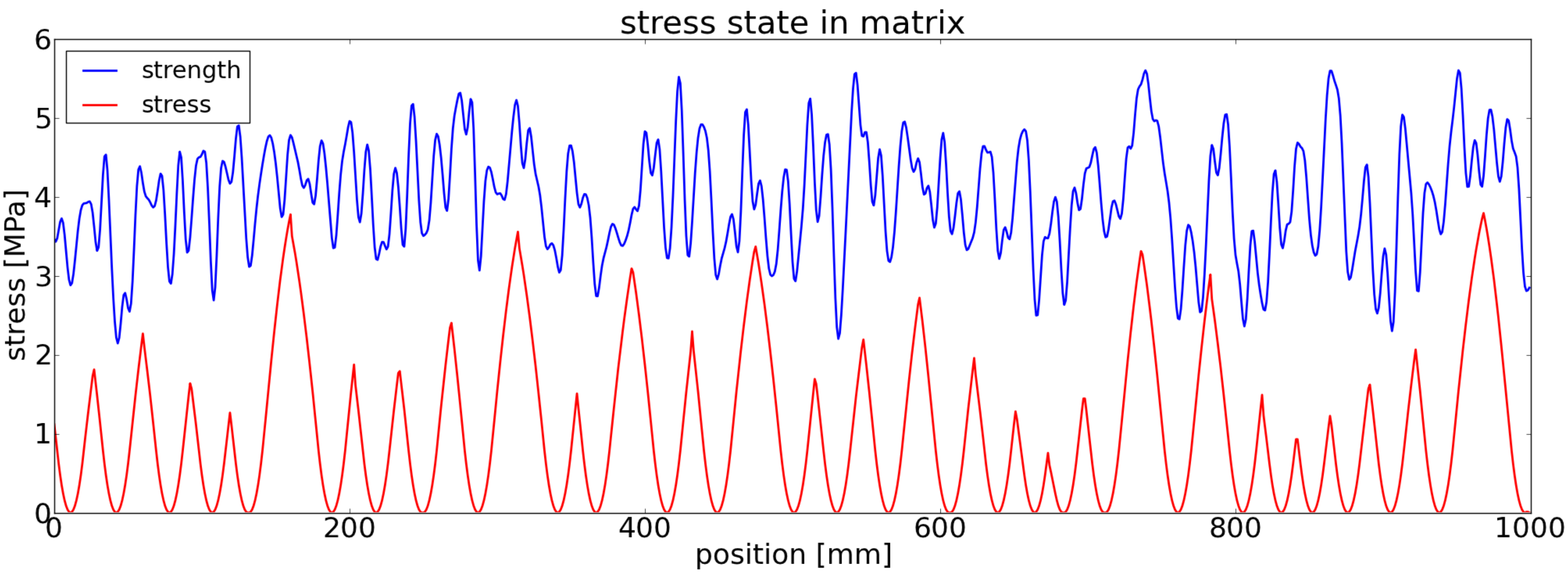

stress – strain curve

the tensile test is load driven

$$\varepsilon_{\mathrm{c}} = \frac{1}{l}\int_0^l \varepsilon_{\mathrm{r}}(P, x, L_l, L_r, \theta)\, dx$$

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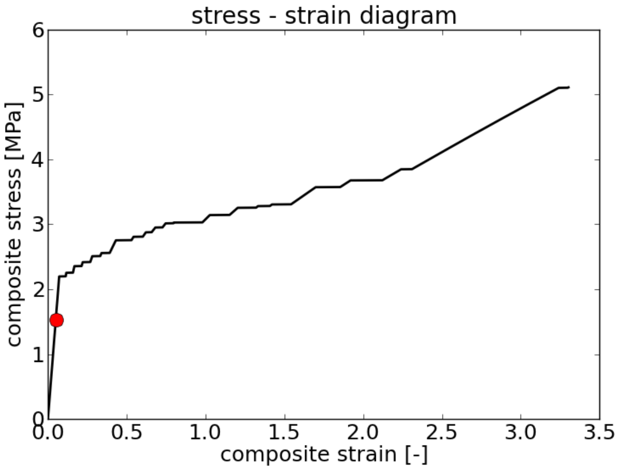

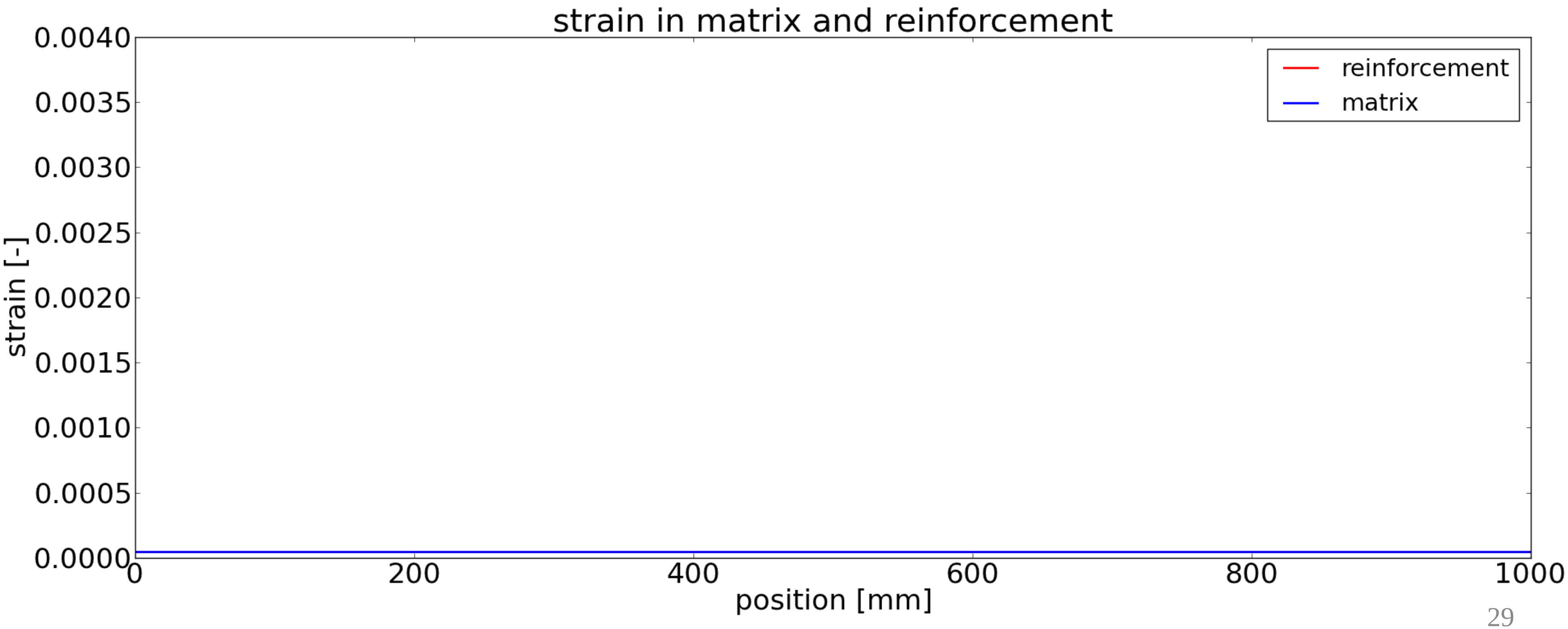

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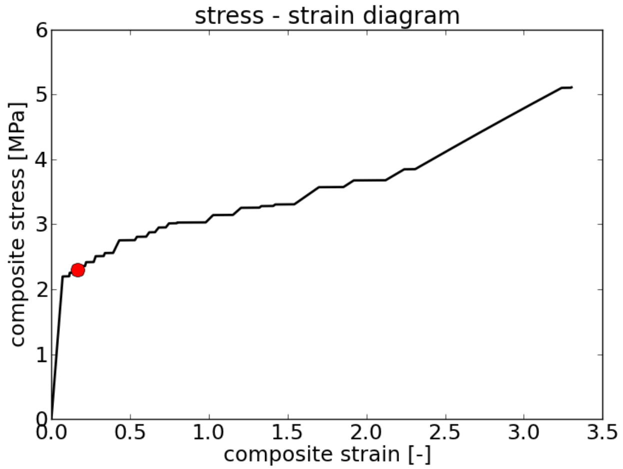

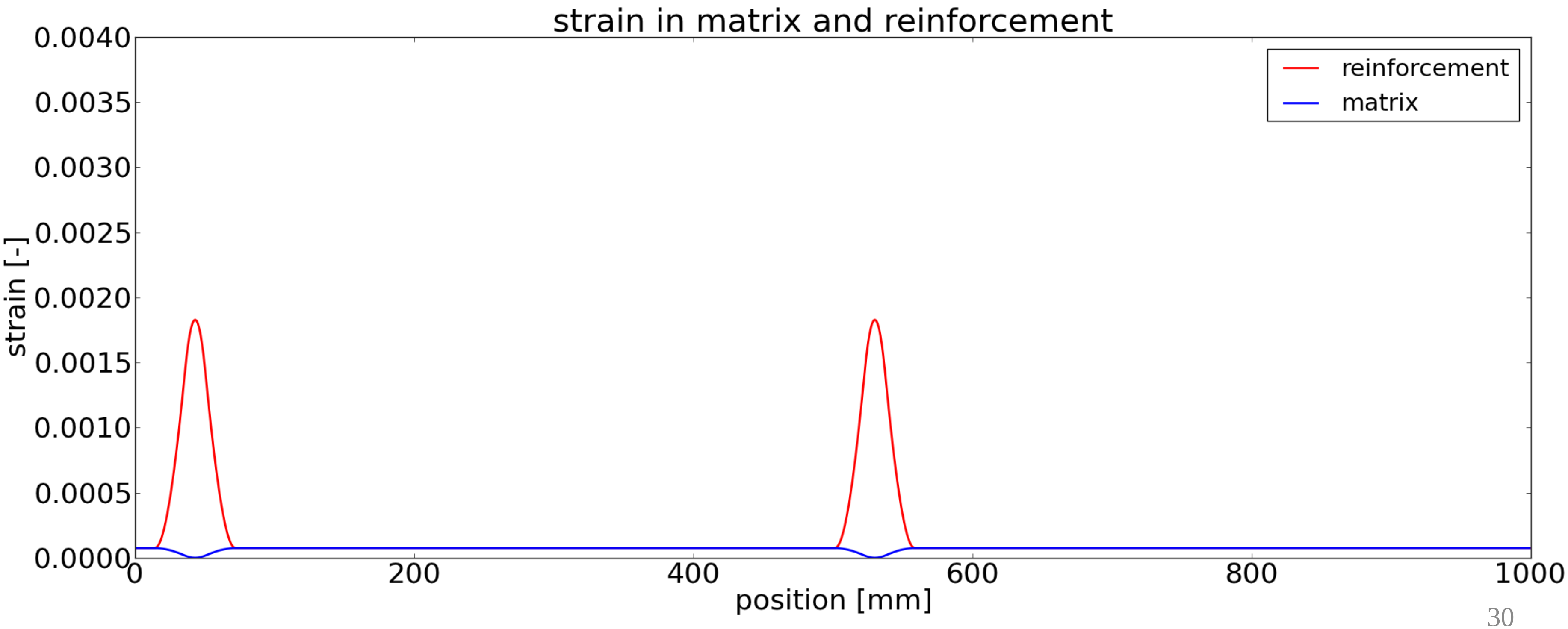

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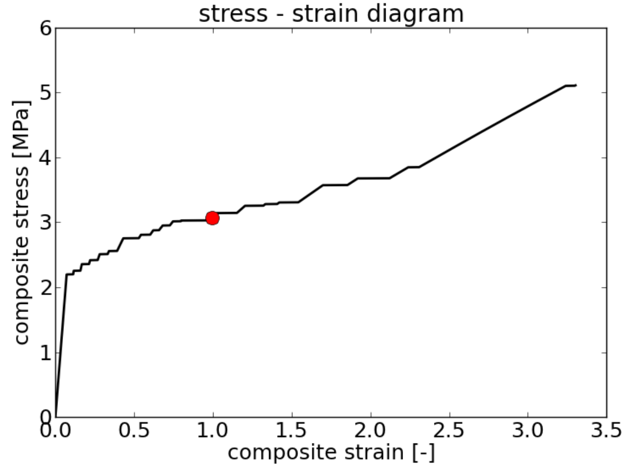

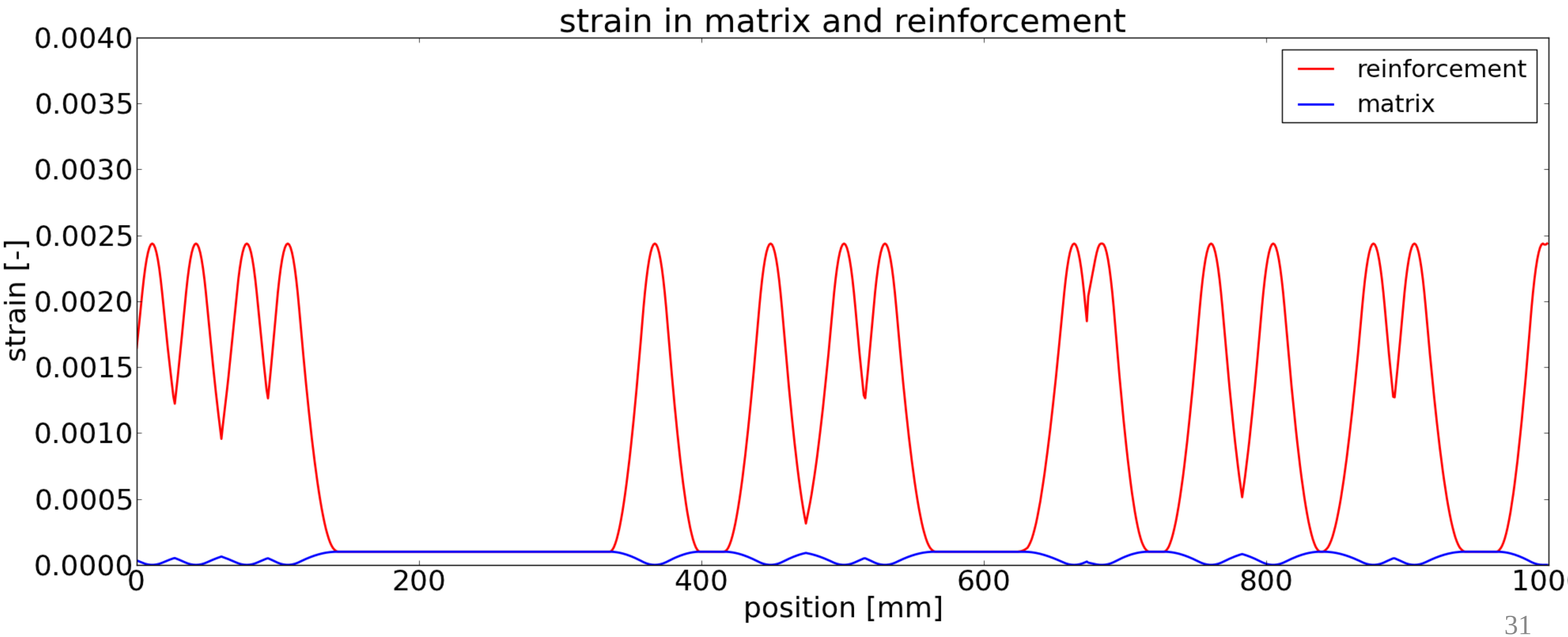

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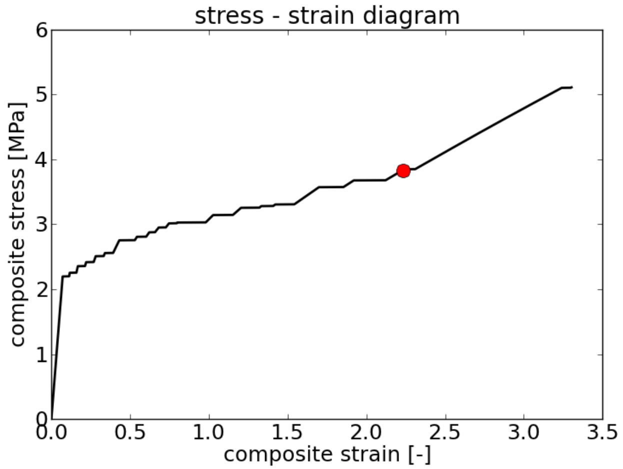

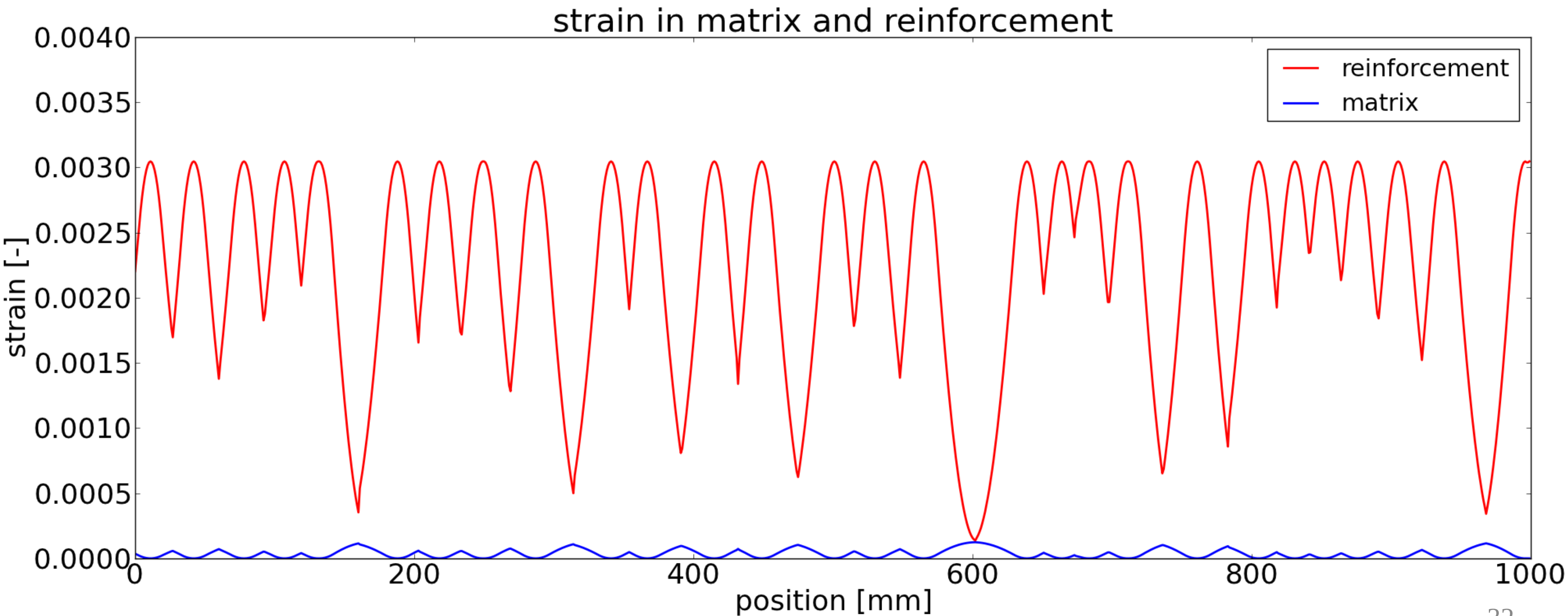

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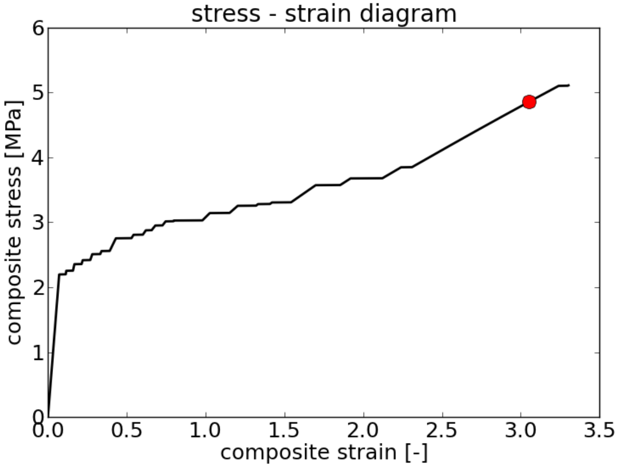
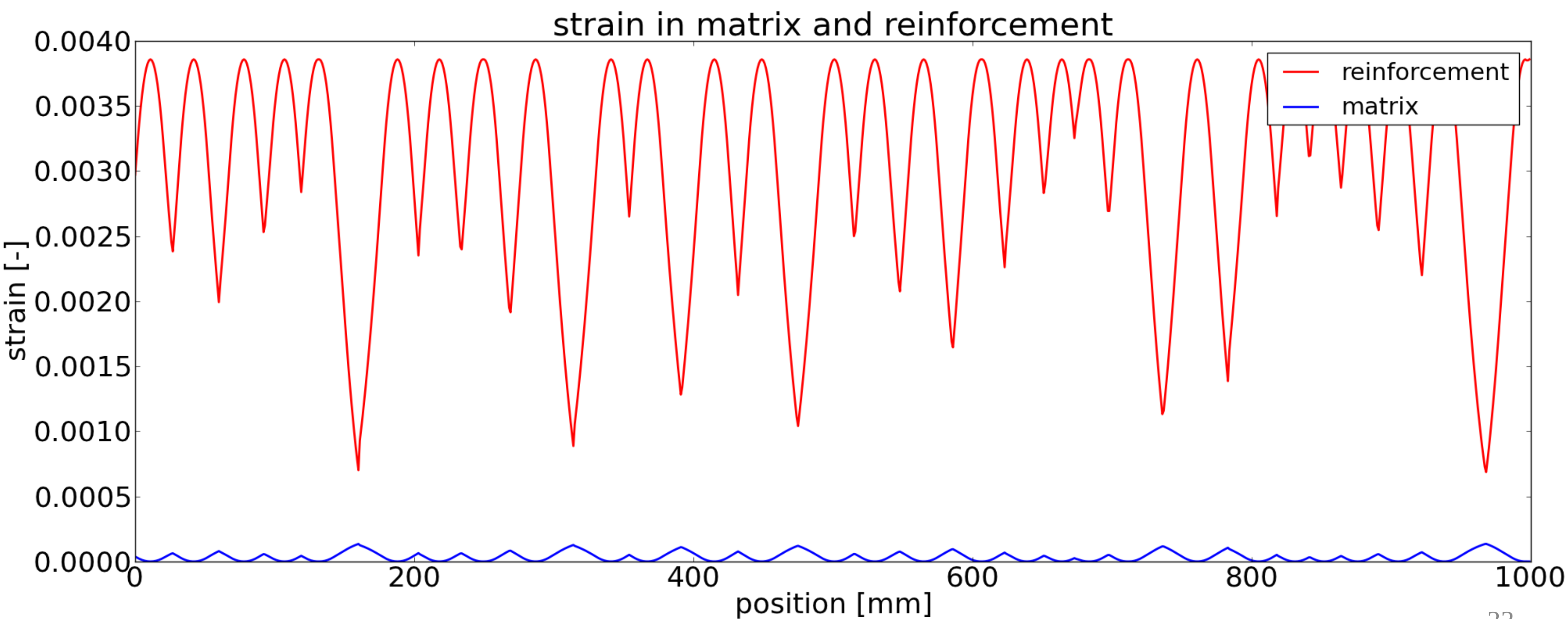
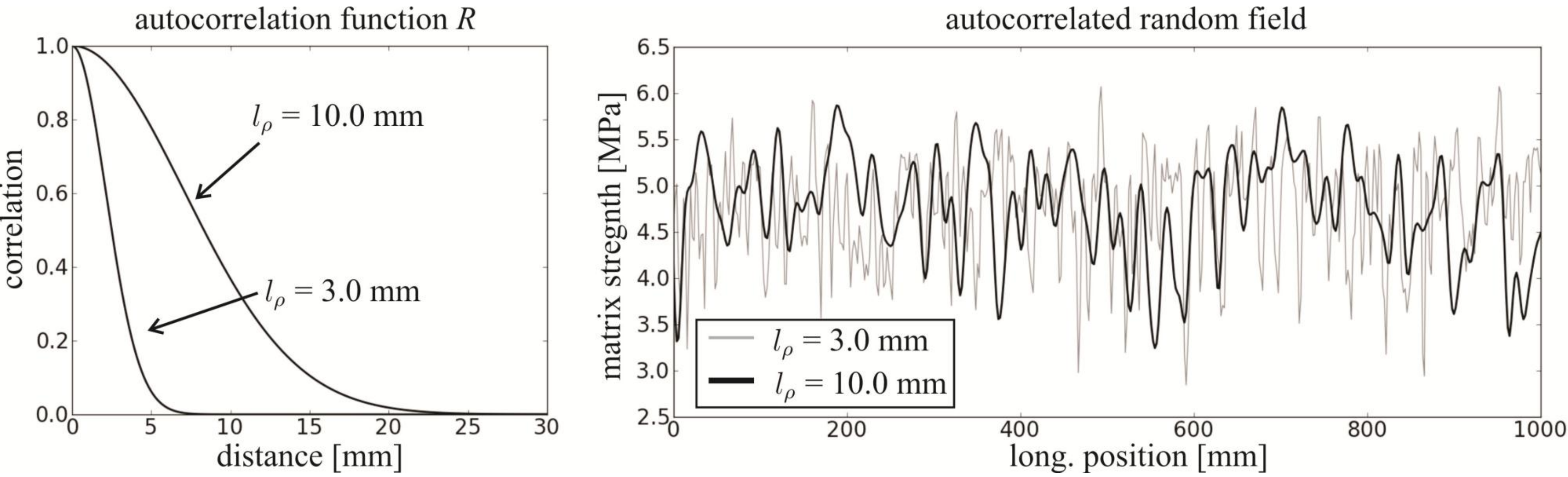
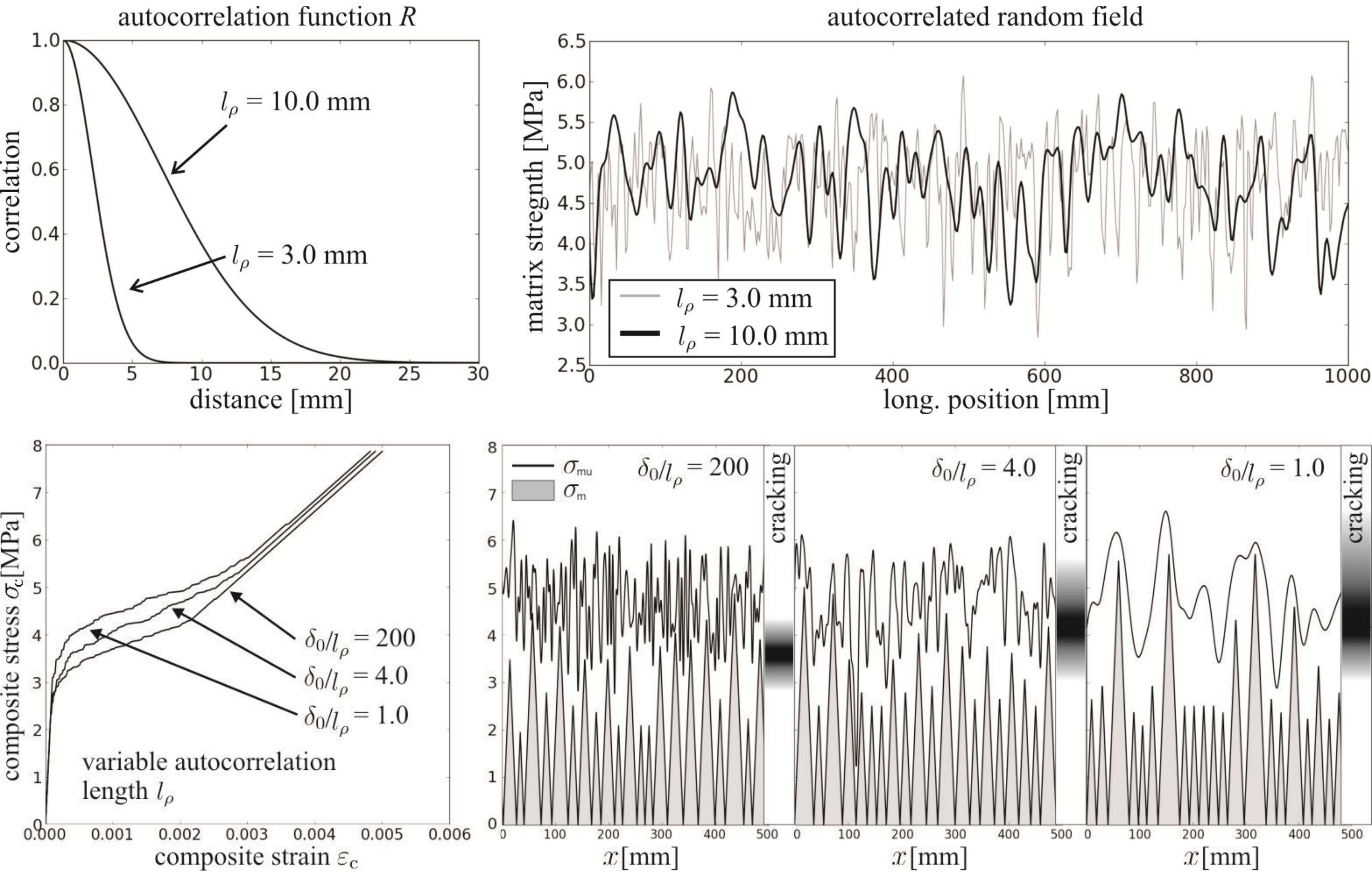




p-study


autocorrelation function R
correlation
distance [mm]
l_ρ = 10.0 mm
l_ρ = 3.0 mm
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
autocorrelated random field
matrix stregnth [MPa]
long. position [mm]
l_ρ = 3.0 mm
l_ρ = 10.0 mm
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
0
200
400
600
800
1000

p-study



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composite stress σ_c [MPa]
composite strain ε_c
δ_0/l_ρ = 200
δ_0/l_ρ = 4.0
δ_0/l_ρ = 1.0
variable autocorrelation
length l_ρ
σ_{mu}
σ_m
cracking
x [mm]

Thank you for your attention