Robust Design Optimization in industrial virtual product development

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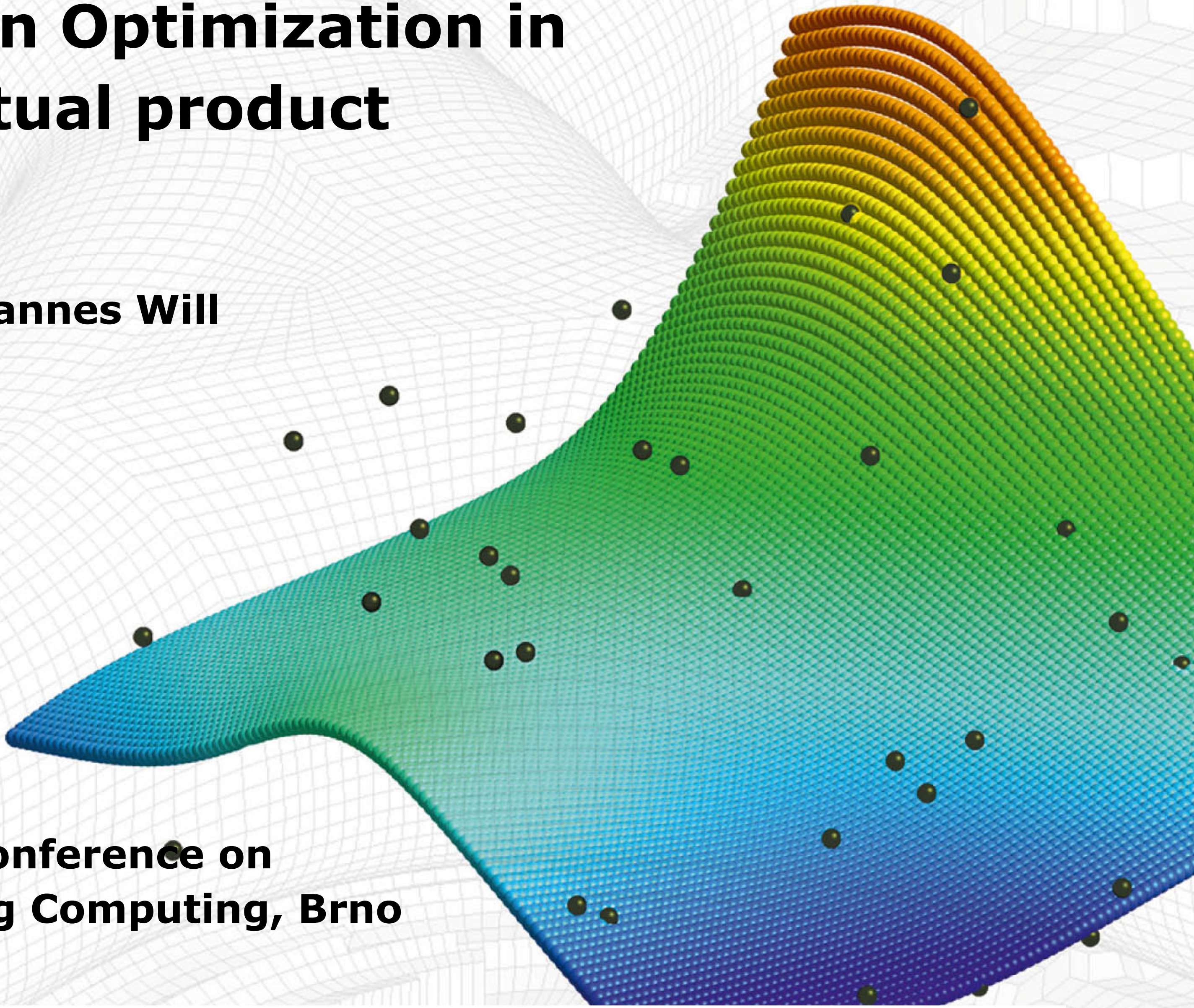


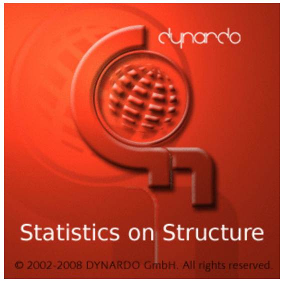
5th International Conference on Reliable Engineering Computing, Brno

Introduction

Dynardo

- Founded: 2001 (Will, Bucher, CADFEM International)
- More than 35 employees, offices at Weimar and Vienna
- Leading technology companies Daimler, Bosch, Eon, Nokia, Siemens, BMW are supported

Software Development



Dynardo is engineering specialist for CAE-based sensitivity analysis, optimization, robustness evaluation and robust design optimization

CAE-Consulting

- Mechanical engineering
- Civil engineering & Geomechanics
- Automotive industry
- Consumer goods industry
- Power generation

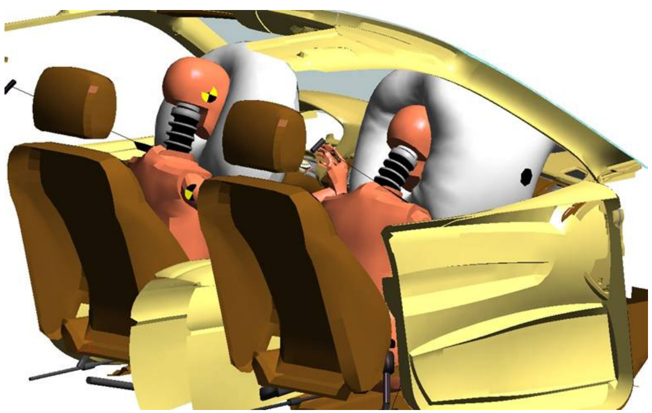
Challenges in Virtual Prototyping

- Virtual prototyping is necessary for cost efficiency
- Test cycles are reduced and placed late in the product development
- CAE-based optimization and CAE-based robustness evaluation becomes more and more important in virtual prototyping

➢ Optimization is introduced into virtual prototyping

➢ Robustness evaluation is the key methodology for safe, reliable and robust products

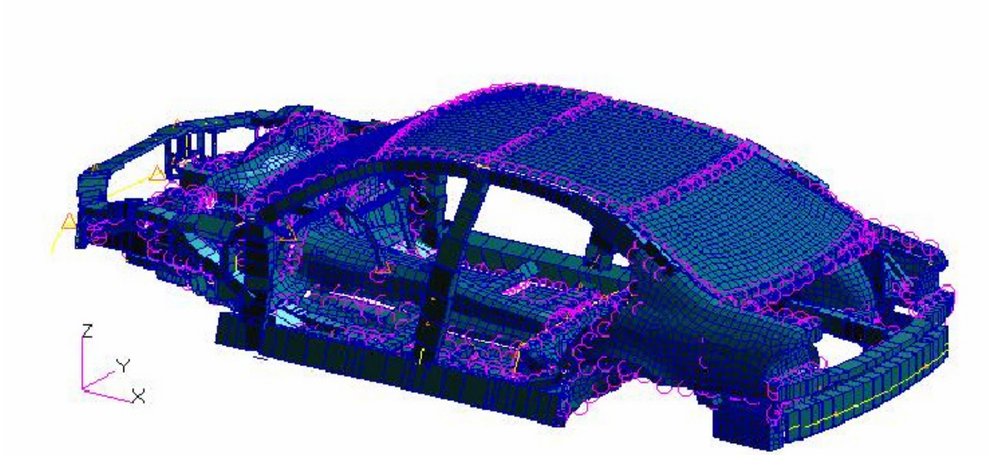
➢ The combination of optimizations and robustness evaluation will lead to robust design optimization strategies

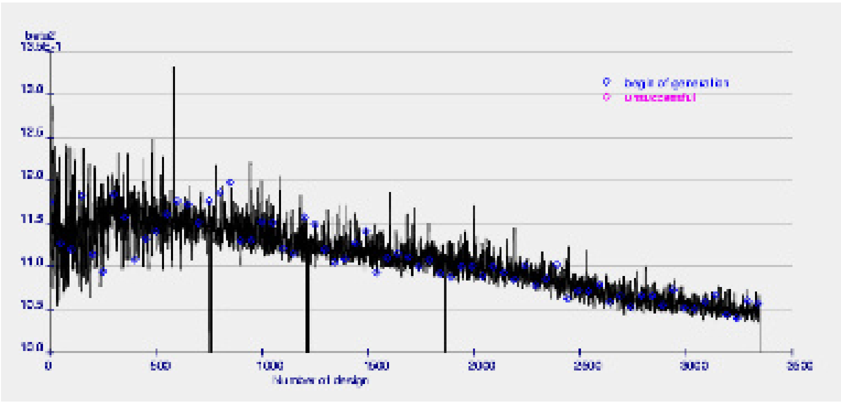


Multidisciplinary Optimization

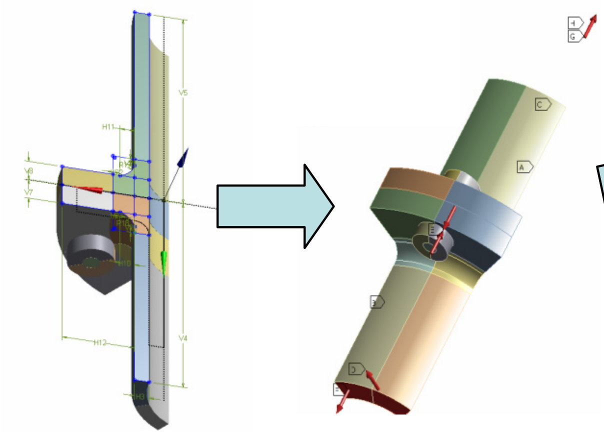
Application of Multidisciplinary Optimization

- Virtual prototyping is an interdisciplinary process
- Multidisciplinary approach requires to run different solvers in parallel and to handle different types of constraints and objectives
- Arbitrary engineering software with complex non-linear analysis have to be connected
- The resulting optimization problem may become very noisy, very sensitive to design changes or ill conditioned for mathematical function analysis (e.g. non-differentiable, non-convex, non-smooth)

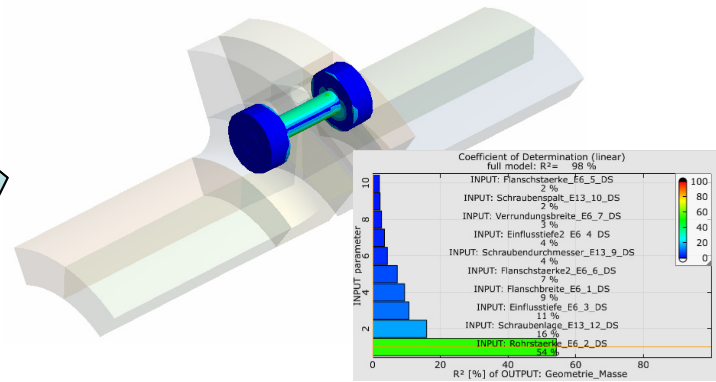




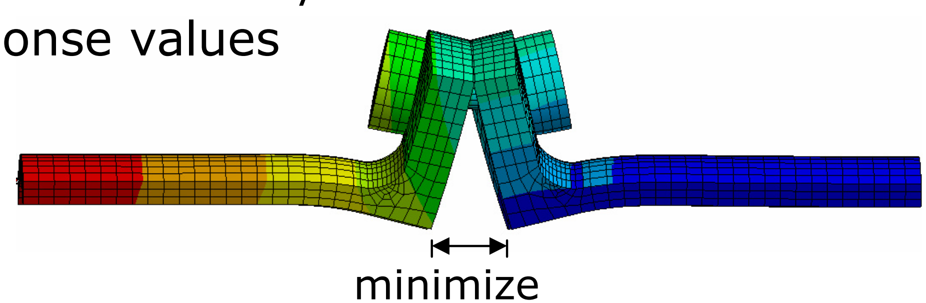
Multidisciplinary Optimization with optiSLang



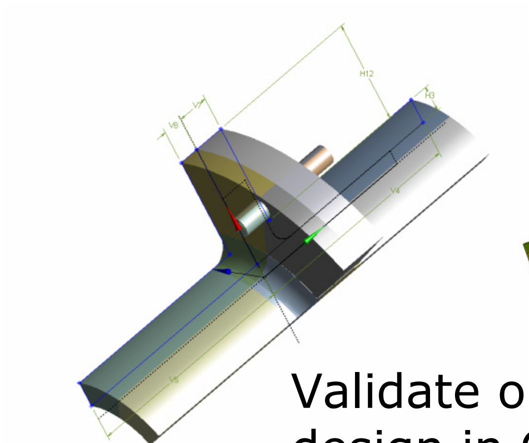
CAD and CAE Parameter definition

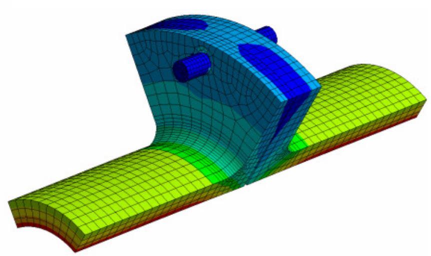


Sensitivity study – identify the most important parameters and check variation/COD of response values



Define optimization goal and optimize

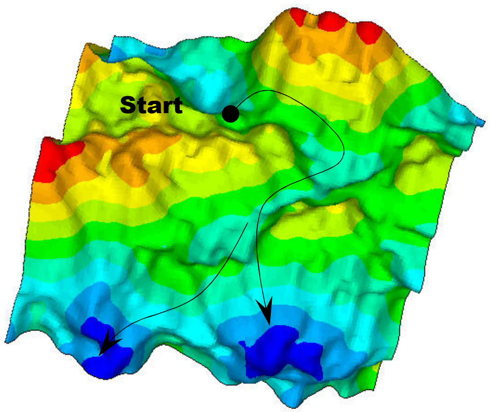




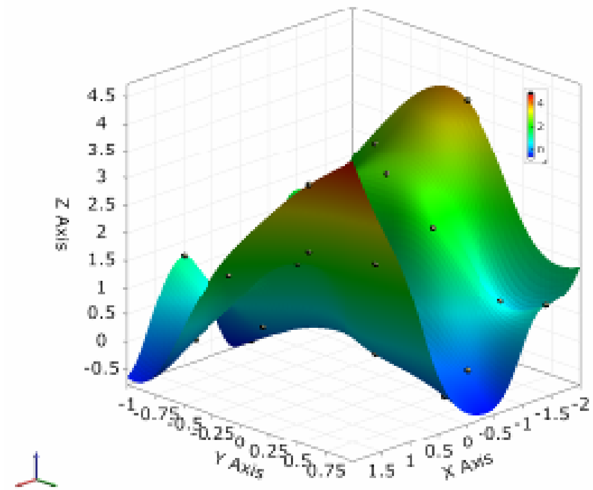
Validate optimized design in CAE and CAD

Optimization Algorithms

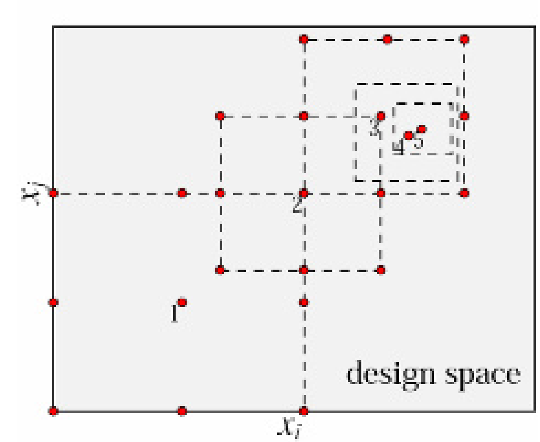
Gradient-based



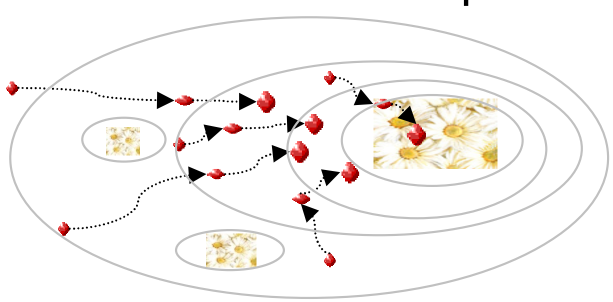
Response surface method

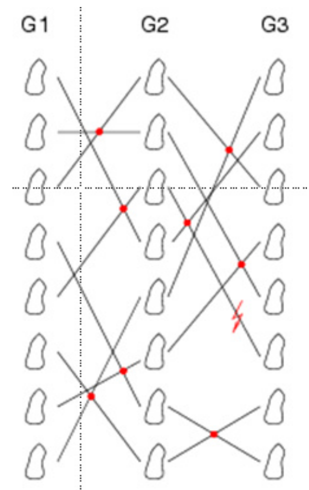


Adaptive RSM

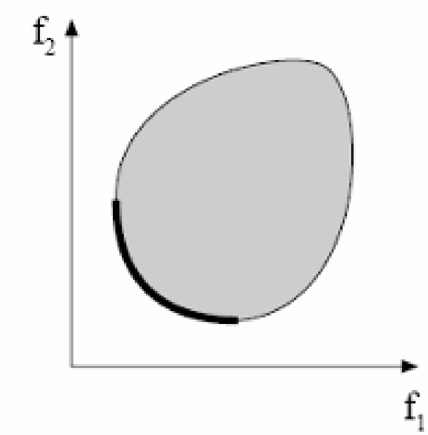


Biological Algorithms:
- Genetic algorithms,
- Evolutionary strategies
- Particle Swarm Optimization



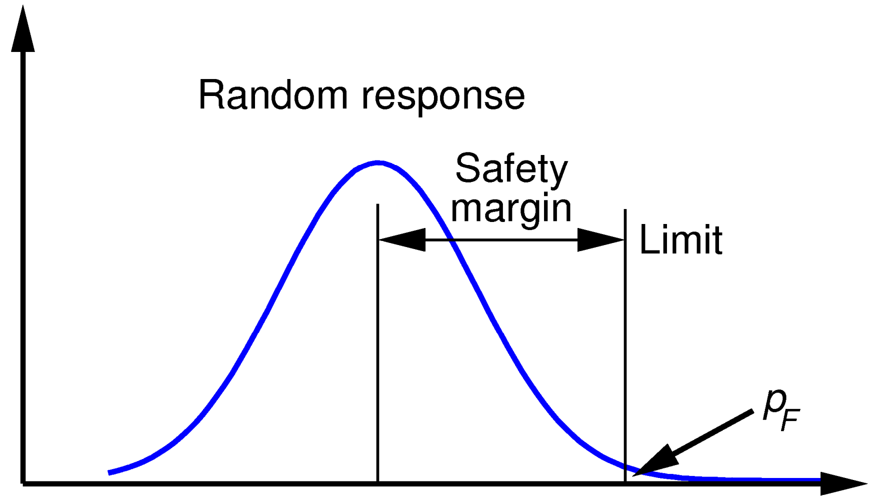


Pareto Optimization



Robustness Analysis

Robustness in terms of constraints



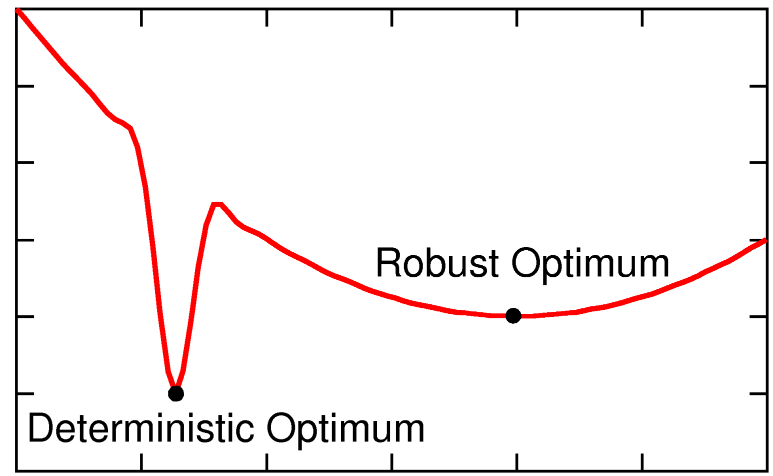
- Safety margin (sigma level) of one or more responses *y*:

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

Robustness in terms of the objective

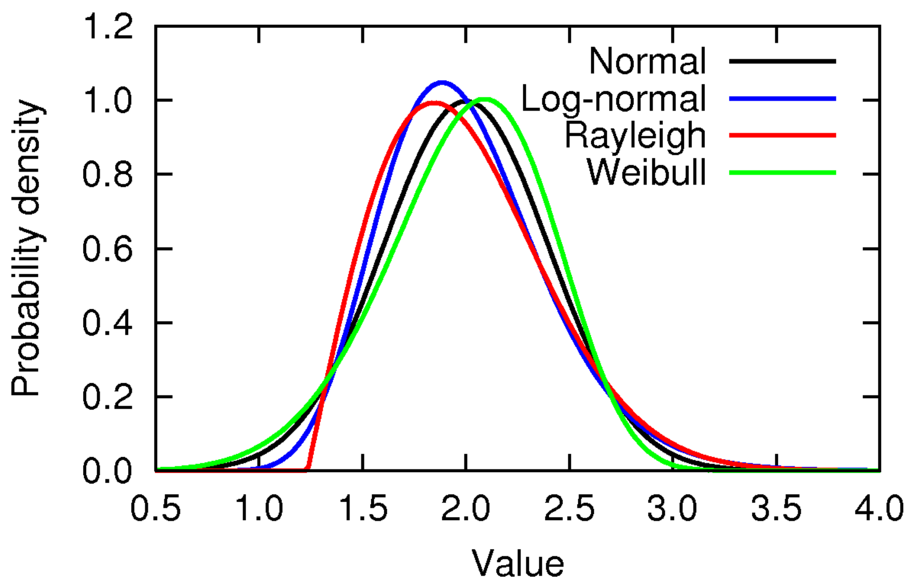


- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

Sigma level vs. failure probability

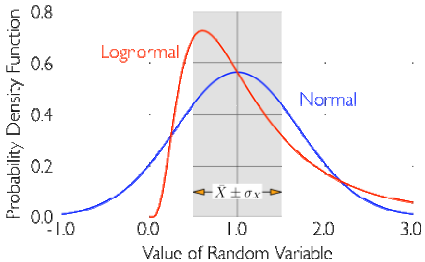
- The sigma level can be used to calculate the probability of exceeding a certain response value
- Since the distribution type is often unknown, this estimate may be very inaccurate for small probabilities
- The sigma level deals with single limit values, whereas the failure probability quantifies the event, that any of several limits is exceeded
- ➢ Reliability analysis should be applied to proof the required safety level

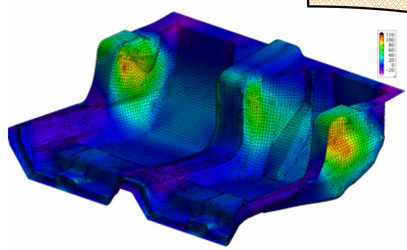


Distribution	Required sigma level (CV=20%)		
	$p_F = 10^{-2}$	$p_F = 10^{-3}$	$p_F = 10^{-6}$
Normal	2.32	3.09	4.75
Log-normal	2.77	4.04	7.57
Rayleigh	2.72	3.76	6.11
Weibull	2.03	2.54	3.49

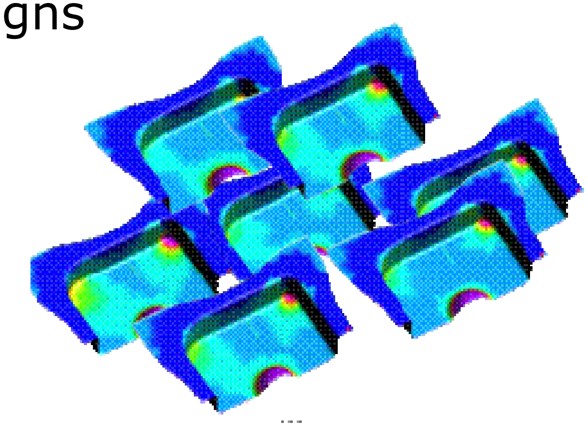
Variance based robustness analysis

1) Define the robustness space using scatter range, distribution and correlation

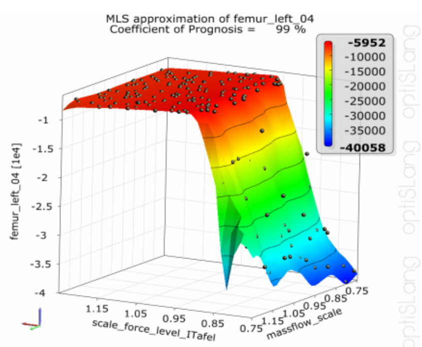




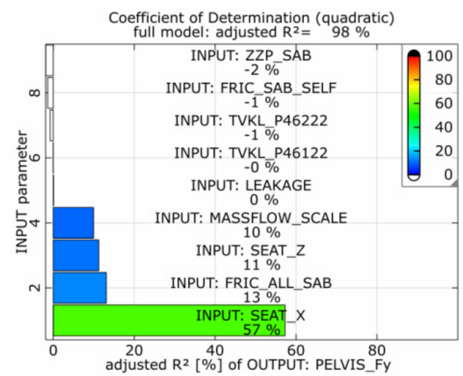
2) Scan the robustness space by producing and evaluating *n* designs



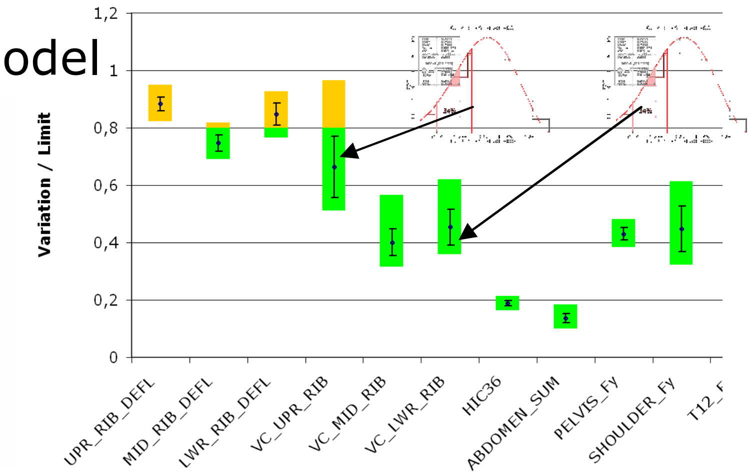
5) Identify the most important scattering variables



4) Check the explainability of the model

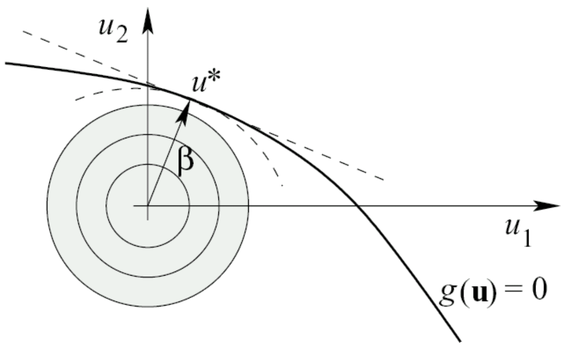


3) Check the variation

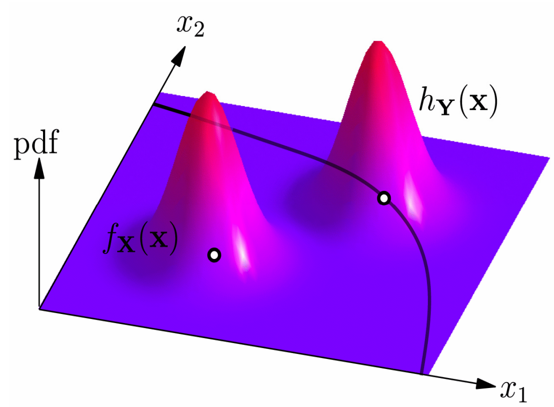


Reliability based robustness analysis

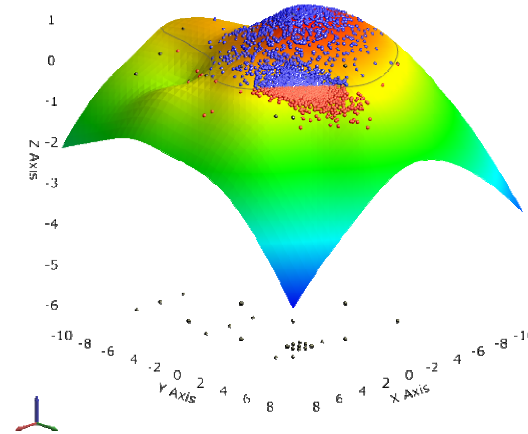
First Order Reliability algorithm (FORM)



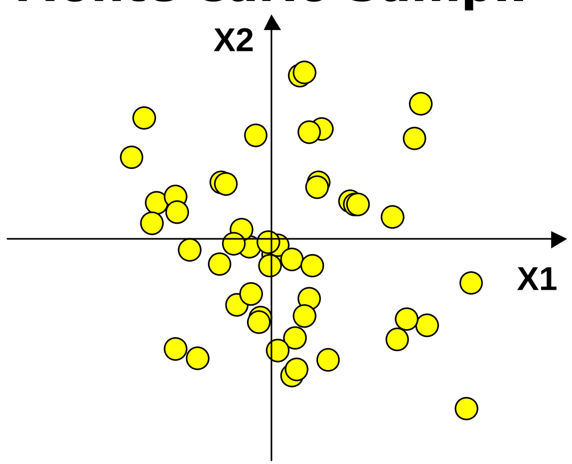
Importance Sampling



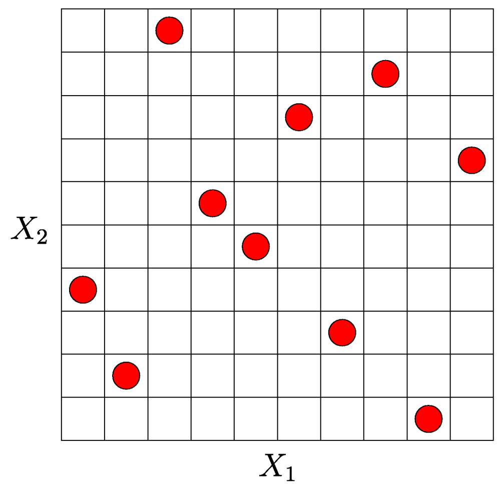
Adaptive Response Surface Method



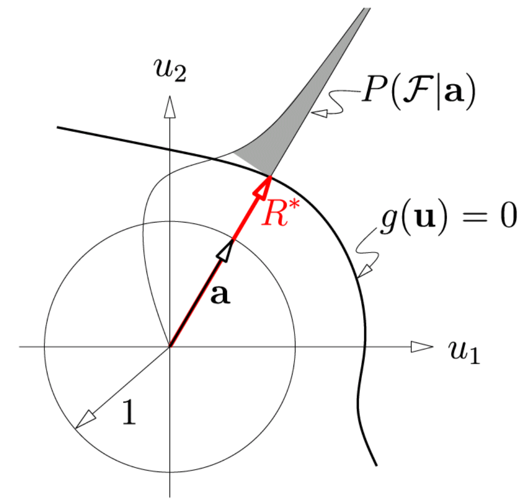
Monte Carlo Sampl.



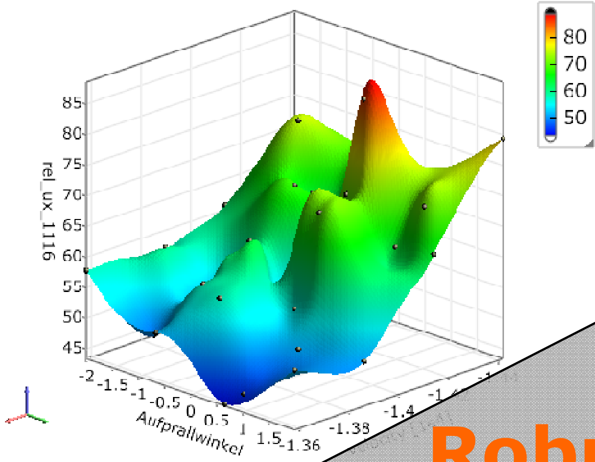
Latin Hypercube Sampl.

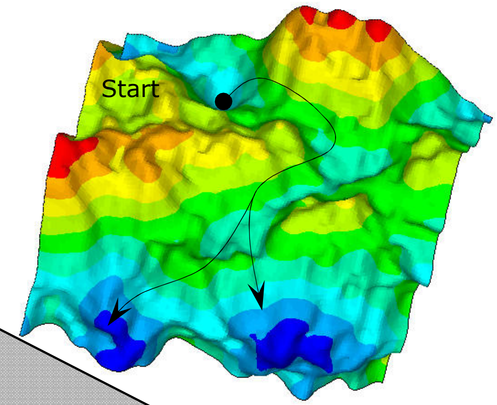


Directional Sampl.



Robust Design Optimization





Robust Design Optimization

Robust Design

Variance based Robustness Evaluation

Probability based Robustness Evaluation, (Reliability analysis)

Optimization

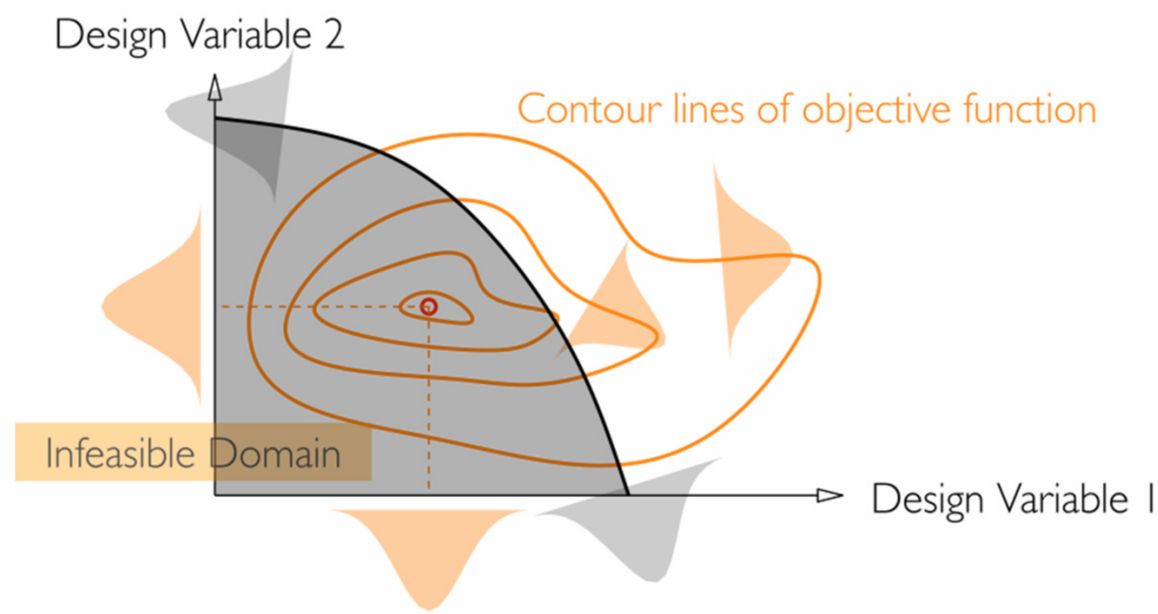
Sensitivity Study

Single & Multi objective (Pareto) optimization

CAE process (FEM, CFD, MBD, Excel, Matlab, etc.)

Robust Design Optimization

- Robust Design Optimization (RDO) optimizes the design performance with consideration of scatter of design (optimization) variables as well as other tolerances or uncertainties
- As a consequence of uncertainties the location of the optima as well as the contour lines of constraints scatter



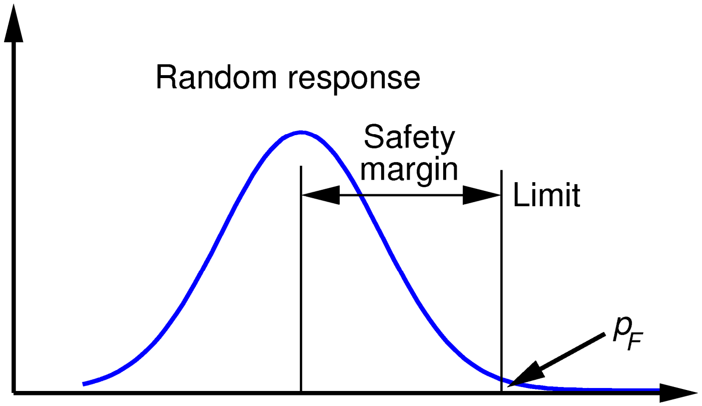
- To proof Robust Designs, safety distances are quantified with variance or probability measures using stochastic analysis

Methods for Robust Design Optimization

Variance-based RDO

- Safety margins of all critical responses are larger than a specified sigma level (e.g. Design for Six Sigma)

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$



Reliability-based RDO

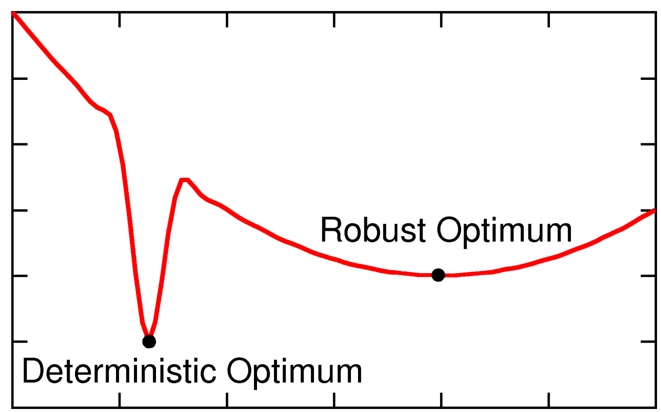
- Failure probability with respect to given limit states is smaller as required value

$$p_F \leq p_F^{target}$$

Taguchi-based RDO

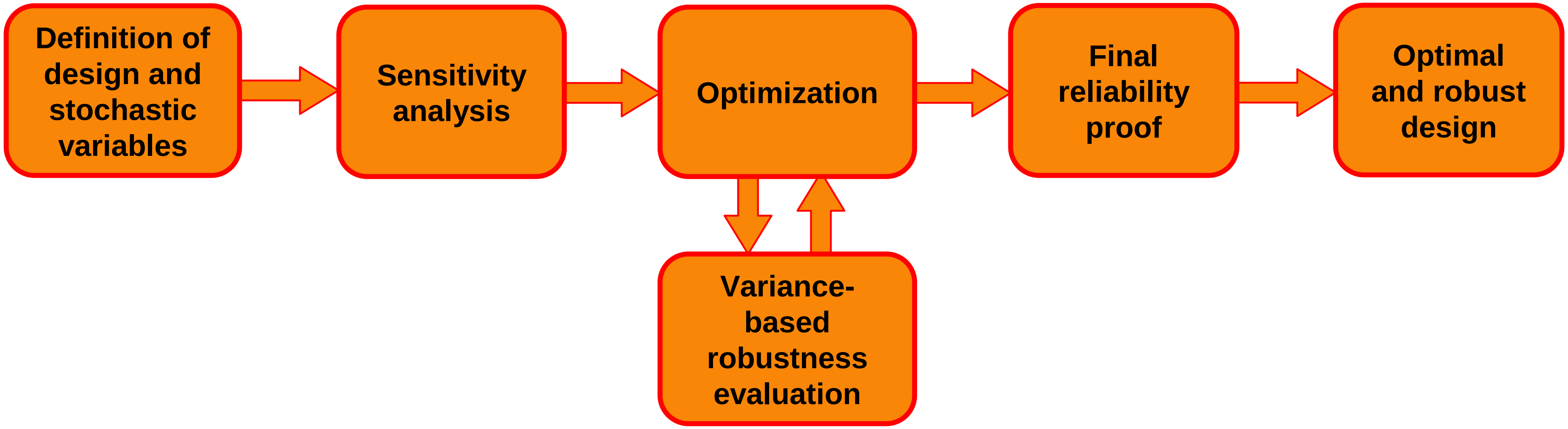
- Taguchi loss functions
- Modified objective function

$$f(y) = \frac{k}{N} \sum y_i^2 = k(\bar{y}^2 + \sigma_y^2)$$



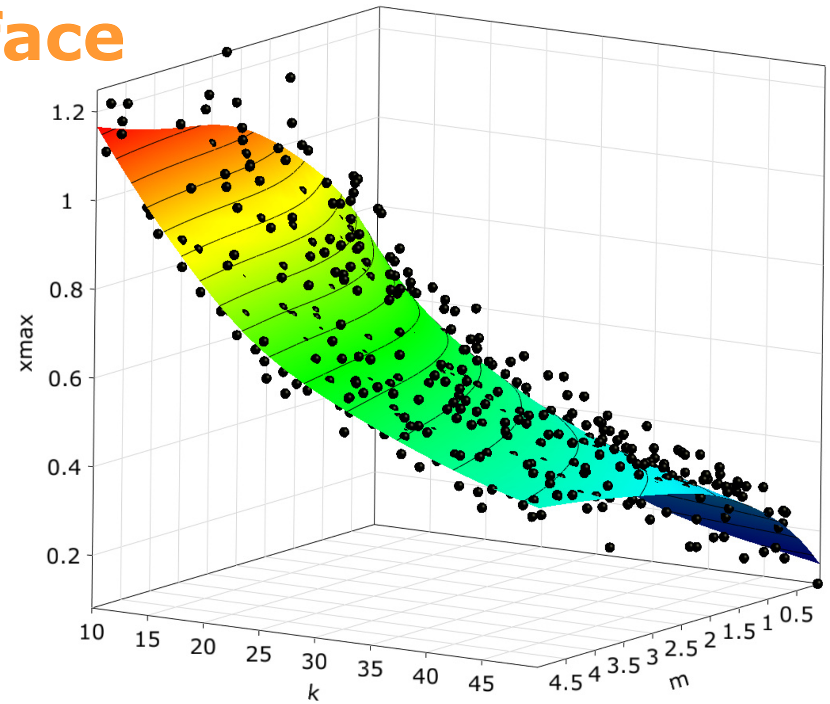
Simultaneous Robust Design Optimization

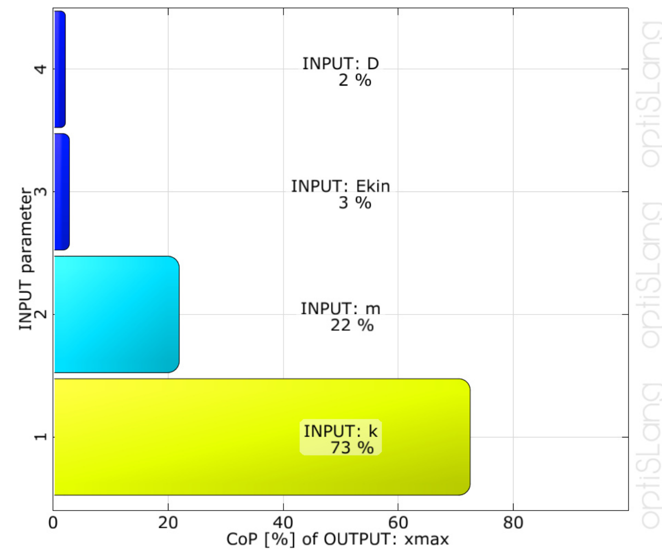
- Fully coupled optimization and robustness/reliability analysis
- For each optimization (nominal) design the robustness/reliability analysis is performed
- Applicable to variance-, reliability- and Taguchi-based RDO
- ➢ Our efficient implementation uses small sample variance-based robustness measures during the optimization and a final (more accurate) reliability proof
- ➢ But still the procedure is often not applicable to complex CAE models



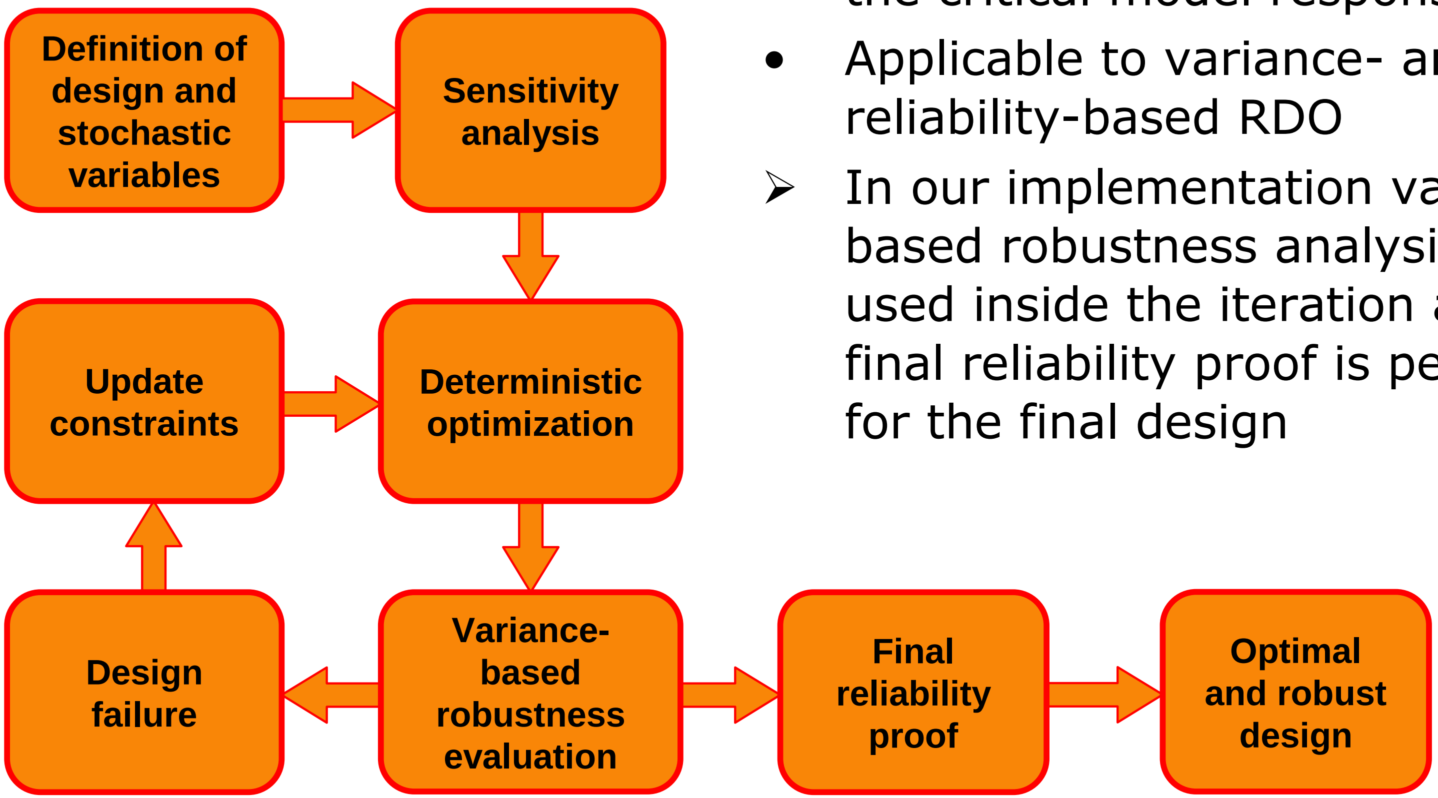
RDO on global response surface

- Approximation of model responses in mixed optimization/stochastic space
- Simultaneous RDO is performed on a global response surface
- Applicable to variance-, reliability- and Taguchi-based RDO
- Approximation quality significantly influences RDO results
- ➢ Final robustness/reliability proof is required
- Pure stochastic variables have small influence compared to design variables
- ➢ Important local effects in the stochastic space may be not represented





Iterative Robust Design Optimization



- Decoupled optimization and robustness/reliability analysis
- For each optimization run the safety factors are adjusted for the critical model responses
- Applicable to variance- and reliability-based RDO
- ➢ In our implementation variance-based robustness analysis is used inside the iteration and a final reliability proof is performed for the final design

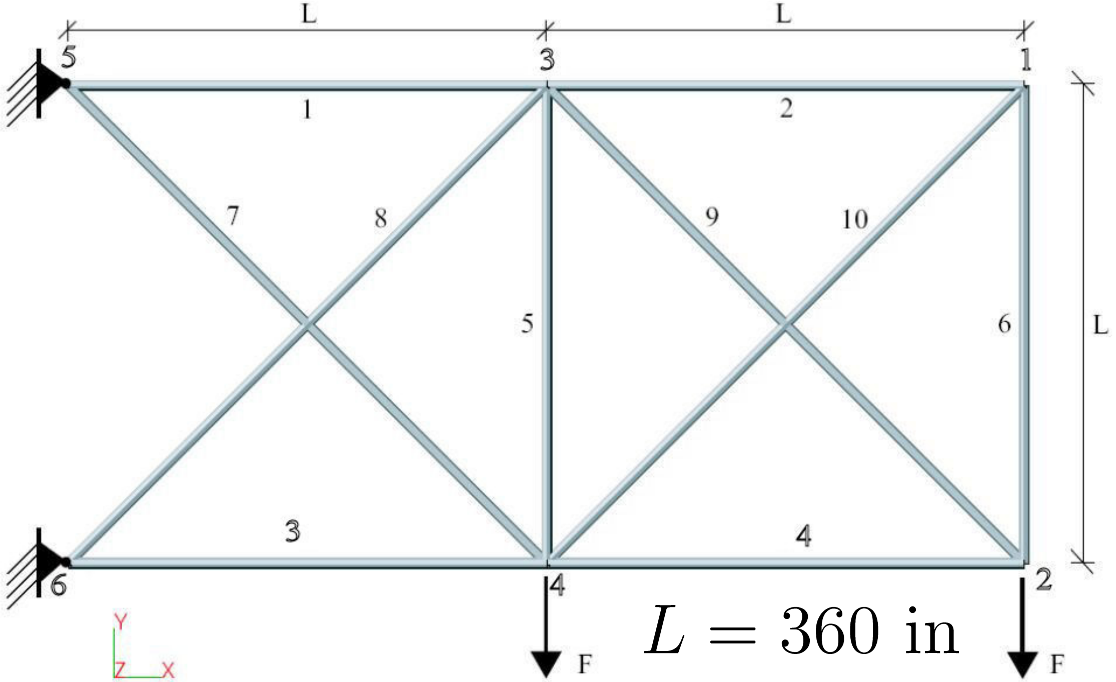
Applications

Example: Truss structure

- Minimization of the total mass (initial mass is 4196.5 lbs)
- Responses from linear finite element analysis are mass, displacements at loading points and stresses for each element
- Probability that max. stress is larger as 30000 psi should be below 10^{-6}
- Cross section areas of the trusses as design variables $0.1 \leq a_i \leq 20$

Random variables:

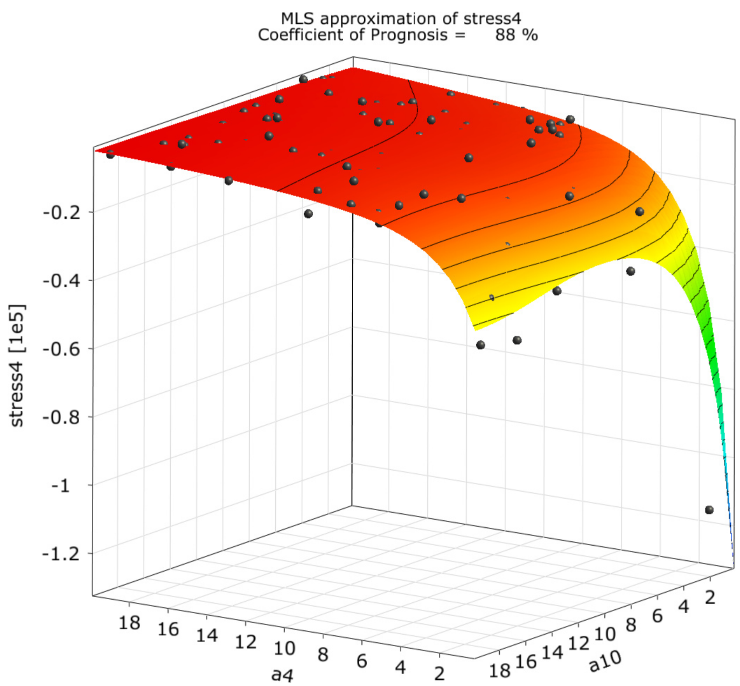
- Cross section areas CV=5%, normal
- Loads, CV=10%, normal $\bar{F} = 100000 \text{ lb}_f$
- Young's modulus, CV=5%, log-normal $\bar{E} = 10^7 \text{ psi}$

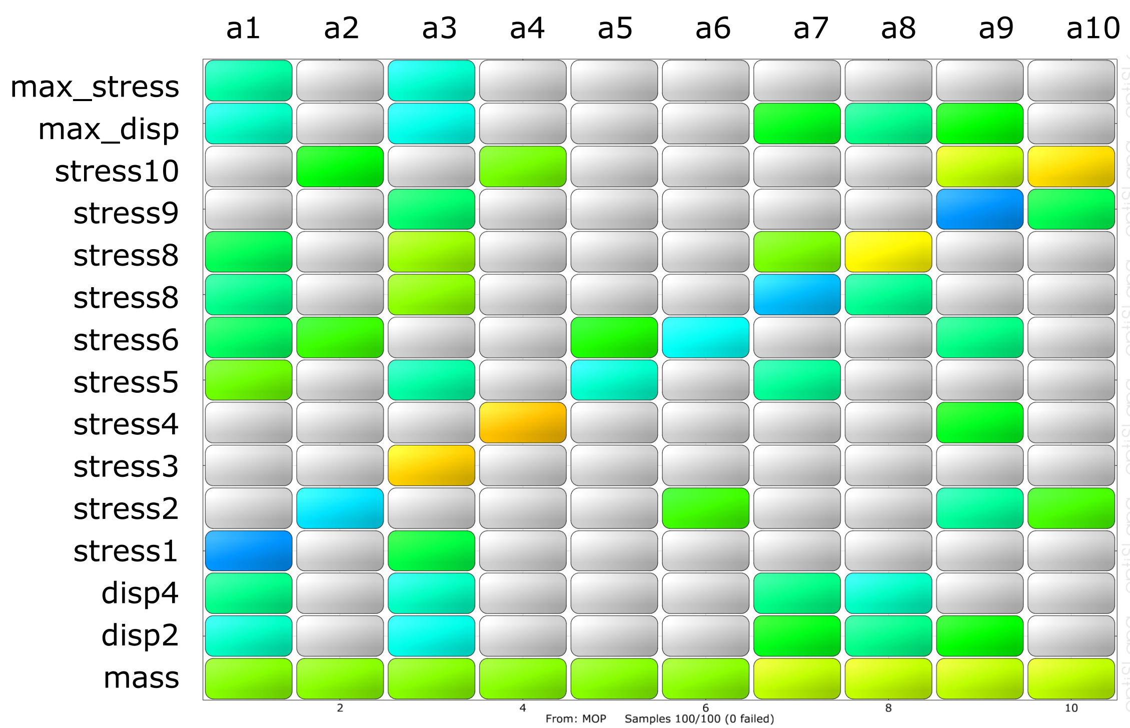


Example: Truss structure

Sensitivity analysis in the optimization space

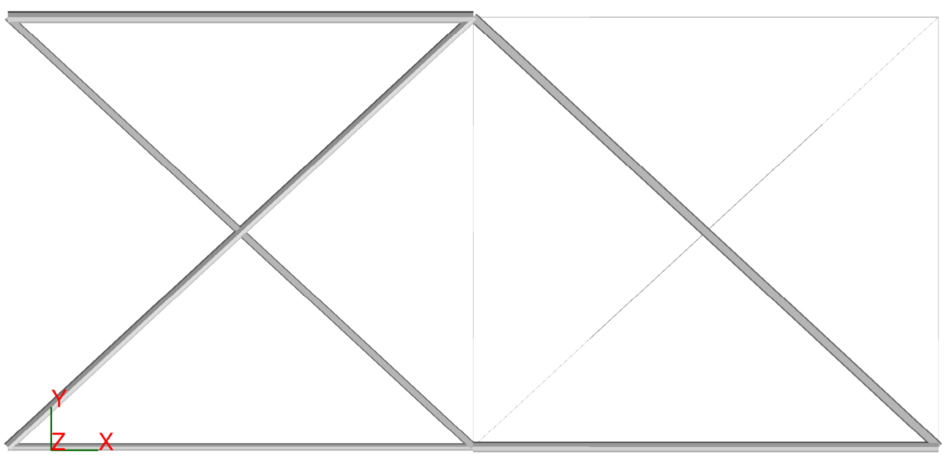
- Sensitivity analysis in the design space carries out, that each cross section area is most important for the corresponding stress
- ➢ Reduction of the number of design variables seems not possible

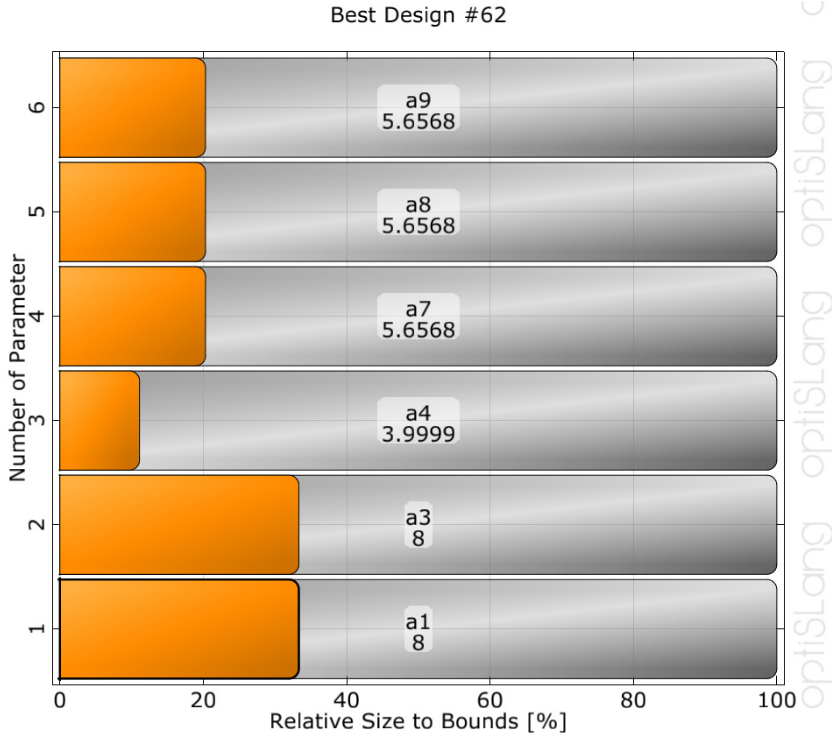




Example: Truss structure
Deterministic optimization

- Global safety factor for the stresses is taken as 1.2
  - ➢ Maximum stress of 25000 psi as constraint
- Gradient-based optimization
  - ➢ Trusses 2, 5, 5, 6 and 10 are removed from the model
  - ➢ Mass of reduced structure is 1584 lbs





Example: Truss structure

Iterative Robust Design Optimization - Varianced-based analysis

- In order to fulfill the failure probability a sigma level of 6.0 is assumed
- After the first deterministic optimization step a robustness analysis is performed which indicates a significant smaller sigma level

➢ Update of the constraint condition by assuming constant coefficient of variation:

$$constraint_{step2} + 6 \cdot CV_{step1} \cdot constraint_{step2} \leq 30000$$

$$\rightarrow \quad constraint_{step2} = 30000/(1 + 6 \cdot CV_{step1})$$

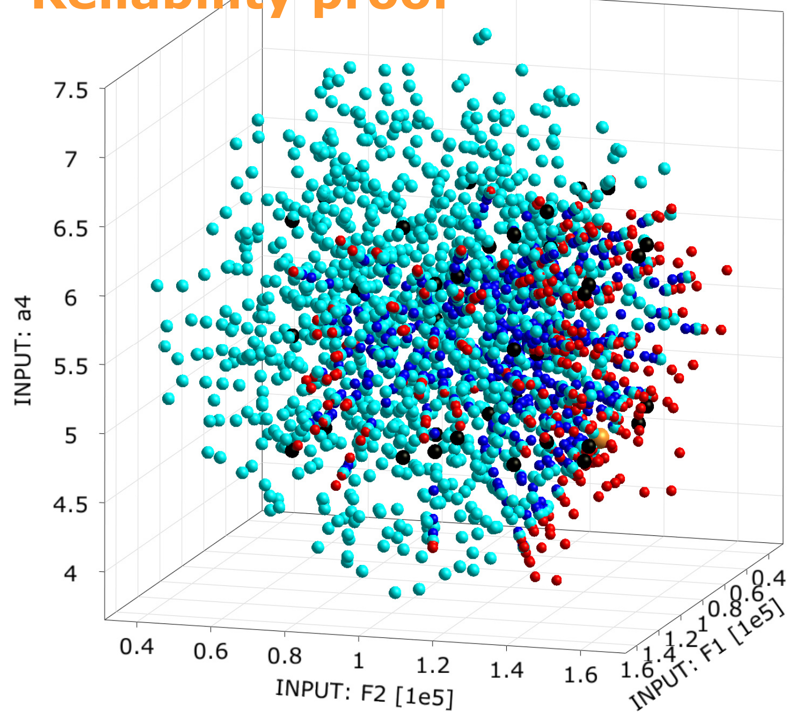
➢ The resulting structure almost fulfills the required sigma level

	Optimization			Robustness	
	Constraint	Mass	No. Designs	Sigma level	No. Samples
Step 1	25000	1584	68	**1.73**	100
Step 2	18000	**2200**	35	**5.80**	100

Example: Truss structure

Iterative Robust Design Optimization - Reliability proof

- ARSM and Directional Sampling on MOP using robustness samples give similar results as reference solution
- FORM with gradient-based search fails
- ARSM is very efficient and can handle multiple failure regions and strongly nonlinear behavior
- Final design fulfills reliability requirements

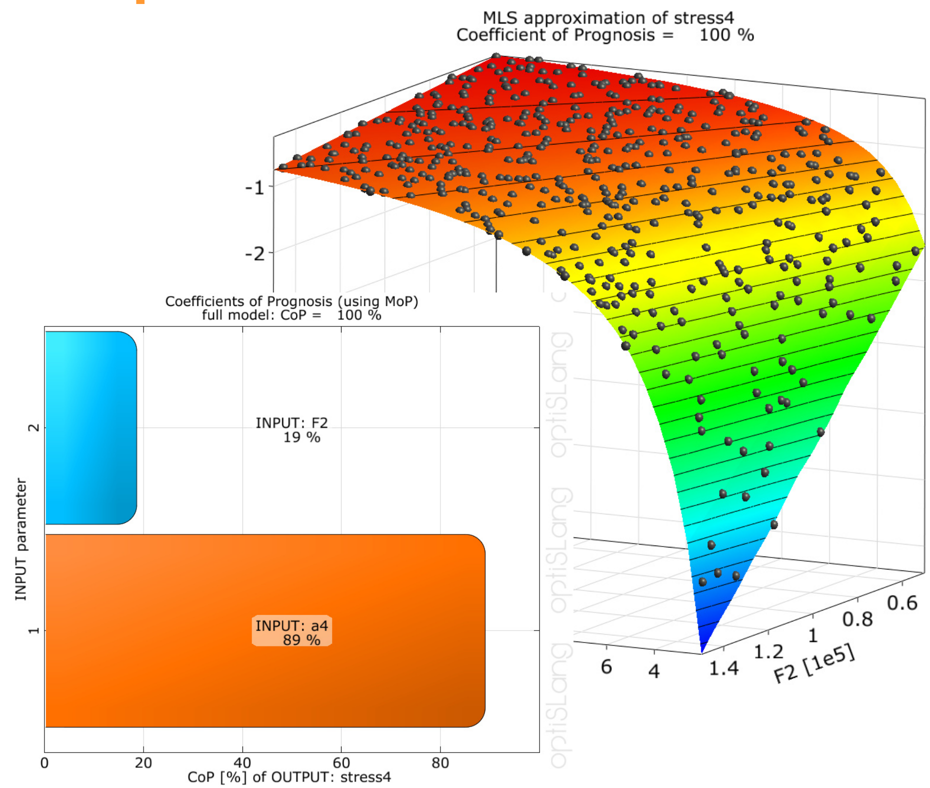


	No. Samples	Failure probability	Reliability index
Directional sampling (reference)	3674	$3.2*10^{-7}$	4.98
FORM	225	-	-
MOP+DS	(100)	$5.1*10^{-7}$	4.89
ARSM+DS	101	$\mathbf{5.8*10^{-7}}$	4.86

Example: Truss structure

Robust Design Optimization on global response surface

- Global approximation of each truss stress by the Metamodel of Optimal Prognosis (MOP)
- For the generation of the support points the mixed variables are varied within the design range bounds
- Pure stochastic variables are varied within +/- 5σ
- ➢ Excellent approximation quality
- ➢ Similar final design as with iterative RDO procedure



	No. Samples	
RDO on MOP	500 supports	Mass = **2211**
Robustness proof	100	Sigma level = **6.13**
Reliability proof ARSM+DS	84	P_f = $\mathbf{1.2 \cdot 10^{-7}}$

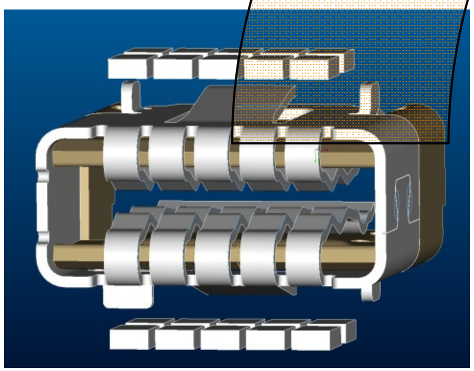
Iterative RDO application - Cable connector

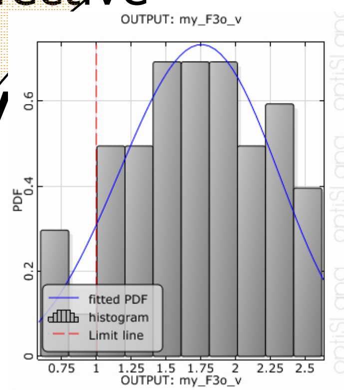
2) The "Six Sigma design" was checked in the space of 36 scattering variables using **optiSLang robustness evaluation**. Some criteria showed high failure probabilities!

1) From the 31 optimization parameter the most effective one were selected with **optiSLang sensitivity analysis**.

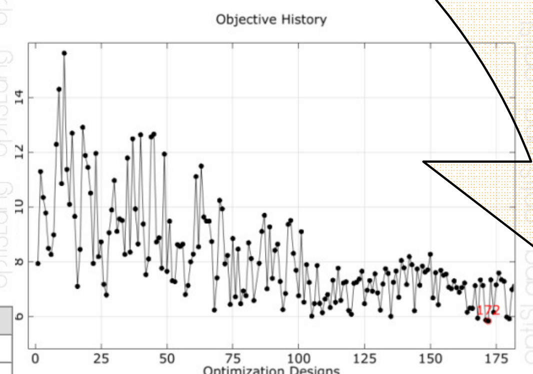
3) From optiSLang Robustness evaluation new safety margins are derived.

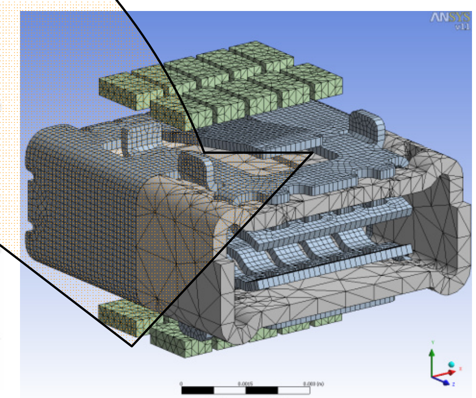
4) Three steps of optimization using **optiSLang ARSM and EA** optimizer improve the design to an optiSLang Six sigma design.





Statistic data			
Min:	0.5997	Max:	2.622
Mean:	1.728	Sigma:	0.525
CV:	0.3038		
Skewness:	-0.2233	Kurtosis:	2.251
Fitted PDF: Weibull (2p)			
Mean:	1.728	Sigma:	0.525
Limit x = 1			
P_rel =	0.08	P_fit =	0.08832
Probability P(X<x) = 0.001			
x_rel =	0.5997	x_fit =	0.2905





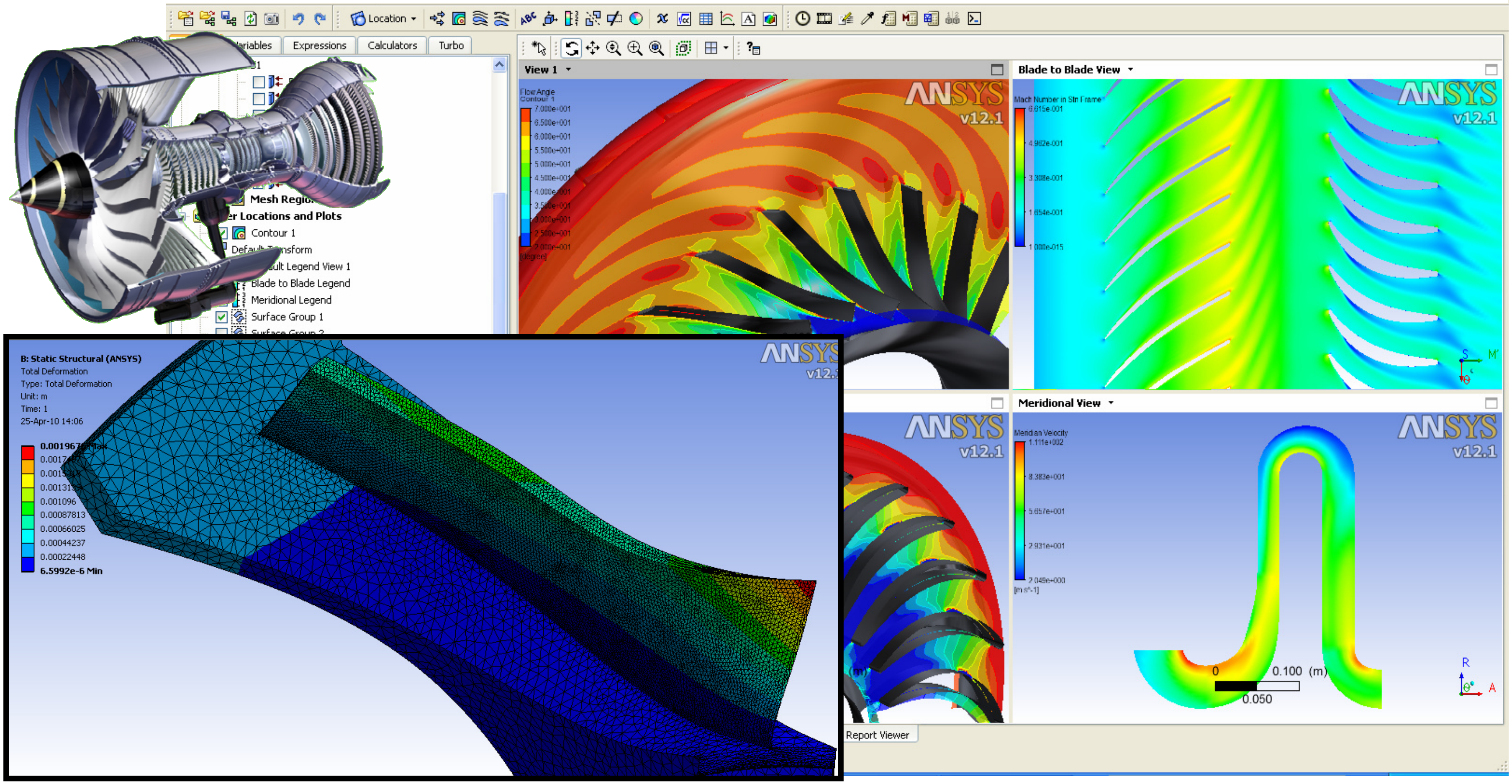
5) Reliability proof using **ARSM** to estimate the failure probability did **proof six sigma quality.**

Start: RDO package using 5 Parameter and global RSM, but then the customer asked: How save is the design?

by courtesy of Tyco Electronics

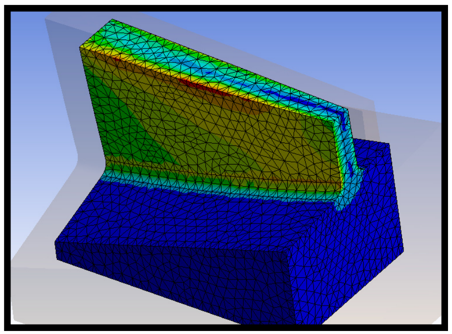
Iterative RDO application - Centrifugal compressor

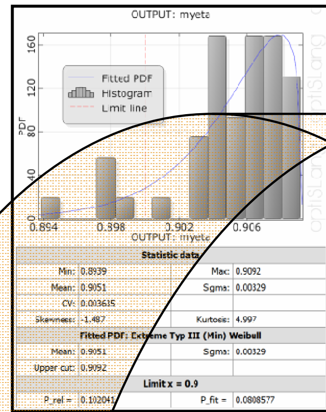
- Geometry definition using ANSYS BladeModeler
- Fluid Structure Interaction using parametric fluid simulation within ANSYS CFX and parametric mechanical setup within ANSYS Workbench



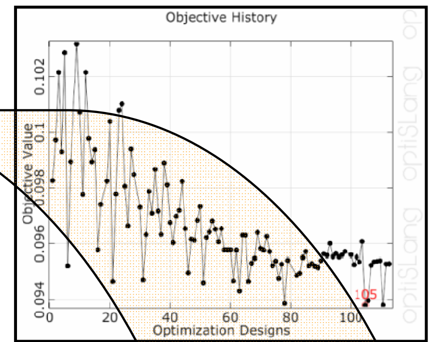
Iterative RDO application - Centrifugal compressor

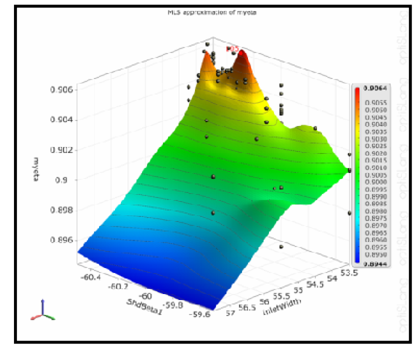
- RDO with respect to 21 design parameters and 20 random geometry parameters, including manufacturing tolerances
- Robust Design was reached after 400+250=650 design evaluations



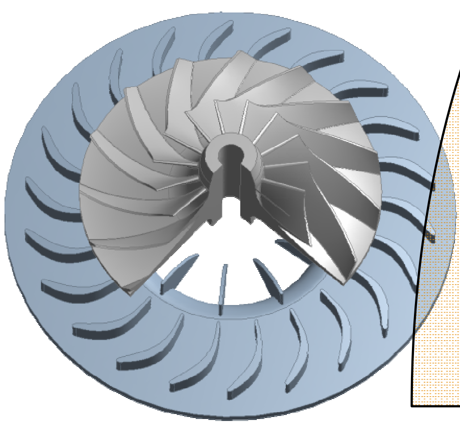


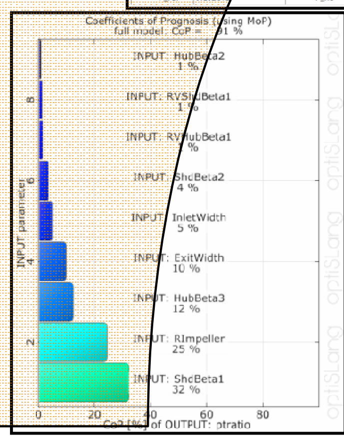
Robustness evaluation

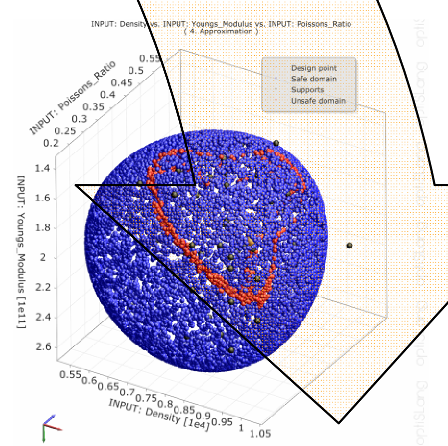


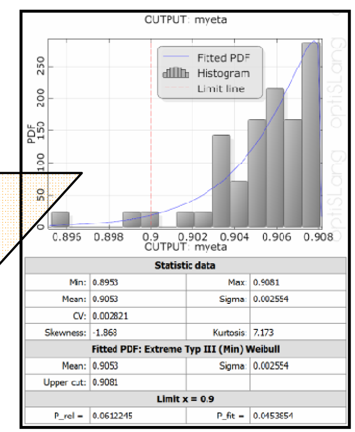


RDO optimization









Sensitivity + first optimization step

Reliability proof

Summary

- Highly optimized structures tend to loose robustness
- Variance-based robustness analysis can estimate small sigma level
- Reliability analysis is necessary to proof small failure probabilities
- Fully coupled optimization and reliability analysis is not applicable to real world problems
- Iterative optimization/variance-based analysis with final reliability proof is applied by Dynardo to industrial problems since several years
- Global response surface approximation may lead to a robust design for sufficient number of support points, but final reliability proof should be performed in any case