On the Effect of **Material** Spatial **Randomness** in Lattice **Simulation** of **Concrete Fracture**

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Deterministic Model description

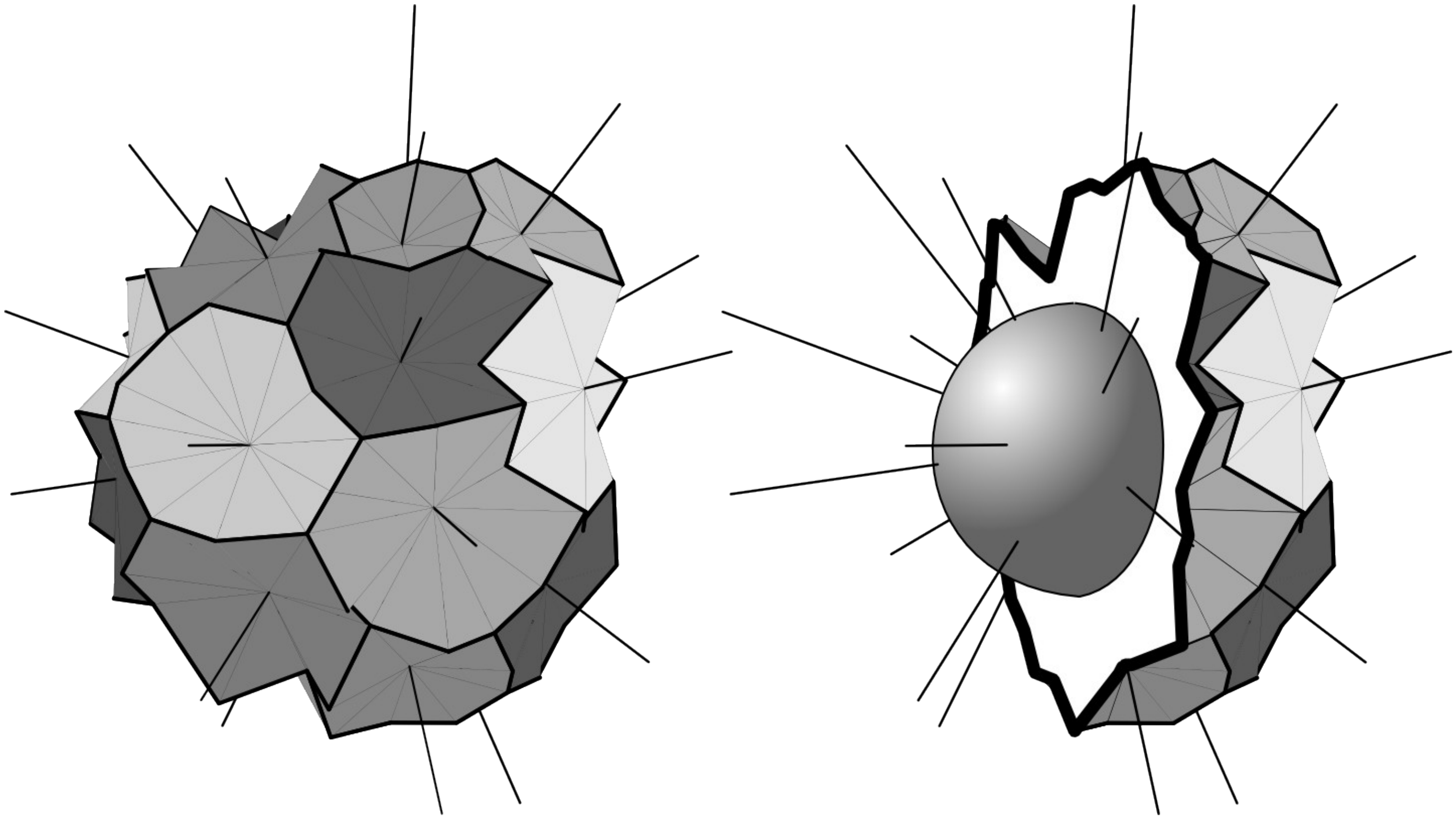
- developed by Gianluca Cusatis

  Cusatis, G., Bažant Z.P. and Cedolin, L., 2003
  Cusatis, G., Bažant Z.P. and Cedolin, L., 2006
  Cusatis, G., Cedolin, L., 2007

- discrete model
- static (time independent)
- geometrically linear
- fixed underlying lattice that determines contacts between cells
- damage model – ignoring frictional slips

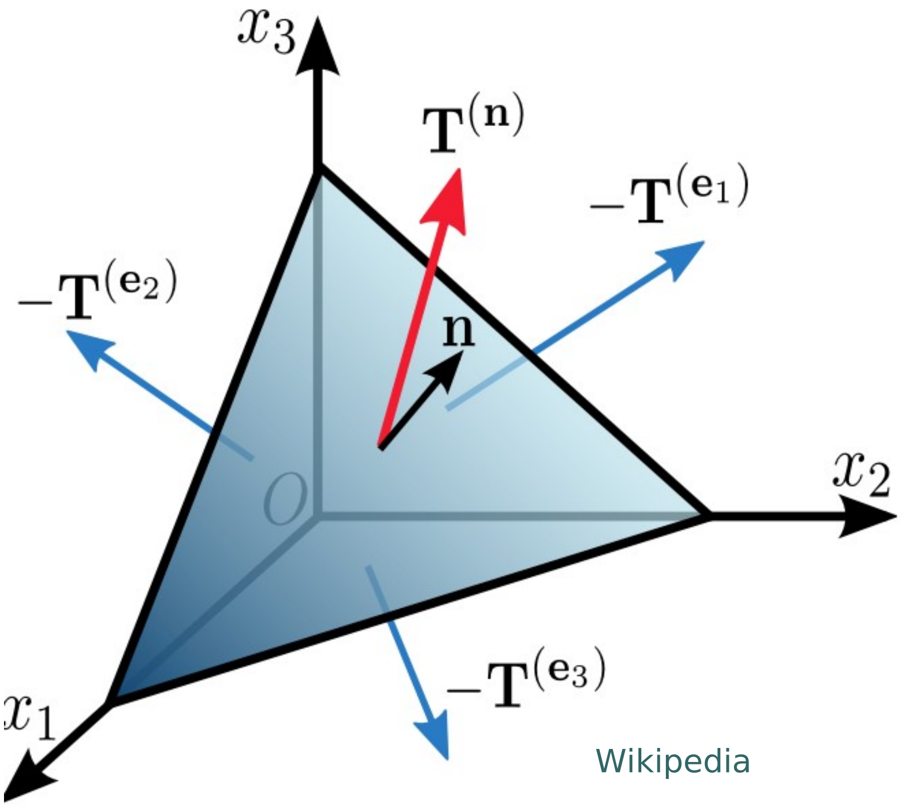
Discretization

- pseudo-random placing of concrete grains with radii respecting given sieve curve
- tessellation into rigid cells



Contact: strain components

$$\varepsilon_{\mathrm{N}} = \frac{\mathbf{n}^{\mathrm{T}}[\![\mathbf{u}_{\mathrm{C}}]\!]}{L}; \quad \varepsilon_{\mathrm{L}} = \frac{\mathbf{l}^{\mathrm{T}}[\![\mathbf{u}_{\mathrm{C}}]\!]}{L}; \quad \varepsilon_{\mathrm{M}} = \frac{\mathbf{m}^{\mathrm{T}}[\![\mathbf{u}_{\mathrm{C}}]\!]}{L}$$



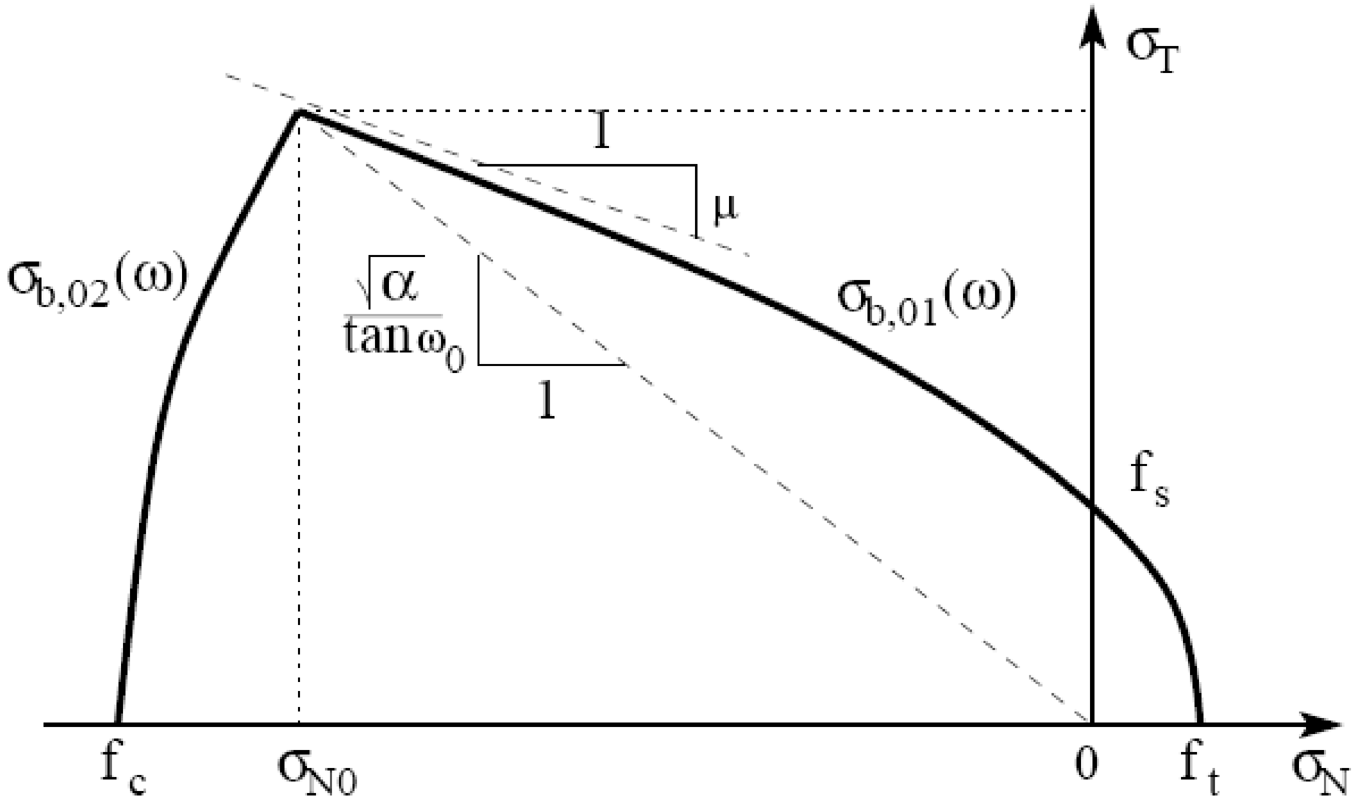
$$\varepsilon_{\mathrm{T}} = \sqrt{\varepsilon_{\mathrm{L}}^2 + \varepsilon_{\mathrm{M}}^2}$$

$$\varepsilon = \sqrt{\varepsilon_{\mathrm{N}}^2 + \alpha\varepsilon_{\mathrm{T}}^2}$$

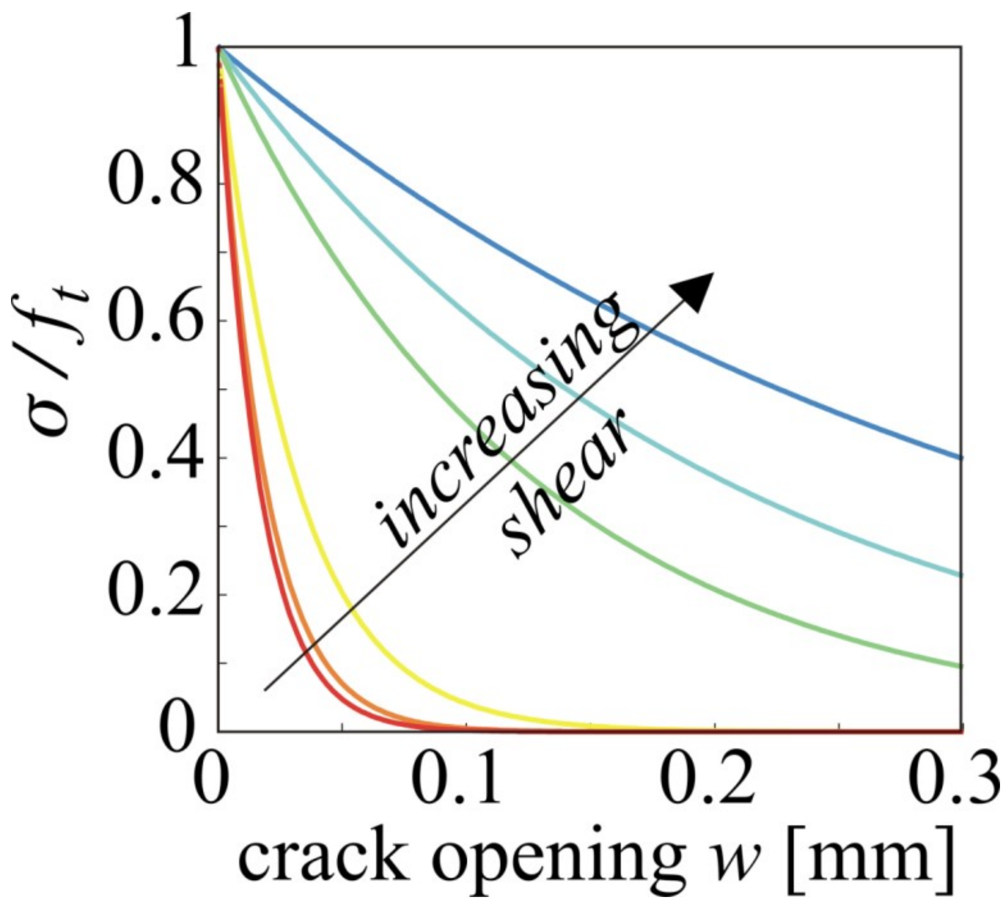
$$\tan\omega = \frac{\varepsilon_{\mathrm{N}}}{\sqrt{\alpha}\varepsilon_{\mathrm{T}}}$$

Elastic stress boundary

- formulated in normal and effective shear stress
- dependent on direction of straining



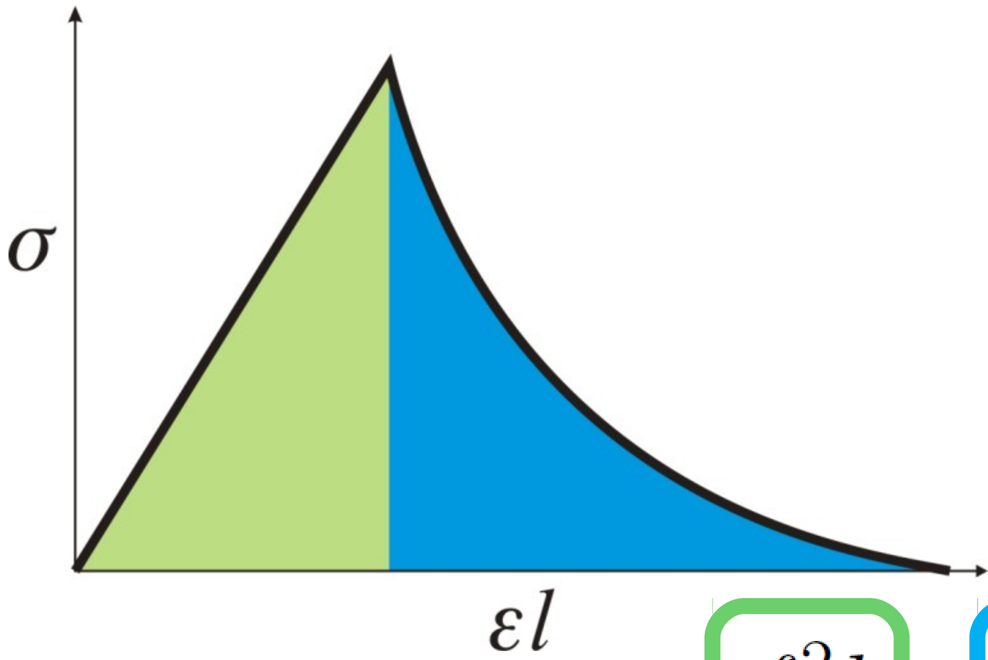
Softening curve



evolution of elastic limit is exponential function developing with increasing maximal reached fracturing strain

$$\sigma_{\mathrm{b}}(\varepsilon, \omega) = \sigma_0(\omega) \exp \left\{ \frac{K(\omega)}{\sigma_0(\omega)} \left\langle \chi(\varepsilon, \omega) - \frac{\sigma_0(\omega)}{E} \right\rangle \right\}$$

Correct energy dissipation



area under contact constitutive law represents energy dissipated between two grains

$$\frac{f_t^2 l}{2E} + \frac{f_t^2 l}{K_t} = G_f^t$$

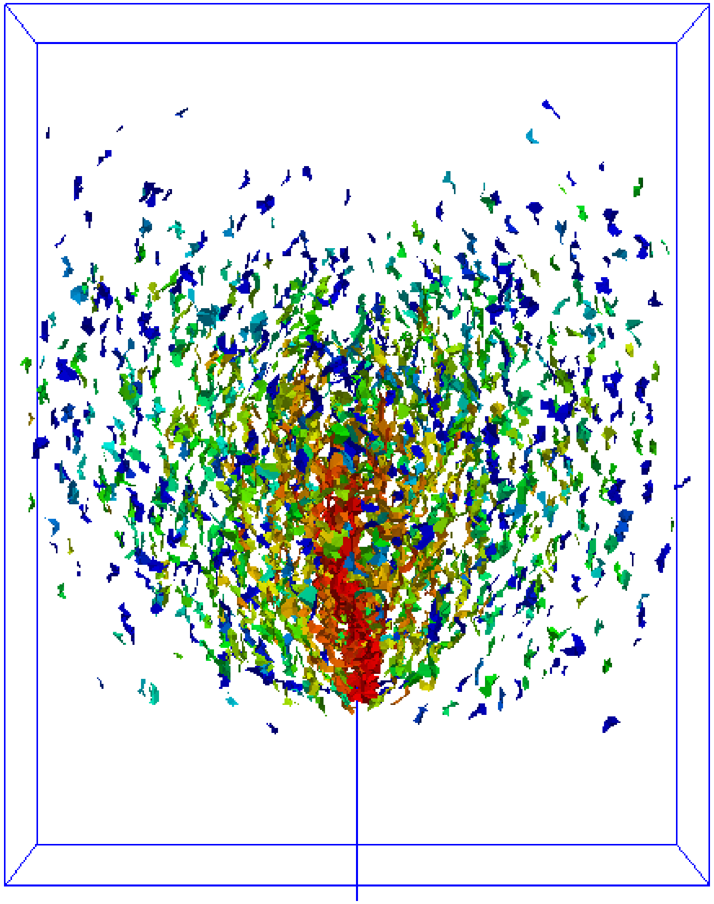
$$\frac{f_s^2 l}{2\alpha E} + \frac{f_s^2 l}{K_s} = G_f^s$$

Stresses at the contacts

$$\sigma_N = (1 - D)E\varepsilon_N$$

$$\sigma_M = \alpha(1 - D)E\varepsilon_M$$

$$\sigma_L = \alpha(1 - D)E\varepsilon_L$$



Material spatial randomness

- every connection has random
  - tensile strength f_t
  - shear strength f_s
  - tensile fracture energy G_t
  - shear fracture energy G_s
- full correlation of these properties and the same coefficient of variation (20%)
- continuous spatial fluctuation
- autocorrelated random field $\boldsymbol{H}(\boldsymbol{x})$ of mean 1 and CoV 20%

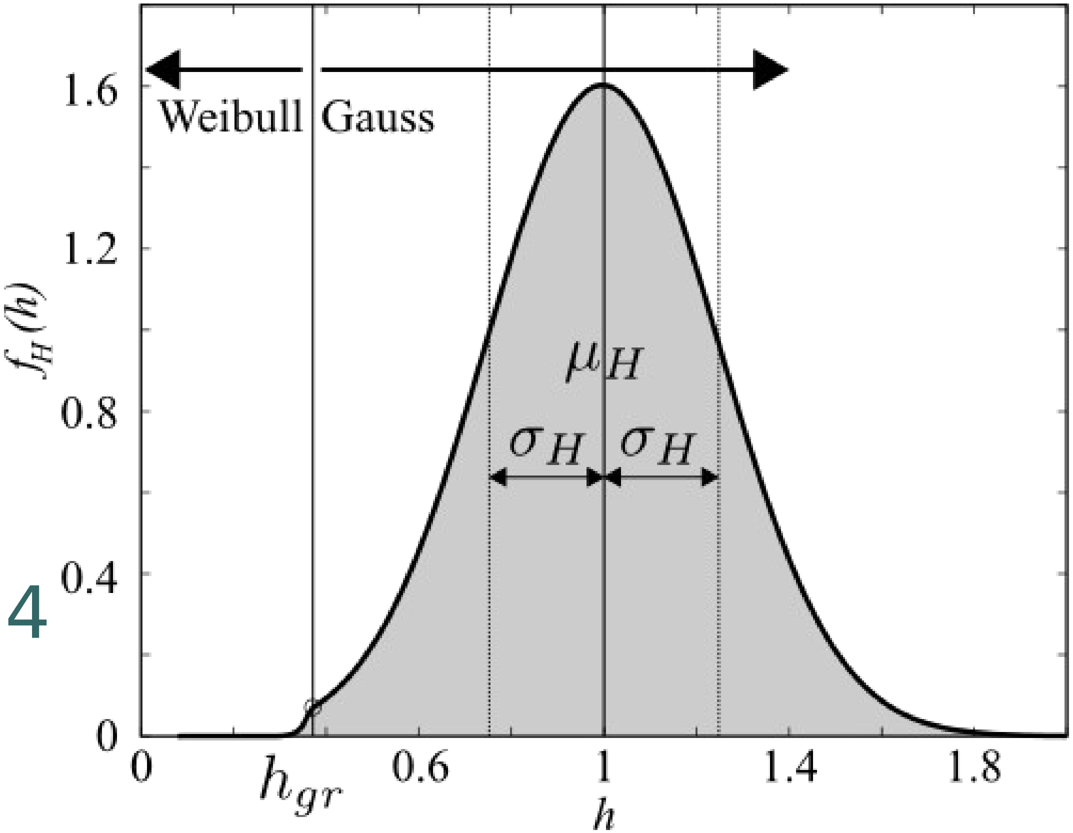
$$X(\boldsymbol{x}) = \bar{X}\boldsymbol{H}(\boldsymbol{x})$$

Random field parameters - CDF

$$F_H(h) = \begin{cases} r_f \left(1 - e^{-\langle h/s_1 \rangle^m}\right) & 0 \le h \le h_{gr} \\ F_H(h_{gr}) + \dfrac{r_f}{\delta_G \sqrt{2\pi}} \displaystyle\int_{h_{gr}}^{h} e^{-(h-\mu_G)^2/2\delta_G^2} \mathrm{d}h & h > h_{gr} \end{cases}$$

- Weibull-Gauss graft derived for strength
- four independent parameters (DOFs)
  - Mean =1
  - CoV = 0.2
  - Weibull mod. $m = 24$
  - grafting prob.

$$F_H(h_{gr}) \approx 10^{-4}$$



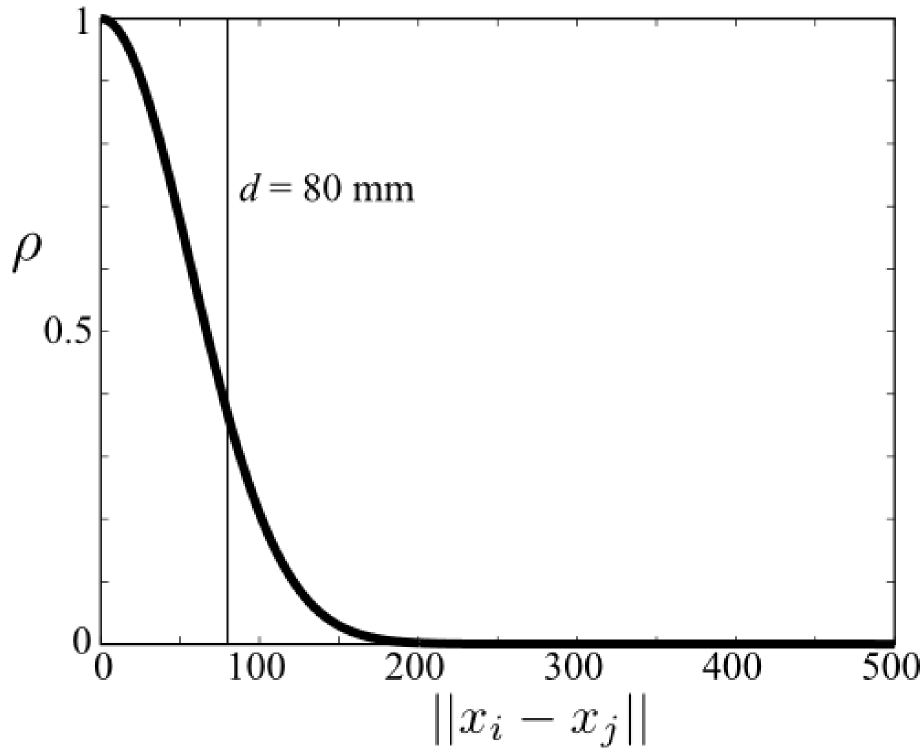
Bažant Z.P. and Pang, S.-D., 2007

Random field parameters

- correlation structure given by

$$\rho_{ij} = \exp\left[-\left(\frac{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|}{d}\right)^2\right]$$

- d is correlation length considered as 40 and 80 mm



Random field generation

- random field value generated in 24 realizations $\boldsymbol{H}^0(\boldsymbol{x}), \boldsymbol{H}^1(\boldsymbol{x}), \ldots, \boldsymbol{H}^{N-1}(\boldsymbol{x})$ at center of every lattice connection
- initially as Gaussian $\widehat{\boldsymbol{H}}^c(\boldsymbol{x})$, then transformed to Weibull-Gauss field $\boldsymbol{H}^c(\boldsymbol{x})$

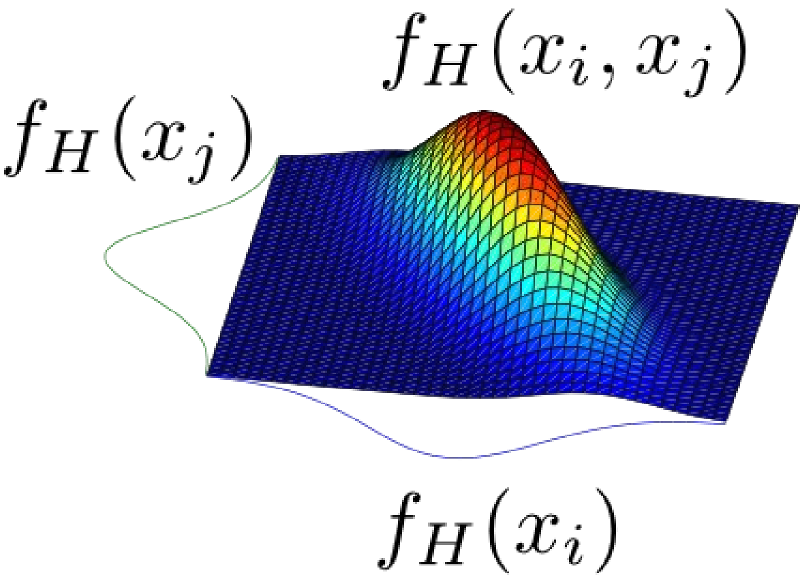
$$\boldsymbol{H}^c(\boldsymbol{x}) = F_H^{-1}(\Phi(\widehat{\boldsymbol{H}}^c(\boldsymbol{x})))$$

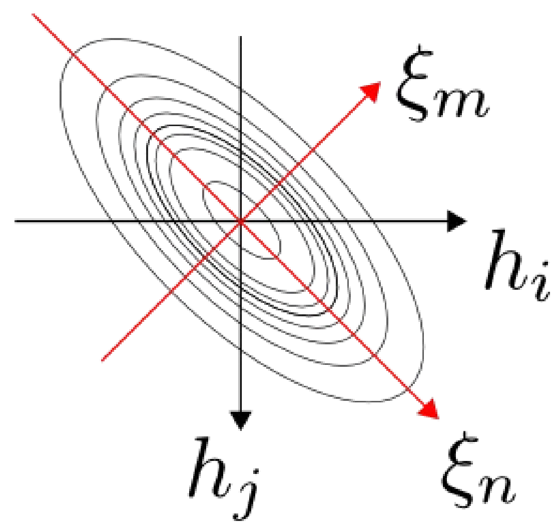
- Isoprobabilistic transformation disturbs field correlation structure.
  This was fixed by Nataf model.

Gaussian random field generation

- Karhunen-Loéve expansion
- considered K eigenvalues λ and corresponding eigenvectors $\boldsymbol{\psi}$ of field covariance matrix

$$\widehat{\boldsymbol{H}}^{c}(\boldsymbol{x}) = \sum_{k=1}^{K} \sqrt{\lambda_k}\,\xi_k^c\,\boldsymbol{\psi}_k(\boldsymbol{x})$$

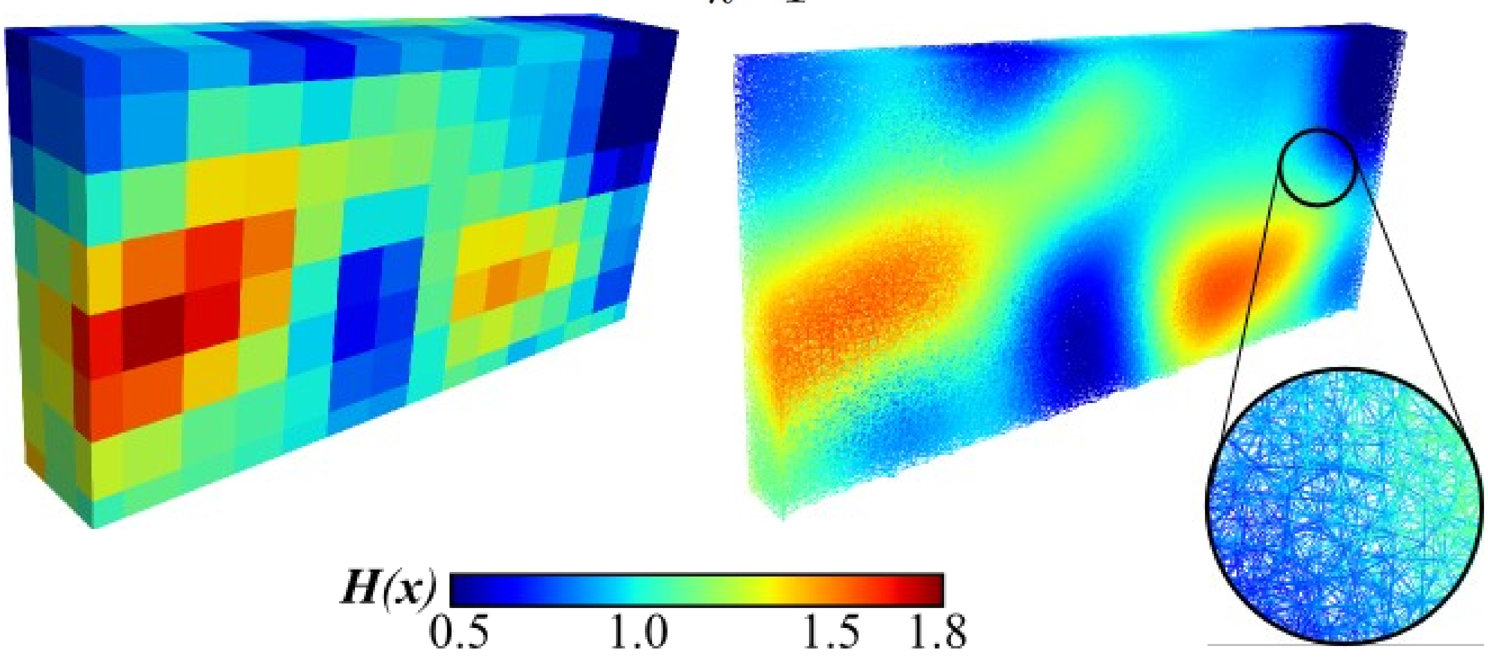




Optimal linear estimation method

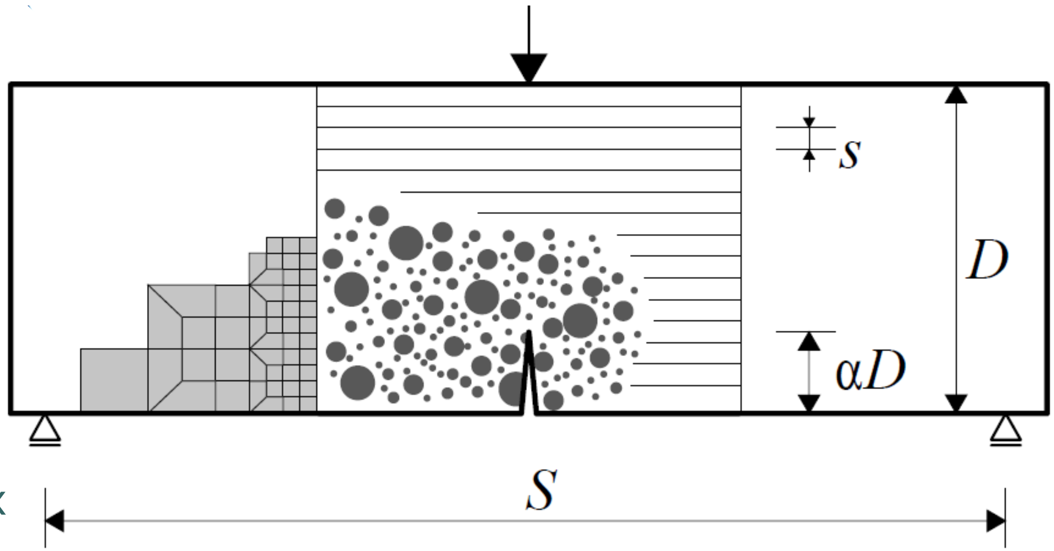
- large covariance m. => slow eigen-decomp.
- random field generated rather on the regular grid and then “projected” on the lattice

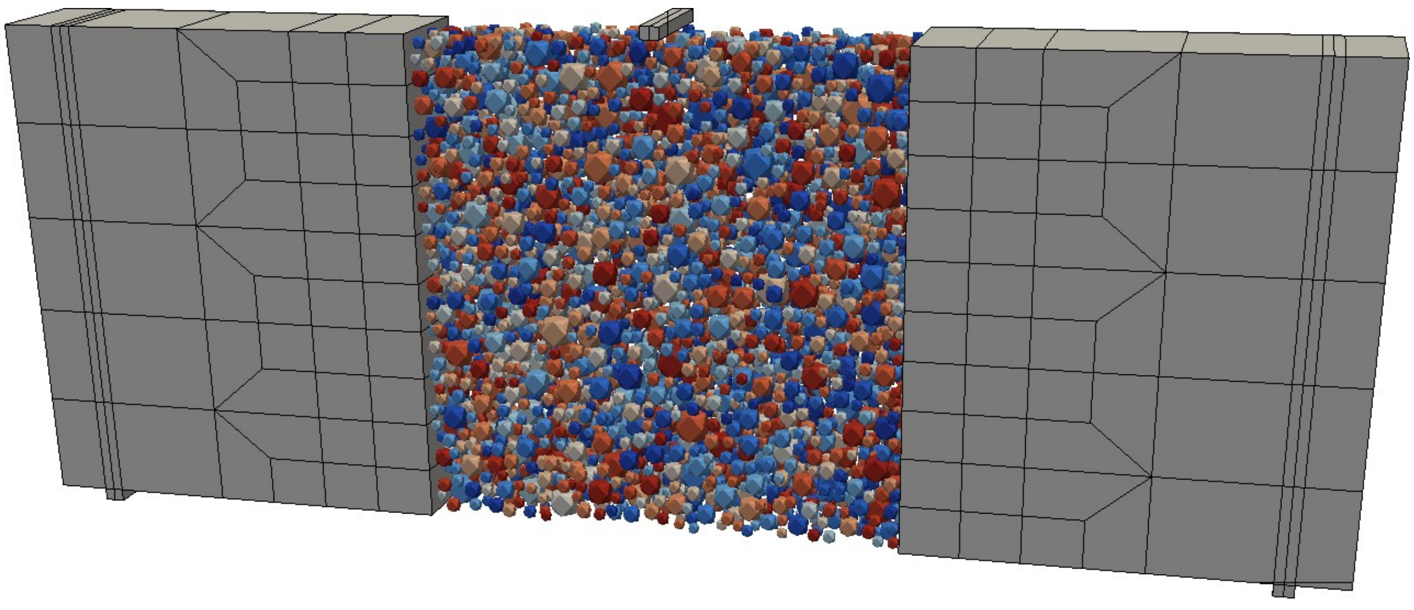
$$\widehat{\boldsymbol{H}}^c(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\xi_k^c}{\sqrt{\lambda_k}} \psi_k^T \boldsymbol{C}_{xg}$$



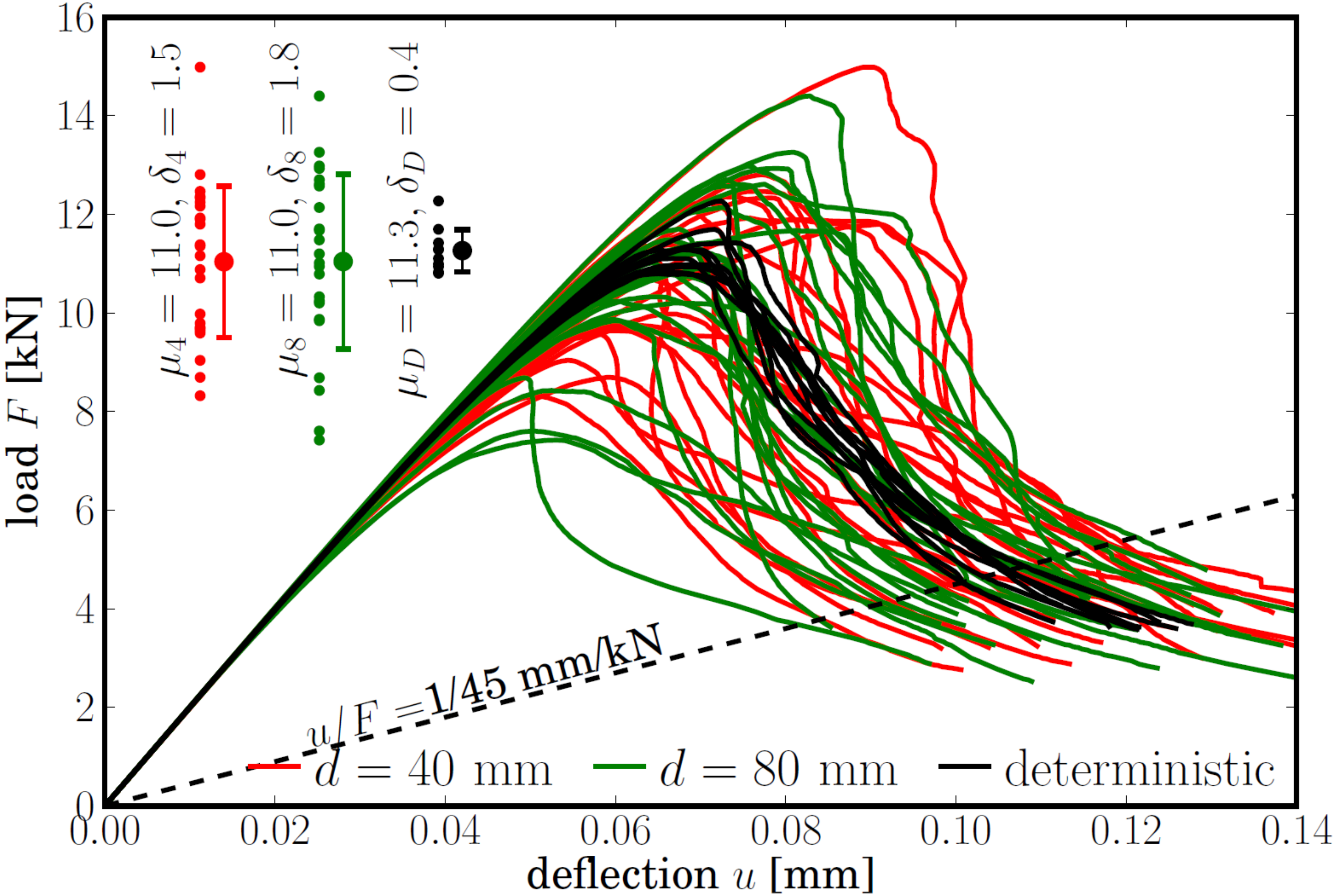
Specimen geometry

D = 300 mm
S = 2.4 D
α = {0,1/6}
terminated at 1/3F_{max}





Notched beams – α = 1/6

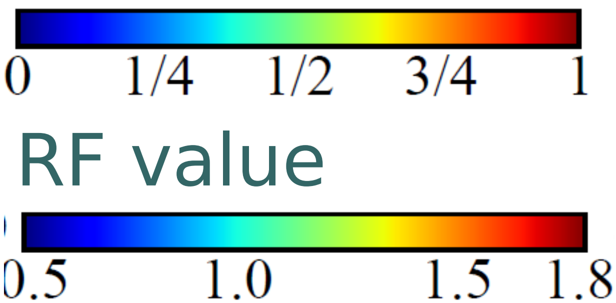


Damage patterns

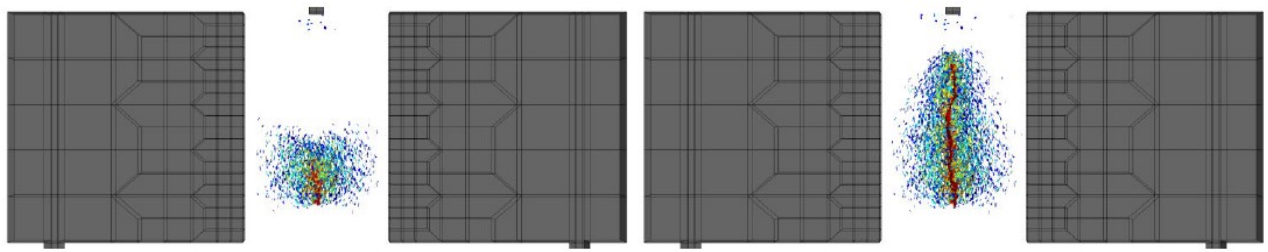
random field | *damage at peak* | *final damage*

damage

RF value

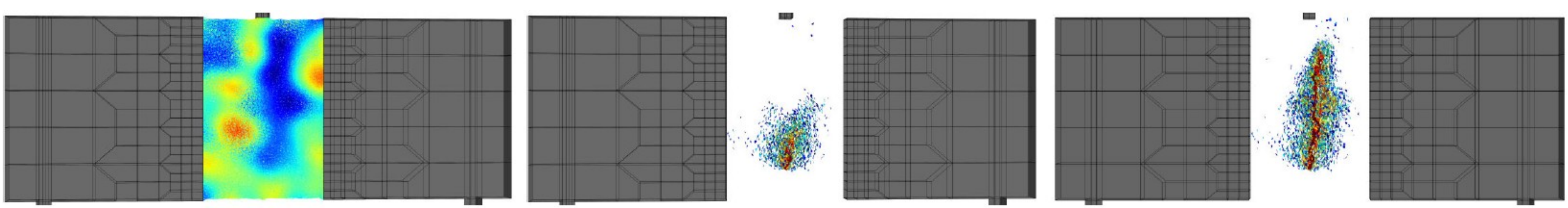


deterministic



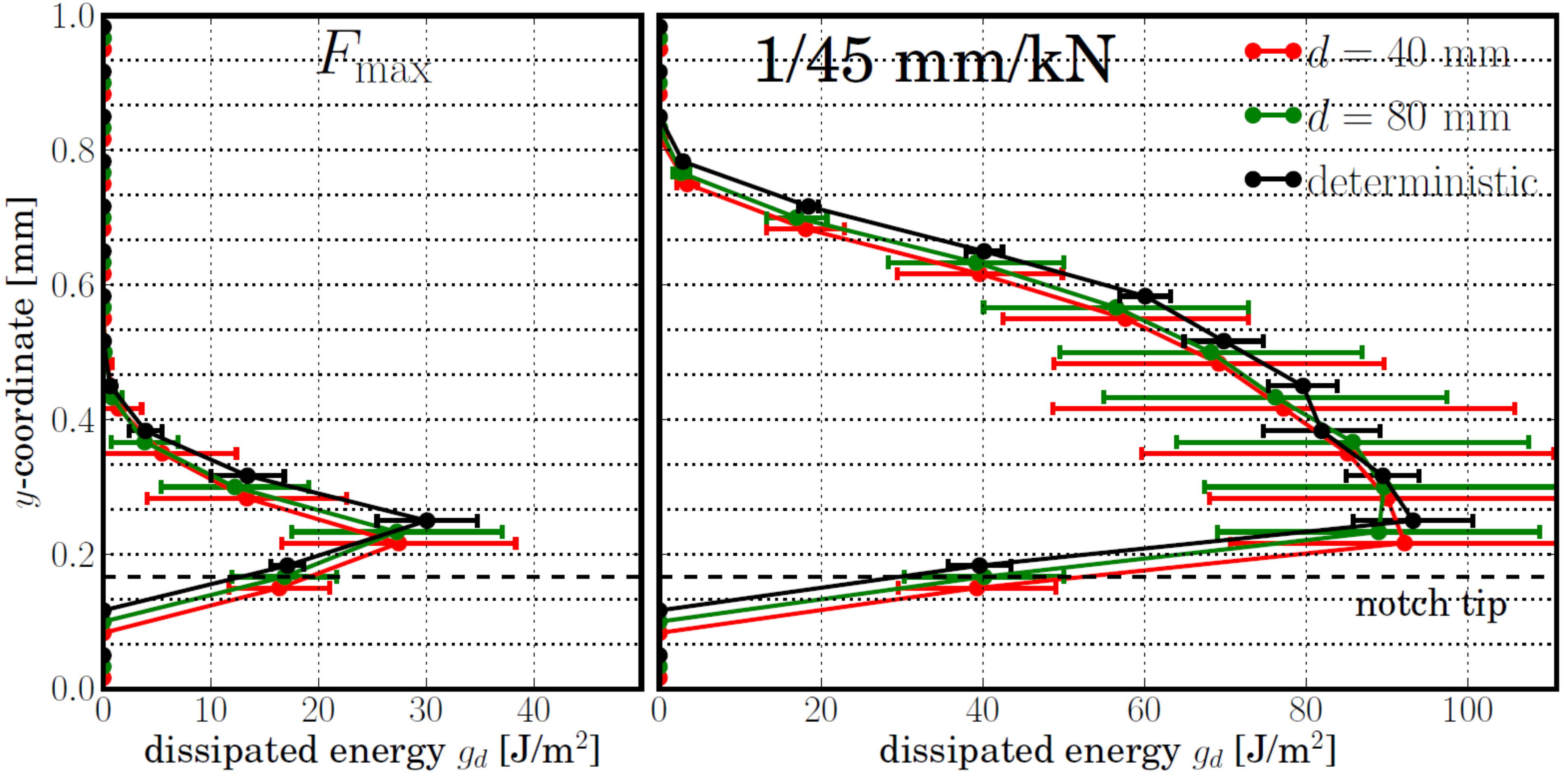
d = 80 mm

d = 40 mm

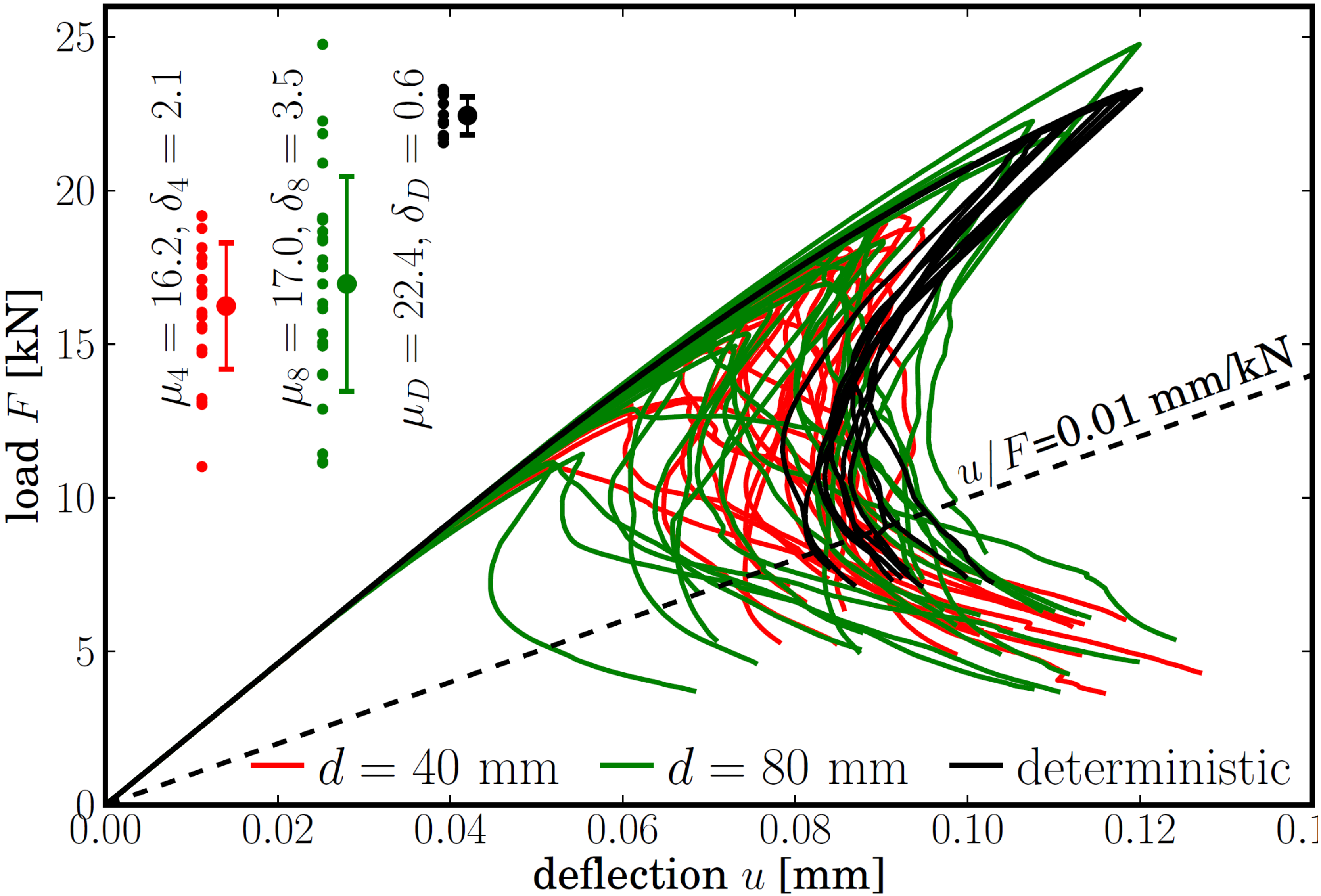


Energy dissipation

- same mean curves but difference in standard deviation



Unnotched beams

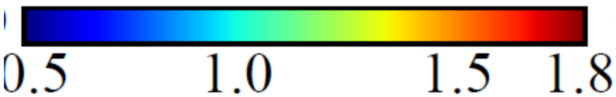


Damage patterns

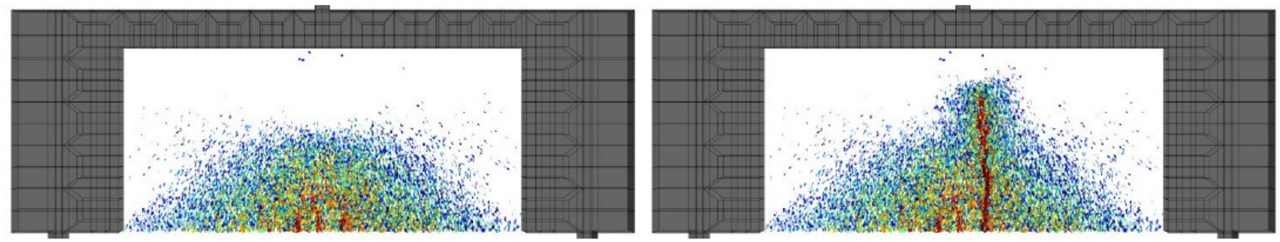
random field *damage at peak* *final damage*

damage

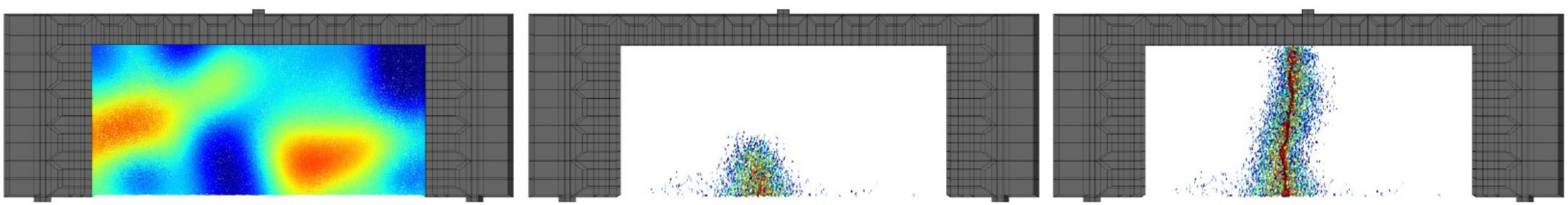
RF value



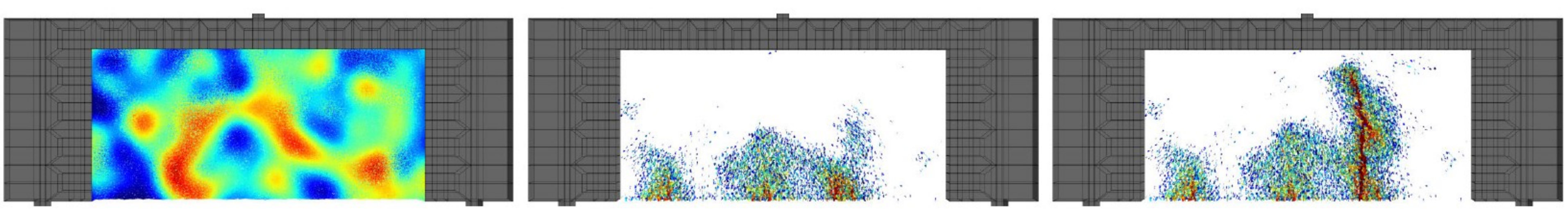
deterministic



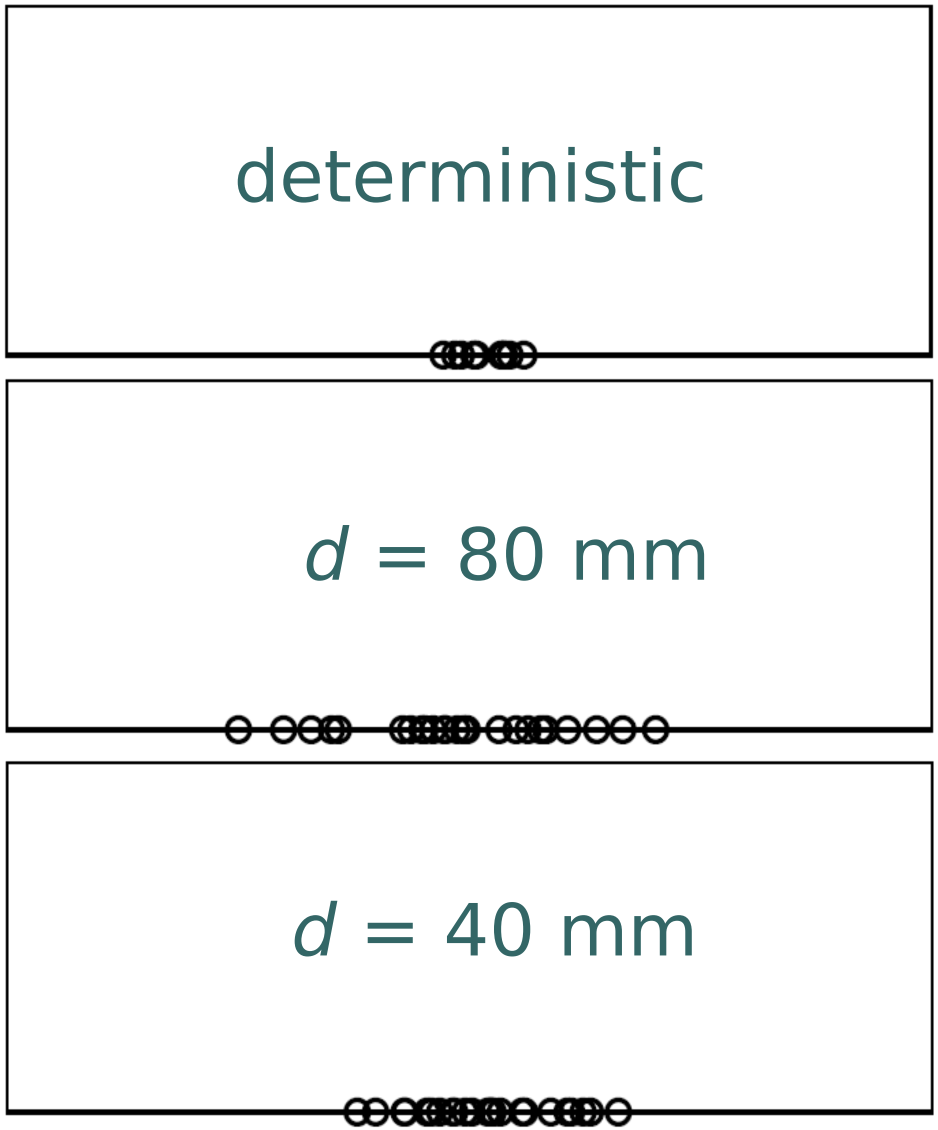
d = 80 mm



d = 40 mm



Location of Crack initiation

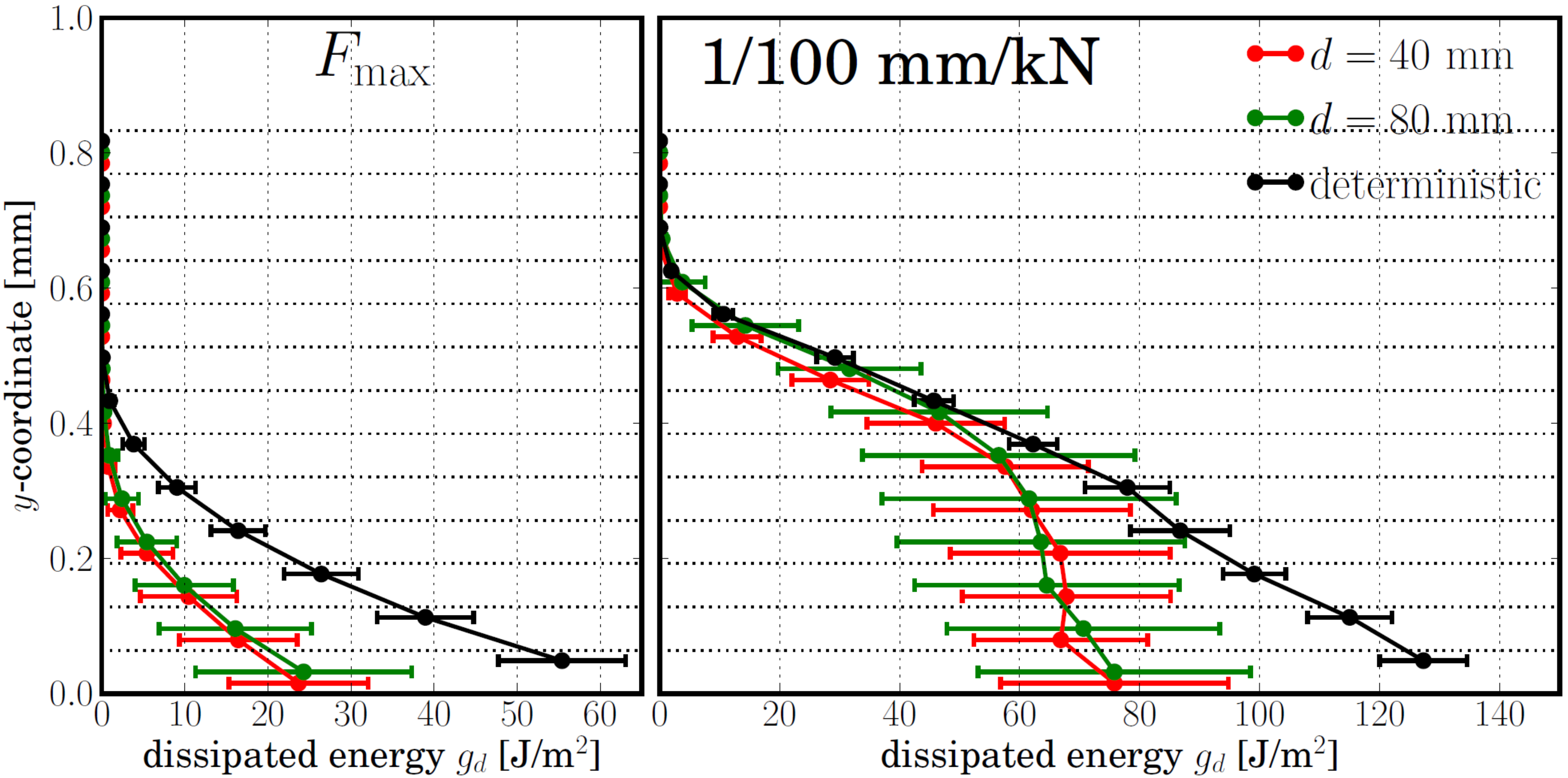


random because
of stress variations
due to grains

weak spot
&
high stress

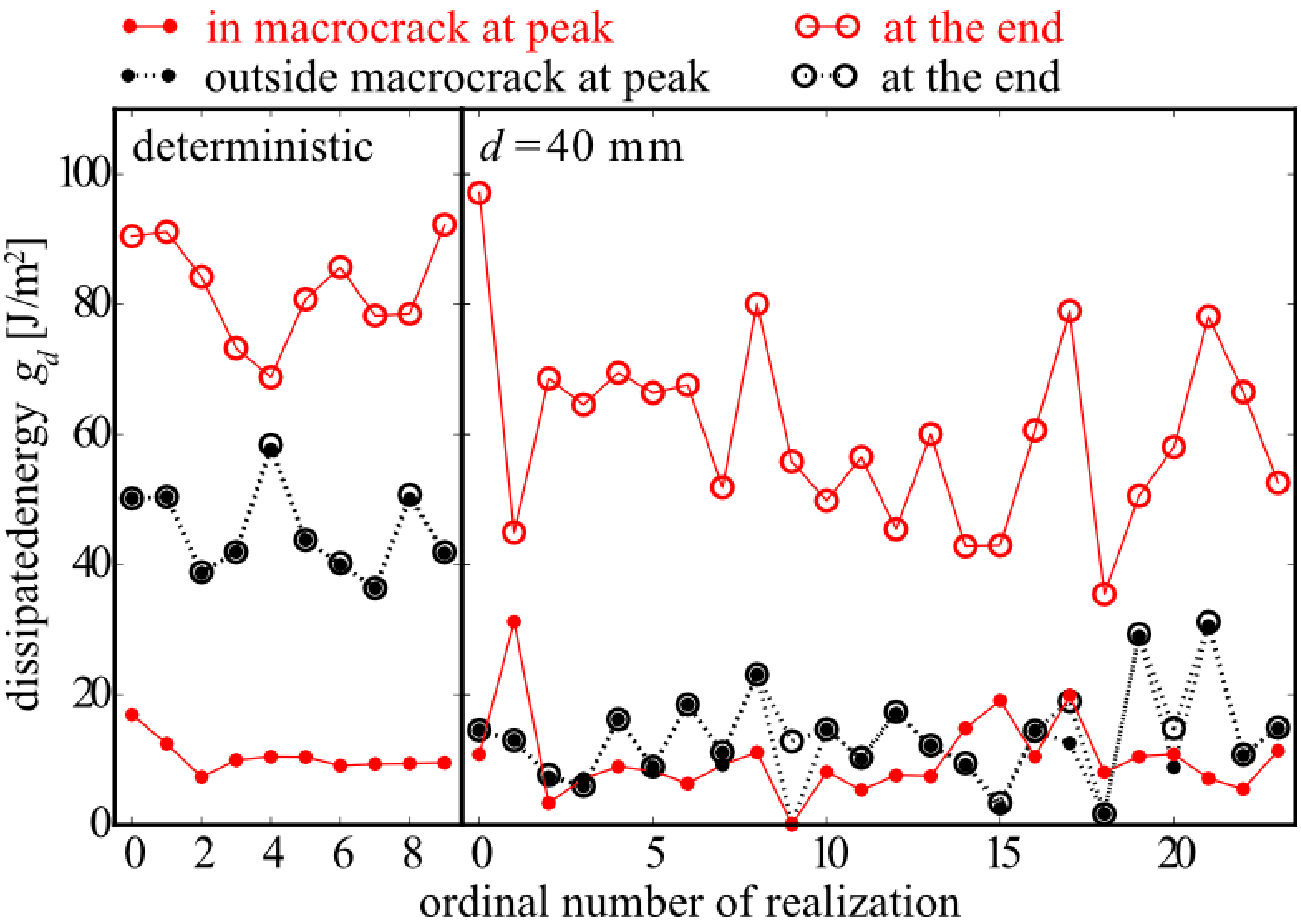
Energy dissipation

- different mean curves and standard deviation



Sources of energy difference

- i) inside and ii) outside the macrocrack



Conclusions

- Results confirmed natural expectations.
- Mean values of deep notch beams (strenth, response curve, dissipated energy, ...) are not infuenced by randomness.
  But the strandard deviations are.
- Mean values of unnotched results depends on randomness. Less energy is dissipated in random case.

- What happen in case of shallow notches?
- What happens when mesoscopic strengths and fracture energies are negatively correlated?