TECHNISCHE UNIVERSITÄT DRESDEN

Georgia Institute of Technology

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A Particle Swarm Optimization Approach for Training Artificial Neural Networks with Uncertain Data

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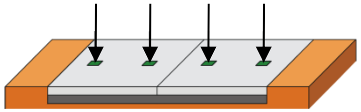
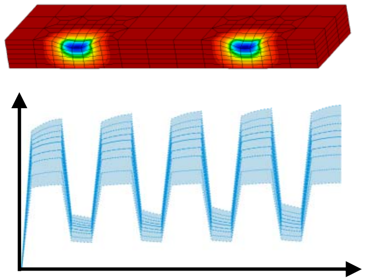
Content

Numerical structural analysis

observation → **modeling** → **numerical analysis**



loading
geometry
material behavior



- artificial neural networks (ANN) for uncertain material behavior
- parameter identification with particle swarm optimization (PSO)
- application within the finite element method (FEM)

Material tests

- test set up (experimental design)

specimen geometry
measurement devices
boundary conditions
initial conditions
loading scenarios
data recording

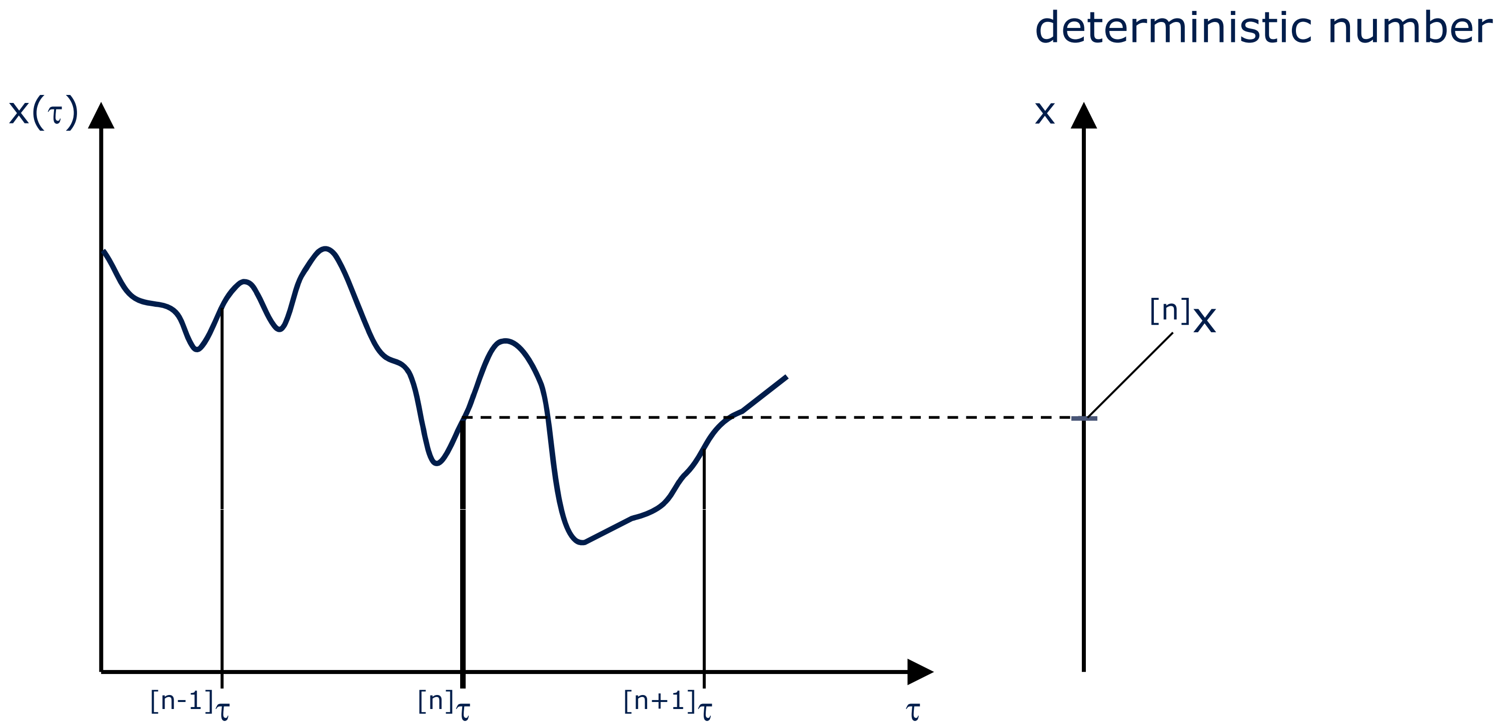
→ **imprecise measurements**

- imprecise measurements of forces and displacements
- uncertain material formulations (stress-strain-time dependencies)

Interval and fuzzy structural processes

- discretization

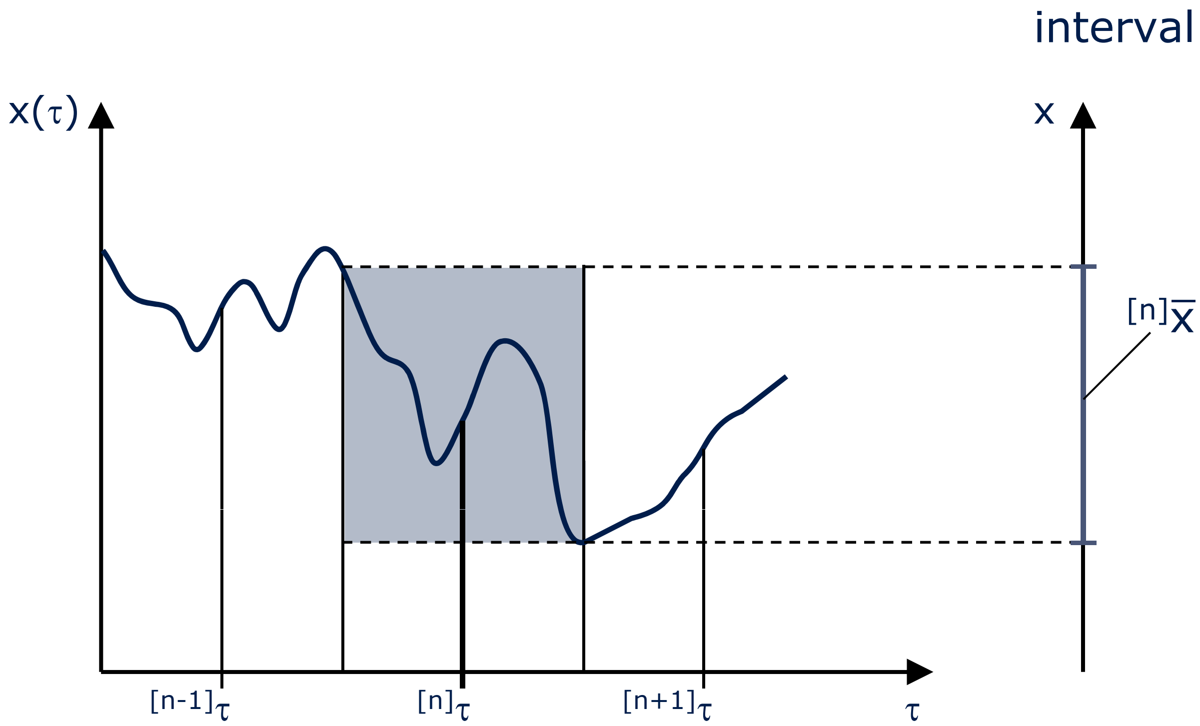
  equidistant time steps n = 1, 2, …, N; with $^{[n]}\Delta\tau = \Delta\tau$



Interval and fuzzy structural processes

- discretization

  equidistant time steps n = 1, 2, …, N; with $^{[n]}\Delta\tau = \Delta\tau$

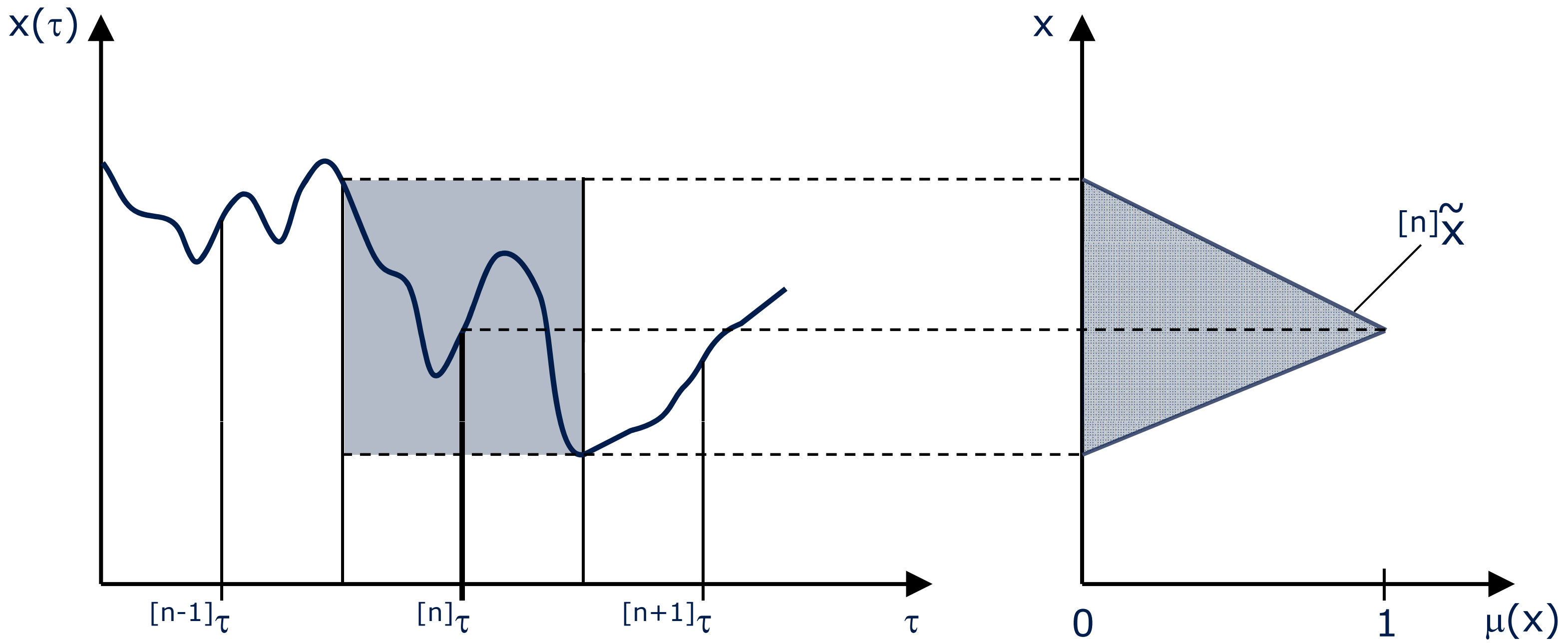


Interval and fuzzy structural processes

- discretization

  equidistant time steps n = 1, 2, …, N; with $^{[n]}\Delta\tau = \Delta\tau$

fuzzy number

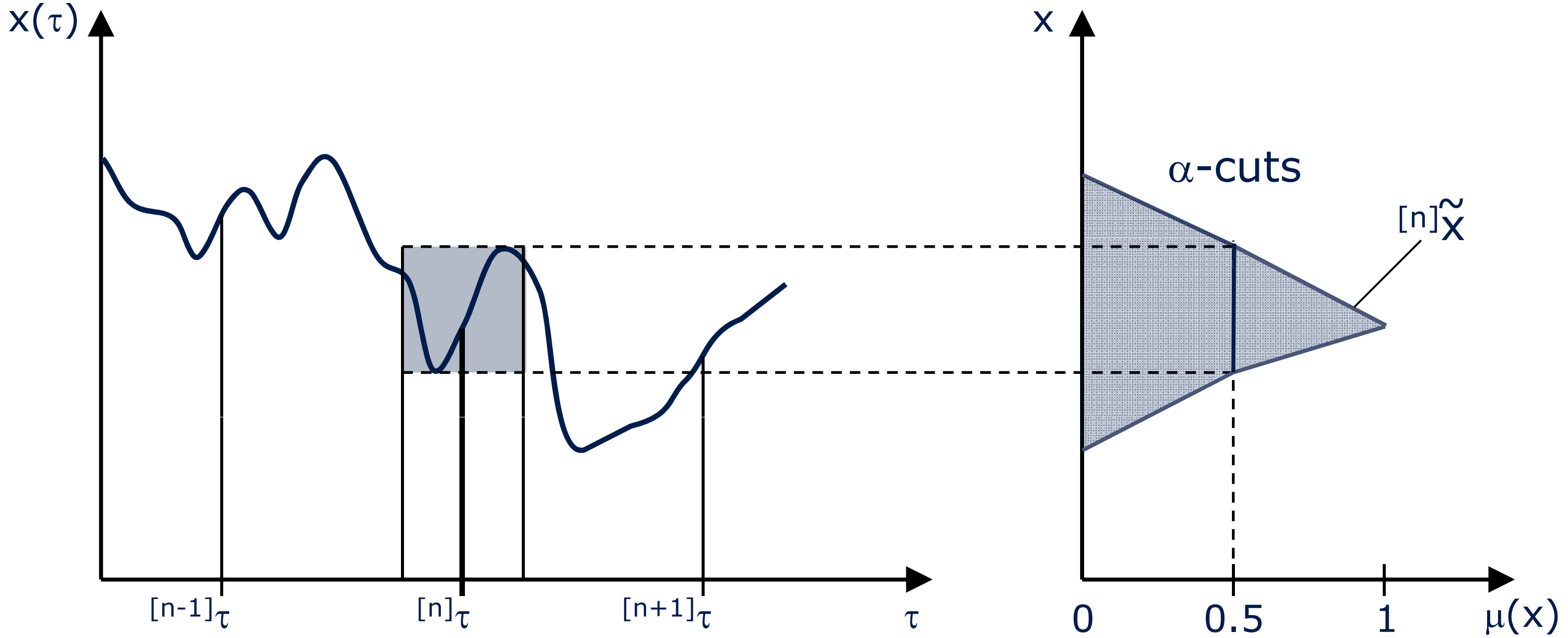


Interval and fuzzy structural processes

- discretization

  equidistant time steps n = 1, 2, …, N; with $^{[n]}\Delta\tau = \Delta\tau$

fuzzy number

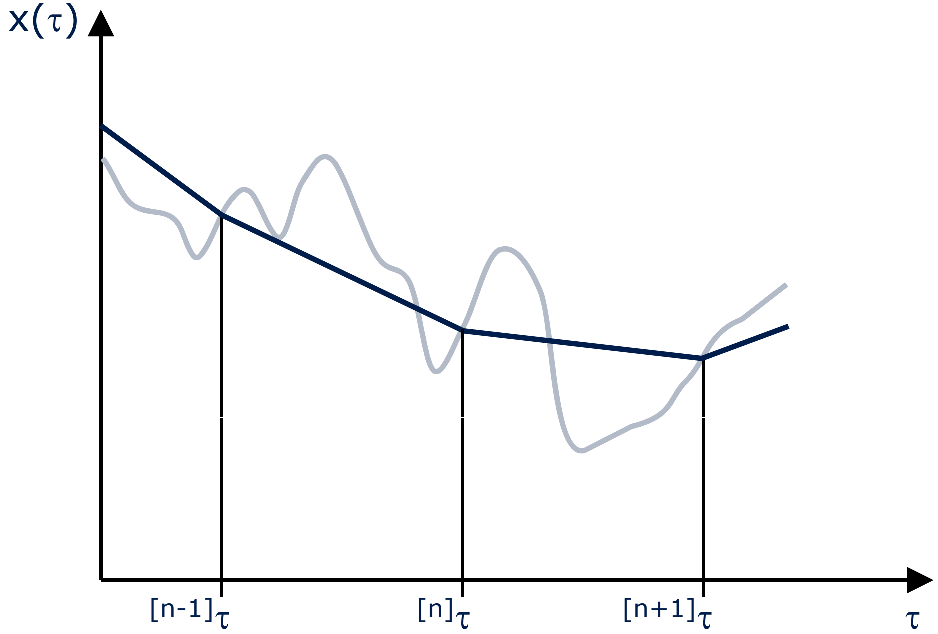


Interval and fuzzy structural processes

- discretization

  equidistant time steps n = 1, 2, …, N; with ${}^{[n]}\Delta\tau = \Delta\tau$

  deterministic process

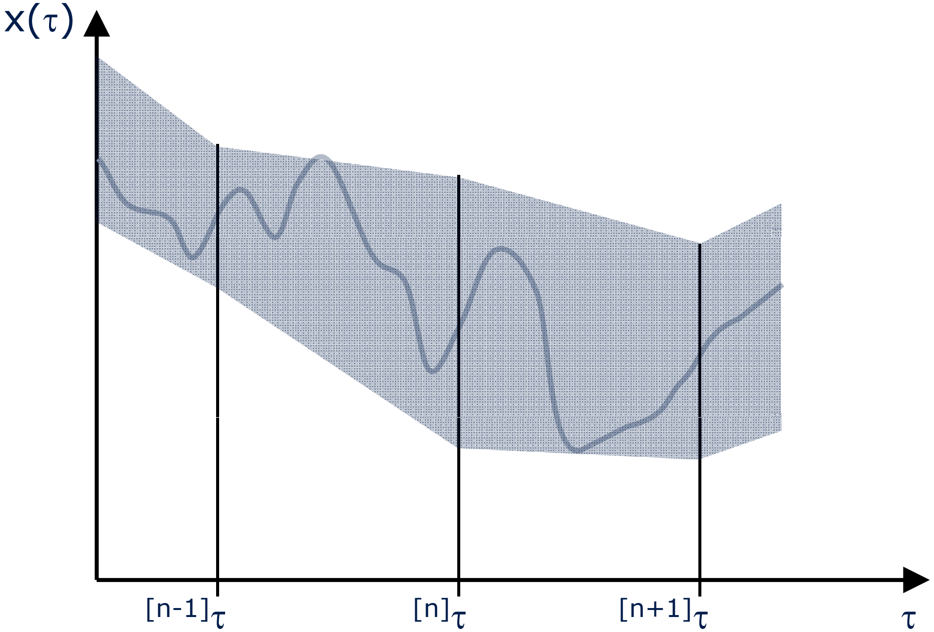


Interval and fuzzy structural processes

- discretization

  equidistant time steps n = 1, 2, …, N; with $^{[n]}\Delta\tau = \Delta\tau$

  interval process

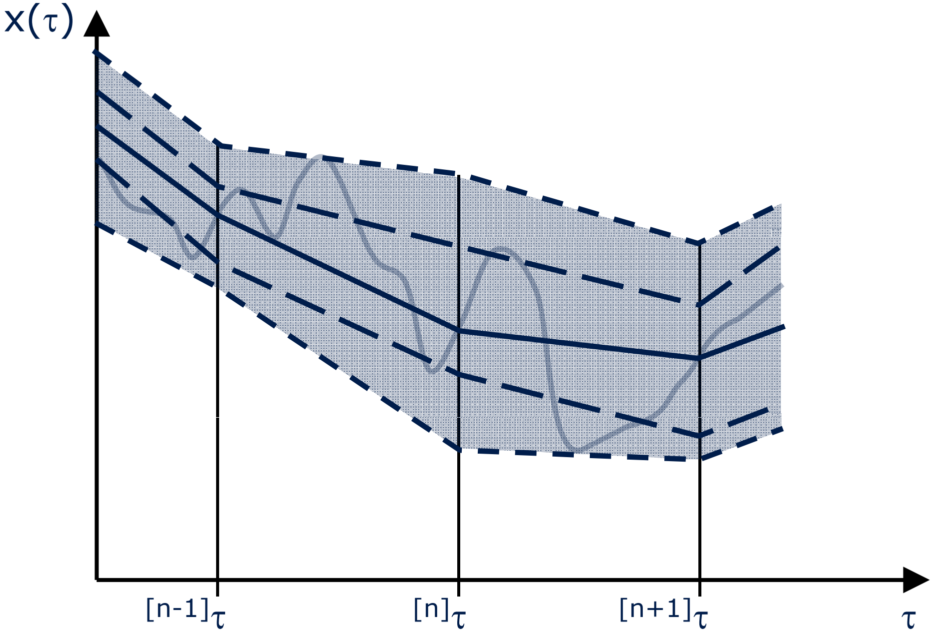


Interval and fuzzy structural processes

- discretization

  equidistant time steps n = 1, 2, …, N; with ${}^{[n]}\Delta\tau = \Delta\tau$

  fuzzy process



Functional relationships between interval processes

- Type 1 mapping: $\overline{\underline{x}}(\tau) \longrightarrow \overline{\underline{z}}(\tau)$

interval processes	deterministic parameters	interval processes

- Type 2 mapping: $\underline{x}(\tau) \Longrightarrow \overline{\underline{z}}(\tau)$

deterministic processes	interval parameters	interval processes

- Type 3 mapping: $\overline{\underline{x}}(\tau) \Longrightarrow \overline{\underline{z}}(\tau)$

interval processes	interval parameters	interval processes

Functional relationships between fuzzy processes

- Type 1 mapping: $\underset{\sim}{\tilde{x}}(\tau) \longrightarrow \underset{\sim}{\tilde{z}}(\tau)$

fuzzy processes	deterministic parameters	fuzzy processes

- Type 2 mapping: $\underset{\sim}{x}(\tau) \overset{\sim}{\longrightarrow} \underset{\sim}{\tilde{z}}(\tau)$

deterministic processes	fuzzy parameters	fuzzy processes

- Type 3 mapping: $\underset{\sim}{\tilde{x}}(\tau) \overset{\sim}{\longrightarrow} \underset{\sim}{\tilde{z}}(\tau)$

fuzzy processes	fuzzy parameters	fuzzy processes

Parameter identification

- difference between collected and computed interval data

$$E^h = \frac{1}{N}\frac{1}{K}\sum_{n=1}^{N}\left[\sum_{k=1}^{K}\left\{\left({}^{[n]}_{l}z_k - {}^{[n]}_{l}z_k^*\right)^2 + \left({}^{[n]}_{u}z_k - {}^{[n]}_{u}z_k^*\right)^2\right\}\right]$$

- difference between collected and computed fuzzy data

$$E^h = \frac{1}{N}\frac{1}{K}\frac{1}{S}\sum_{n=1}^{N}\left[\sum_{k=1}^{K}\left\{\sum_{s=1}^{S}\left[\left({}^{[n]}_{sl}z_k - {}^{[n]}_{sl}z_k^*\right)^2 + \left({}^{[n]}_{su}z_k - {}^{[n]}_{su}z_k^*\right)^2\right]\right\}\right]$$

- optimization task with deterministic, interval or fuzzy parameters
  objective function

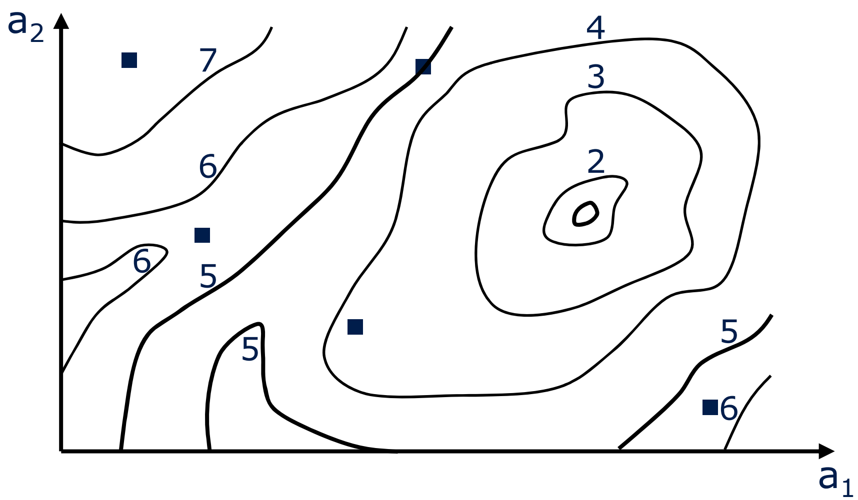
$$E^{av} = \frac{1}{H}\sum_{h=1}^{H}\left[E^h\right] \qquad E^{av} \rightarrow \min$$

PSO – deterministic parameters

- biosocial motivated search algorithm

$$^{(r+1)}a_q^i = {}^{(r)}a_q^i + {}^{(r)}\Delta a_q^i$$

$$^{(r)}\Delta a_q^i = \underbrace{c_3 \cdot {}^{(r-1)}\Delta a_q^i}_{\text{history}} + \underbrace{c_1 \cdot d \cdot \left(p_q^i - {}^{(r)}a_q^i\right)}_{\text{individual best}} + \underbrace{c_2 \cdot e \cdot \left(g_q - {}^{(r)}a_q^i\right)}_{\text{group best}}$$



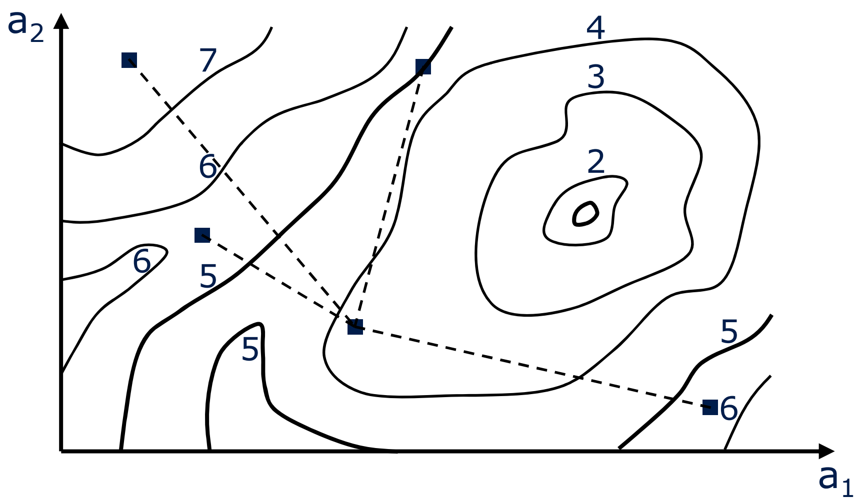
random initial particle positions

PSO – deterministic parameters

- biosocial motivated search algorithm

$$ {}^{(r+1)}a_q^i = {}^{(r)}a_q^i + {}^{(r)}\Delta a_q^i $$

$$ {}^{(r)}\Delta a_q^i = \underbrace{c_1 \cdot {}^{(r-1)}\Delta a_q^i}_{\text{history}} + \underbrace{c_2 \cdot d \cdot \left(p_q^i - {}^{(r)}a_q^i\right)}_{\text{individual best}} + \underbrace{c_3 \cdot e \cdot \left(g_q - {}^{(r)}a_q^i\right)}_{\text{group best}} $$



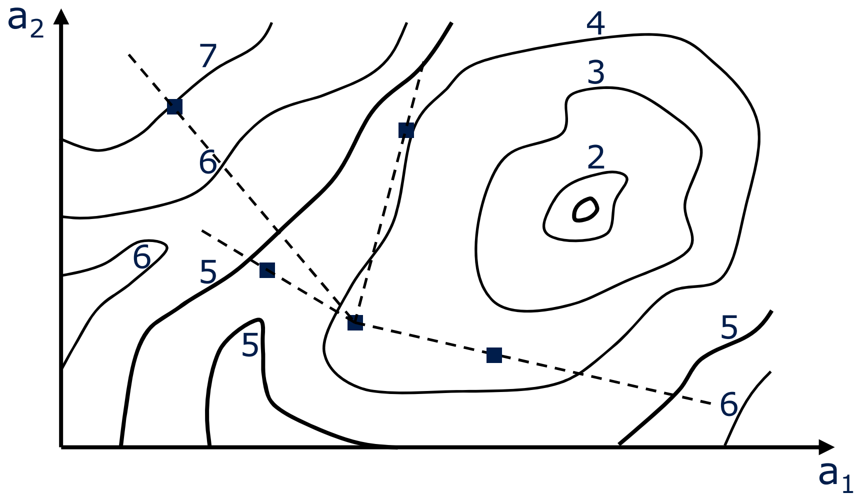
r=1

PSO – deterministic parameters

- biosocial motivated search algorithm

$$^{(r+1)}a_q^i = {}^{(r)}a_q^i + {}^{(r)}\Delta a_q^i$$

$$^{(r)}\Delta a_q^i = \underbrace{c_3 \cdot {}^{(r-1)}\Delta a_q^i}_{\text{history}} + \underbrace{c_1 \cdot d \cdot \left(p_q^i - {}^{(r)}a_q^i\right)}_{\text{individual best}} + \underbrace{c_2 \cdot e \cdot \left(g_q - {}^{(r)}a_q^i\right)}_{\text{group best}}$$



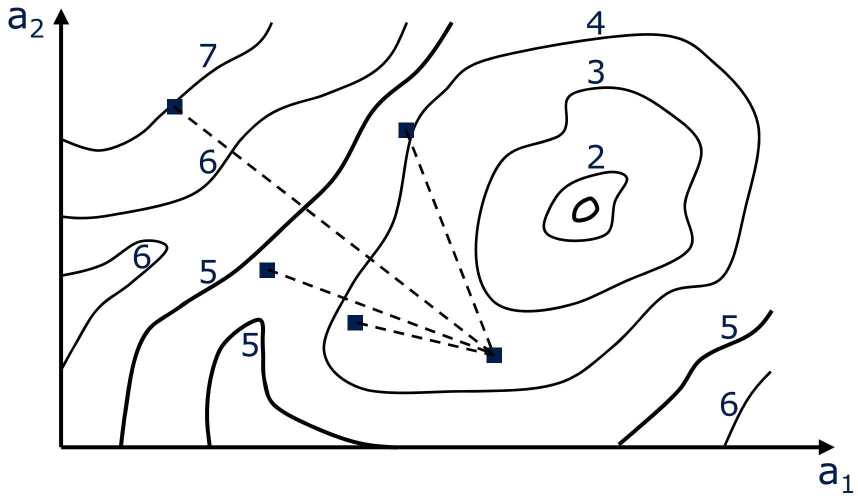
r=2

PSO – deterministic parameters

- biosocial motivated search algorithm

$$ {}^{(r+1)}a_q^i = {}^{(r)}a_q^i + {}^{(r)}\Delta a_q^i $$

$$ {}^{(r)}\Delta a_q^i = \underbrace{c_3 \cdot {}^{(r-1)}\Delta a_q^i}_{\text{history}} + \underbrace{c_1 \cdot d \cdot \left(p_q^i - {}^{(r)}a_q^i\right)}_{\text{individual best}} + \underbrace{c_2 \cdot e \cdot \left(g_q - {}^{(r)}a_q^i\right)}_{\text{group best}} $$



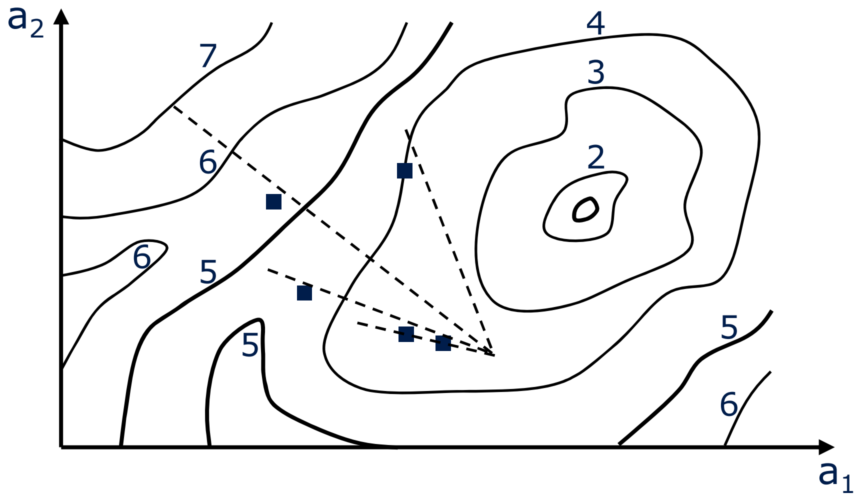
r=2

PSO – deterministic parameters

- biosocial motivated search algorithm

$$ {}^{(r+1)}a_q^i = {}^{(r)}a_q^i + {}^{(r)}\Delta a_q^i $$

$$ {}^{(r)}\Delta a_q^i = \underbrace{c_3 \cdot {}^{(r-1)}\Delta a_q^i}_{\text{history}} + \underbrace{c_1 \cdot d \cdot \left(p_q^i - {}^{(r)}a_q^i\right)}_{\text{individual best}} + \underbrace{c_2 \cdot e \cdot \left(g_q - {}^{(r)}a_q^i\right)}_{\text{group best}} $$



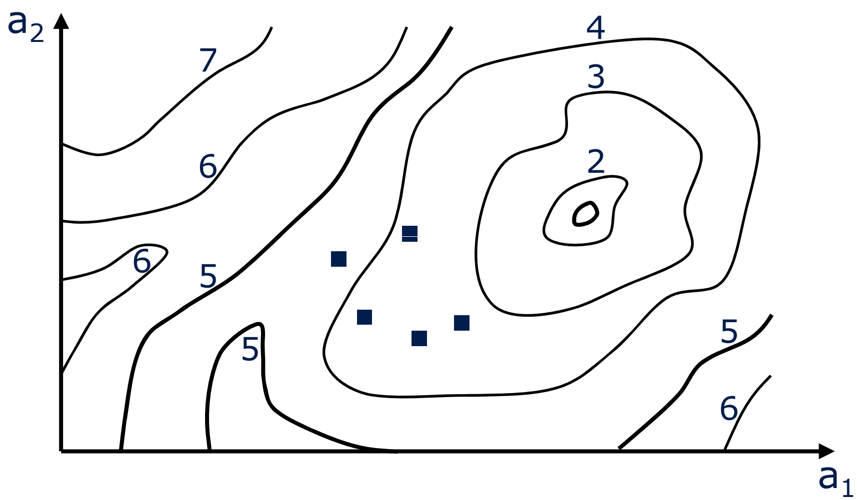
r=3

PSO – deterministic parameters

- biosocial motivated search algorithm

$$^{(r+1)}a_q^i = {}^{(r)}a_q^i + {}^{(r)}\Delta a_q^i$$

$$^{(r)}\Delta a_q^i = \underbrace{c_3 \cdot {}^{(r-1)}\Delta a_q^i}_{\text{history}} + \underbrace{c_1 \cdot d \cdot \left(p_q^i - {}^{(r)}a_q^i\right)}_{\text{individual best}} + \underbrace{c_2 \cdot e \cdot \left(g_q - {}^{(r)}a_q^i\right)}_{\text{group best}}$$



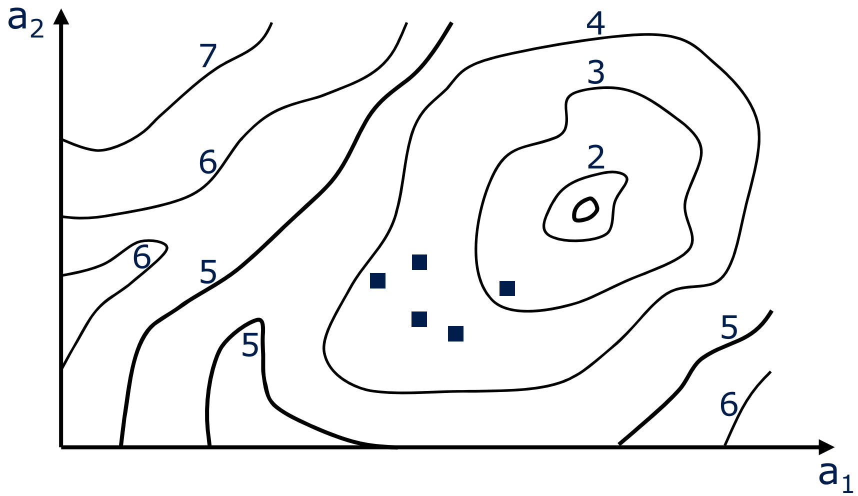
r=4

PSO – deterministic parameters

- biosocial motivated search algorithm

$$^{(r+1)}a_q^i = {}^{(r)}a_q^i + {}^{(r)}\Delta a_q^i$$

$$^{(r)}\Delta a_q^i = \underbrace{c_3 \cdot {}^{(r-1)}\Delta a_q^i}_{\text{history}} + \underbrace{c_1 \cdot d \cdot \left(p_q^i - {}^{(r)}a_q^i\right)}_{\text{individual best}} + \underbrace{c_2 \cdot e \cdot \left(g_q - {}^{(r)}a_q^i\right)}_{\text{group best}}$$



r=5

PSO – interval parameters

- particles are interval numbers

$$ {}^{(r+1)}_{l}a^{i}_{q} = {}^{(r+1)}_{m}a^{i}_{q} - {}^{(r+1)}_{r}a^{i}_{q} \qquad {}^{(r+1)}_{u}a^{i}_{q} = {}^{(r+1)}_{m}a^{i}_{q} + {}^{(r+1)}_{r}a^{i}_{q} $$

- update of midpoint

$$ {}^{(r+1)}_{m}a^{i}_{q} = {}^{(r)}_{m}a^{i}_{q} + {}^{(r)}_{m}\Delta a^{i}_{q} $$

$$ {}^{(r)}_{m}\Delta a^{i}_{q} = c_3 \cdot {}^{(r-1)}_{m}\Delta a^{i}_{q} + c_1 \cdot d \cdot \left({}_{m}p^{i}_{q} - {}^{(r)}_{m}a^{i}_{q} \right) + c_2 \cdot e \cdot \left({}_{m}g_{q} - {}^{(r)}_{m}a^{i}_{q} \right) $$

- update of radius

$$ {}^{(r+1)}_{r}a^{i}_{q} = {}^{(r)}_{r}a^{i}_{q} + {}^{(r)}_{r}\Delta a^{i}_{q} \qquad \text{constraint} \quad {}^{(r+1)}_{r}a^{i}_{q} \geq 0 $$

$$ {}^{(r)}_{r}\Delta a^{i}_{q} = c_3 \cdot {}^{(r-1)}_{r}\Delta a^{i}_{q} + c_1 \cdot d \cdot \left({}_{r}p^{i}_{q} - {}^{(r)}_{r}a^{i}_{q} \right) + c_2 \cdot e \cdot \left({}_{r}g_{q} - {}^{(r)}_{r}a^{i}_{q} \right) $$

PSO – fuzzy parameters

- particles are fuzzy numbers

$$ {}^{(r+1)}_{sl}a^{i}_{q} = {}^{(r+1)}_{sm}a^{i}_{q} - {}^{(r+1)}_{sr}a^{i}_{q} \qquad {}^{(r+1)}_{su}a^{i}_{q} = {}^{(r+1)}_{sm}a^{i}_{q} + {}^{(r+1)}_{sr}a^{i}_{q} $$

- update of midpoint for α-cut s=1

$$ {}^{(r+1)}_{1m}a^{i}_{q} = {}^{(r)}_{1m}a^{i}_{q} + {}^{(r)}_{1m}\Delta a^{i}_{q} $$

$$ {}^{(r)}_{1m}\Delta a^{i}_{q} = c_3 \cdot {}^{(r-1)}_{1m}\Delta a^{i}_{q} + c_1 \cdot d \cdot \left({}_{1m}p^{i}_{q} - {}^{(r)}_{1m}a^{i}_{q} \right) + c_2 \cdot e \cdot \left({}_{1m}g_{q} - {}^{(r)}_{1m}a^{i}_{q} \right) $$

- update of radius for α-cut s=1

$$ {}^{(r+1)}_{1r}a^{i}_{q} = {}^{(r)}_{1r}a^{i}_{q} + {}^{(r)}_{1r}\Delta a^{i}_{q} \qquad \text{constraint} \quad {}^{(r+1)}_{1r}a^{i}_{q} \geq 0 $$

$$ {}^{(r)}_{1r}\Delta a^{i}_{q} = c_3 \cdot {}^{(r-1)}_{1r}\Delta a^{i}_{q} + c_1 \cdot d \cdot \left({}_{1r}p^{i}_{q} - {}^{(r)}_{1r}a^{i}_{q} \right) + c_2 \cdot e \cdot \left({}_{1r}g_{q} - {}^{(r)}_{1r}a^{i}_{q} \right) $$

PSO – fuzzy parameters

- particles are fuzzy numbers

$$ {}^{(r+1)}_{sl}a^i_q = {}^{(r+1)}_{sm}a^i_q - {}^{(r+1)}_{sr}a^i_q \qquad {}^{(r+1)}_{su}a^i_q = {}^{(r+1)}_{sm}a^i_q + {}^{(r+1)}_{sr}a^i_q $$

- update of midpoint for α-cut s>1

$$ {}^{(r+1)}_{sm}a^i_q = {}^{(r)}_{sm}a^i_q + {}^{(r)}_{sm}\Delta a^i_q \qquad \text{constraint} \quad {}^{(r+1)}_{s-1l}a^i_q \le {}^{(r+1)}_{sm}a^i_q \le {}^{(r+1)}_{s-1u}a^i_q $$

$$ {}^{(r)}_{sm}\Delta a^i_q = c_3 \cdot {}^{(r-1)}_{sm}\Delta a^i_q + c_1 \cdot d \cdot \left({}_{sm}p^i_q - {}^{(r)}_{sm}a^i_q \right) + c_2 \cdot e \cdot \left({}_{sm}g_q - {}^{(r)}_{sm}a^i_q \right) $$

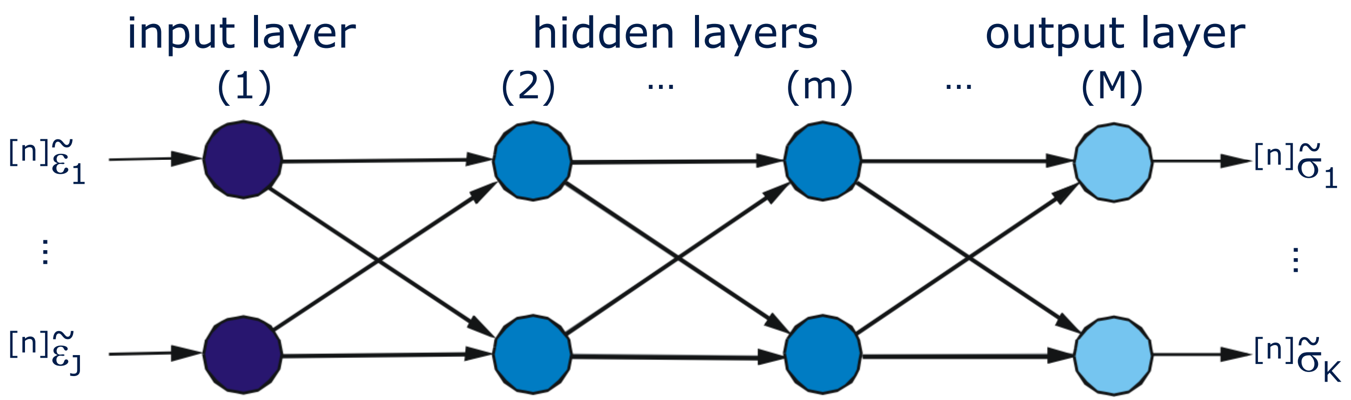
- update of radius for α-cut s>1

$$ {}^{(r+1)}_{sr}a^i_q = {}^{(r)}_{sr}a^i_q + {}^{(r)}_{sr}\Delta a^i_q \qquad \text{constraints} \quad {}^{(r+1)}_{sr}a^i_q \ge 0 $$

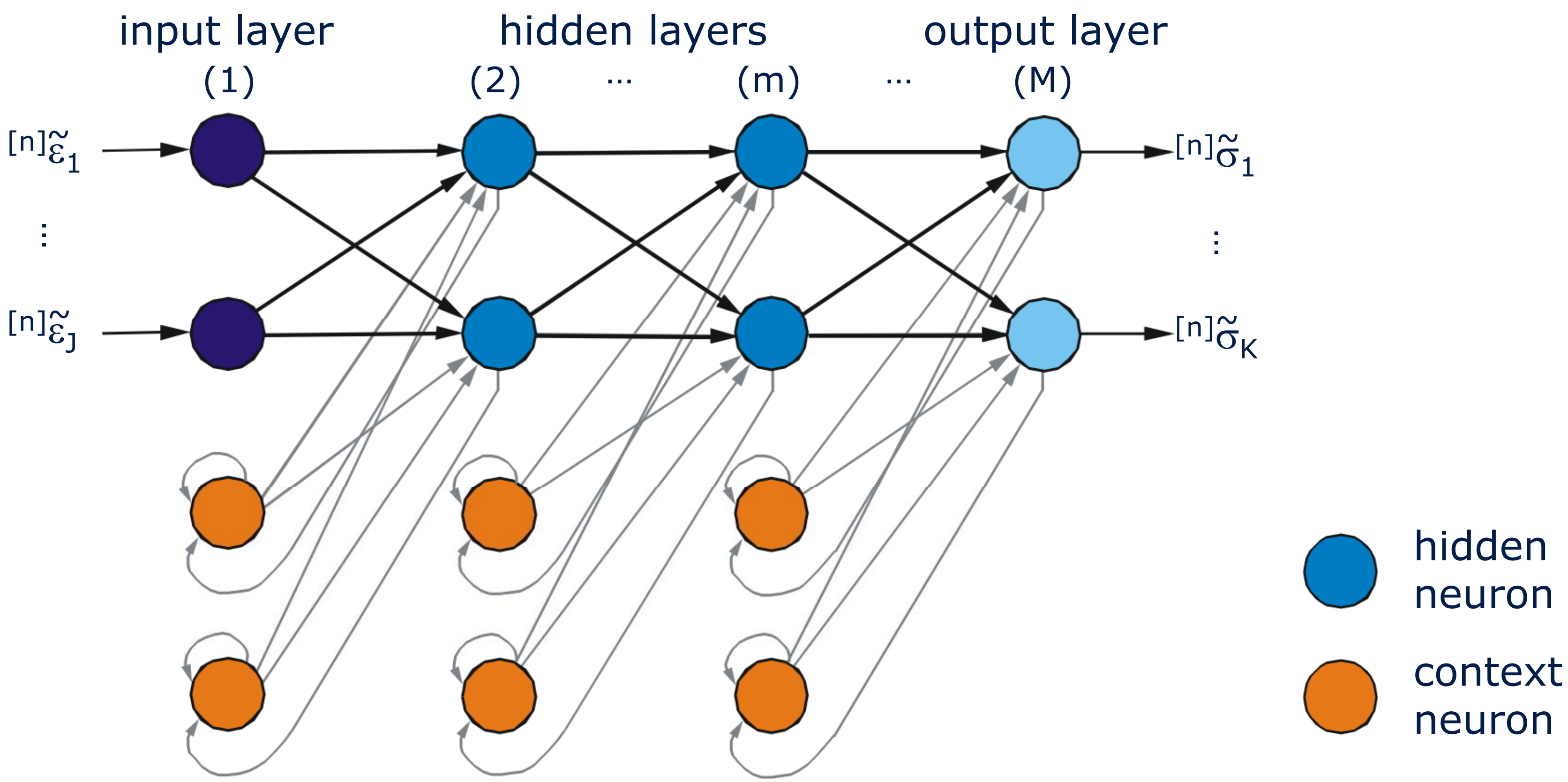
$$ {}^{(r+1)}_{sr}a^i_q \le \min\left[\left({}^{(r+1)}_{sm}a^i_q - {}^{(r+1)}_{s-1l}a^i_q \right), \left({}^{(r+1)}_{s-1u}a^i_q - {}^{(r+1)}_{sm}a^i_q \right) \right] $$

$$ {}^{(r)}_{sr}\Delta a^i_q = c_3 \cdot {}^{(r-1)}_{sr}\Delta a^i_q + c_1 \cdot d \cdot \left({}_{sr}p^i_q - {}^{(r)}_{sr}a^i_q \right) + c_2 \cdot e \cdot \left({}_{sr}g_q - {}^{(r)}_{sr}a^i_q \right) $$

Feed forward neural networks for interval or fuzzy data



Recurrent neural networks for interval or fuzzy data



- interval analysis or α-level optimization

FE analysis – incremental formulation

- nonlinear system of equations

$$^{[n]}\underline{\tilde{K}}_T \cdot {}^{[n]}\Delta\underline{\tilde{v}} = {}^{[n]}\Delta\underline{\tilde{f}}$$

$^{[n]}\underline{\tilde{K}}_T$ uncertain tangential system stiffness matrix
$^{[n]}\Delta\underline{\tilde{v}}$ uncertain incremental nodal displacements
$^{[n]}\Delta\underline{\tilde{f}}$ uncertain incremental nodal forces

- iterative solution

- uncertain tangential element stiffness matrix

$$^{[n]}\underline{\tilde{K}}_T^e = \int_{\tilde{V}} {}^{[n]}\underline{\tilde{B}}^T \cdot {}^{[n]}\underline{\tilde{C}} \cdot {}^{[n]}\underline{\tilde{B}} \, d\tilde{V}$$

$^{[n]}\underline{\tilde{B}}$ uncertain gradient matrix
$^{[n]}\underline{\tilde{C}}$ uncertain tangential stiffness matrix at material point

Tangential stiffness

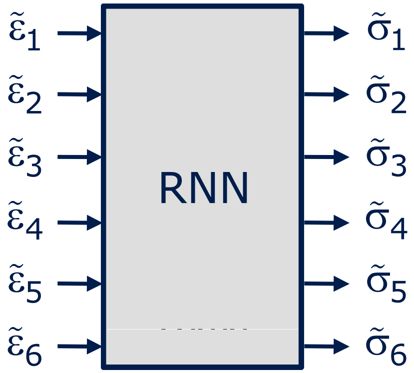
- tangential stiffness matrix at material point – linearized form

$$ {}^{[n]}\tilde{C}_{kj} = \frac{\partial^{[n]}\Delta\tilde{\sigma}_k}{\partial^{[n]}\Delta\tilde{\varepsilon}_j} = \frac{\partial^{[n]}\Delta\tilde{\sigma}_k}{\partial^{[n]}\tilde{x}_k^{(M)}} \cdot \frac{\partial^{[n]}\tilde{x}_k^{(M)}}{\partial^{[n]}\tilde{x}_j^{(1)}} \cdot \frac{\partial^{[n]}\tilde{x}_j^{(1)}}{\partial^{[n]}\Delta\tilde{\varepsilon}_j} = \sigma_k^{sc} \cdot \frac{\partial^{[n]}\tilde{x}_k^{(M)}}{\partial^{[n]}\tilde{x}_j^{(1)}} \cdot \frac{1}{\varepsilon_j^{sc}} $$

derivative of k-th output signal with respect to j-th input signal

- special network structures

symmetric



nonlinear anisotropic

$$ {}^{[n]}\underline{C} = \begin{bmatrix} \tilde{c}_1 & \tilde{c}_2 & \tilde{c}_3 & \tilde{c}_4 & \tilde{c}_5 & \tilde{c}_6 \\ \tilde{c}_2 & \tilde{c}_7 & \tilde{c}_8 & \tilde{c}_9 & \tilde{c}_{10} & \tilde{c}_{11} \\ \tilde{c}_3 & \tilde{c}_8 & \tilde{c}_{12} & \tilde{c}_{13} & \tilde{c}_{14} & \tilde{c}_{15} \\ \tilde{c}_4 & \tilde{c}_9 & \tilde{c}_{13} & \tilde{c}_{16} & \tilde{c}_{17} & \tilde{c}_{18} \\ \tilde{c}_5 & \tilde{c}_{10} & \tilde{c}_{14} & \tilde{c}_{17} & \tilde{c}_{19} & \tilde{c}_{20} \\ \tilde{c}_6 & \tilde{c}_{11} & \tilde{c}_{15} & \tilde{c}_{18} & \tilde{c}_{20} & \tilde{c}_{21} \end{bmatrix} $$

Tangential stiffness

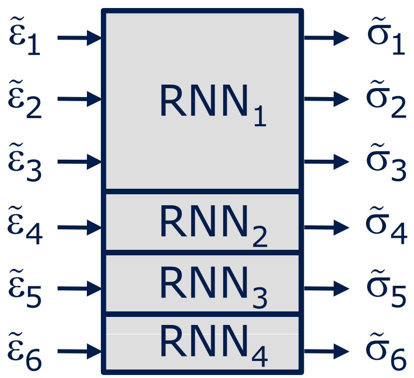
- tangential stiffness matrix at material point – linearized form

$$ {}^{[n]}\tilde{C}_{kj} = \frac{\partial^{[n]}\Delta\tilde{\sigma}_k}{\partial^{[n]}\Delta\tilde{\varepsilon}_j} = \frac{\partial^{[n]}\Delta\tilde{\sigma}_k}{\partial^{[n]}\tilde{x}_k^{(M)}} \cdot \frac{\partial^{[n]}\tilde{x}_k^{(M)}}{\partial^{[n]}\tilde{x}_j^{(1)}} \cdot \frac{\partial^{[n]}\tilde{x}_j^{(1)}}{\partial^{[n]}\Delta\tilde{\varepsilon}_j} = \sigma_k^{sc} \cdot \frac{\partial^{[n]}\tilde{x}_k^{(M)}}{\partial^{[n]}\tilde{x}_j^{(1)}} \cdot \frac{1}{\varepsilon_j^{sc}} $$

derivative of k-th output signal with respect to j-th input signal

- special network structures

partially connected, RNN_1 symmetric



nonlinear orthotropic

$$ {}^{[n]}\underline{C} = \begin{bmatrix} \tilde{c}_1 & \tilde{c}_2 & \tilde{c}_3 & 0 & 0 & 0 \\ \tilde{c}_2 & \tilde{c}_4 & \tilde{c}_5 & 0 & 0 & 0 \\ \tilde{c}_3 & \tilde{c}_5 & \tilde{c}_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{c}_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{c}_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{c}_9 \end{bmatrix} $$

Tangential stiffness

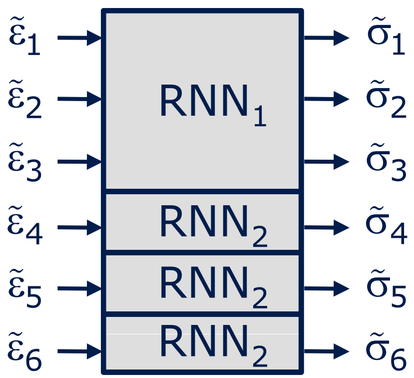
- tangential stiffness matrix at material point – linearized form

$$ {}^{[n]}\tilde{C}_{kj} = \frac{\partial^{[n]}\Delta\tilde{\sigma}_k}{\partial^{[n]}\Delta\tilde{\varepsilon}_j} = \frac{\partial^{[n]}\Delta\tilde{\sigma}_k}{\partial^{[n]}\tilde{x}_k^{(M)}} \cdot \frac{\partial^{[n]}\tilde{x}_k^{(M)}}{\partial^{[n]}\tilde{x}_j^{(1)}} \cdot \frac{\partial^{[n]}\tilde{x}_j^{(1)}}{\partial^{[n]}\Delta\tilde{\varepsilon}_j} = \sigma_k^{sc} \cdot \frac{\partial^{[n]}\tilde{x}_k^{(M)}}{\partial^{[n]}\tilde{x}_j^{(1)}} \cdot \frac{1}{\varepsilon_j^{sc}} $$

derivative of k-th output signal
with respect to j-th input signal

- special network structures

partially connected, RNN_1 double symmetric



nonlinear isotropic

$$ {}^{[n]}\underline{C} = \begin{bmatrix} \tilde{c}_1 & \tilde{c}_2 & \tilde{c}_2 & 0 & 0 & 0 \\ \tilde{c}_2 & \tilde{c}_1 & \tilde{c}_2 & 0 & 0 & 0 \\ \tilde{c}_2 & \tilde{c}_2 & \tilde{c}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{c}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{c}_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{c}_3 \end{bmatrix} $$

Linear elastic material model – Type 1 mapping

- system of equations $\tilde{\underline{\sigma}} = \underline{C} \cdot \tilde{\underline{\varepsilon}}$
- parameters for fuzzy analysis

$E = 20000\,\text{MPa}$ $\upsilon = 0.2$

- tangential stiffness matrix

$$\underline{C} = \begin{bmatrix} 22222 & 5556 & 5556 & 0 & 0 & 0 \\ 5556 & 22222 & 5556 & 0 & 0 & 0 \\ 5556 & 5556 & 22222 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8333 \end{bmatrix} \text{MPa}$$

Linear elastic material model – Type 1 mapping

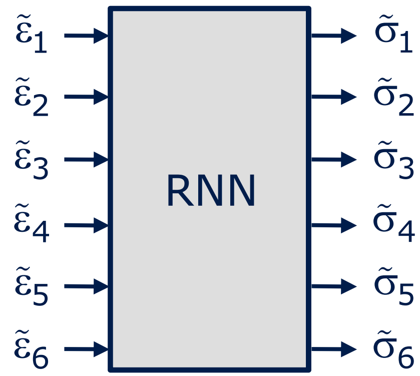
- recurrent neural network architecture

$\underline{\tilde{\varepsilon}}(\tau) \longrightarrow \underline{\tilde{\sigma}}(\tau)$ **6** – 6 – **6**

- randomized fuzzy processes with 100 time steps for network **training**

- tangential stiffness matrix **training** (mean values)

$$\underline{C}^{Tr} = \begin{bmatrix} 21119 & 5535 & 5587 & 351 & -237 & -86 \\ 5017 & 21708 & 5828 & -189 & 22 & 4 \\ 5458 & 5490 & 22073 & -58 & 163 & -132 \\ 217 & 63 & -19 & 7947 & 61 & -16 \\ -193 & 53 & -36 & -57 & 8147 & 44 \\ 72 & -19 & -33 & -147 & 25 & 8195 \end{bmatrix} \text{MPa}$$



Linear elastic material model – Type 1 mapping

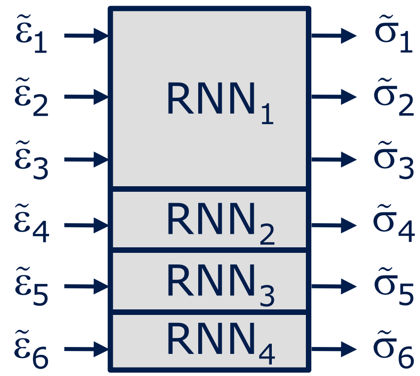
- recurrent neural network architecture

$\tilde{\underline{\varepsilon}}(\tau) \longrightarrow \tilde{\underline{\sigma}}(\tau)$ **6** – 6 – **6**

- randomized fuzzy processes with 100 time steps for network **training**

- tangential stiffness matrix **training** (mean values)

$$\underline{C}^{Tr} = \begin{bmatrix} 21469 & 5561 & 5475 & 0 & 0 & 0 \\ 5561 & 21764 & 5483 & 0 & 0 & 0 \\ 5475 & 5483 & 21424 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8298 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8210 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8195 \end{bmatrix} \text{MPa}$$



Linear elastic material model – Type 1 mapping

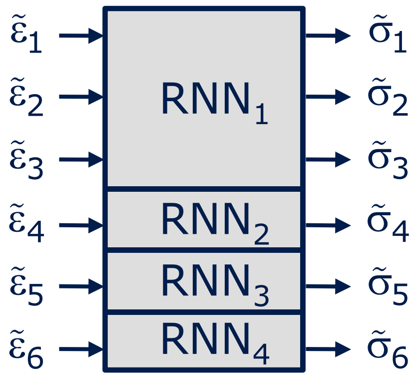
- recurrent neural network architecture

$\tilde{\underline{\varepsilon}}(\tau) \longrightarrow \tilde{\underline{\sigma}}(\tau)$ **6** – 6 – **6**

- randomized fuzzy processes with 100 time steps for network **validation**

- tangential stiffness matrix **validation** (mean values)

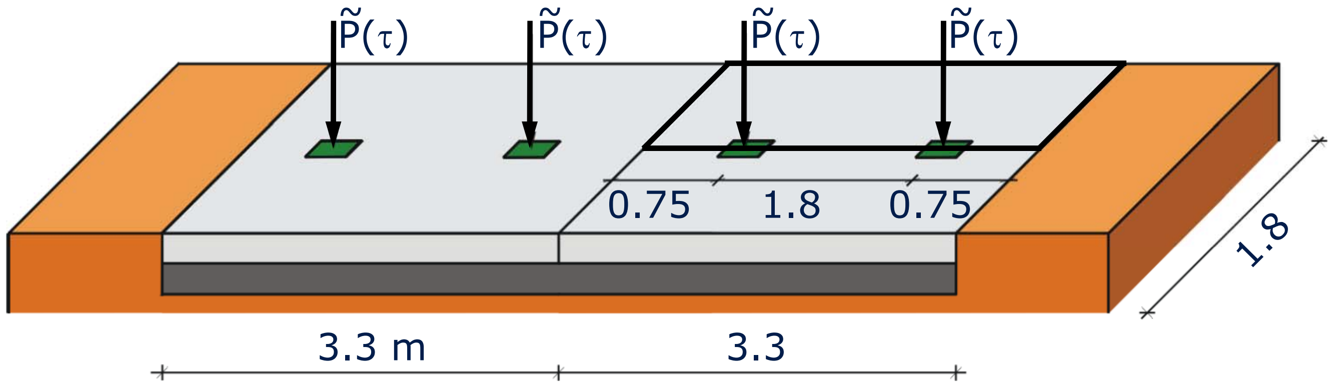
$$\underline{C}^{V} = \begin{bmatrix} 21748 & 5684 & 5551 & 0 & 0 & 0 \\ 5684 & 21862 & 5535 & 0 & 0 & 0 \\ 5551 & 5535 & 21674 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8298 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8226 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8205 \end{bmatrix} \text{MPa}$$



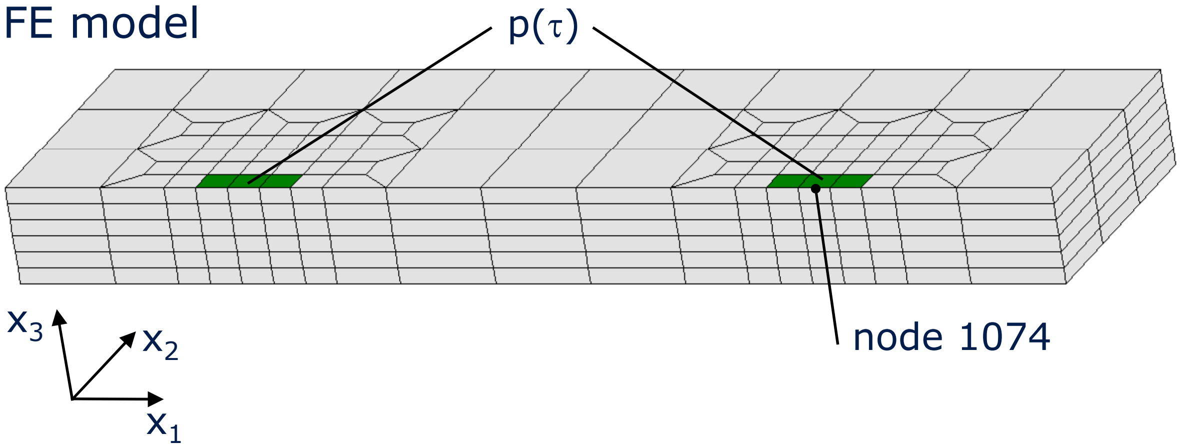
- history independence of fuzzy stress processes identified

Pavement structure

- geometry and loading

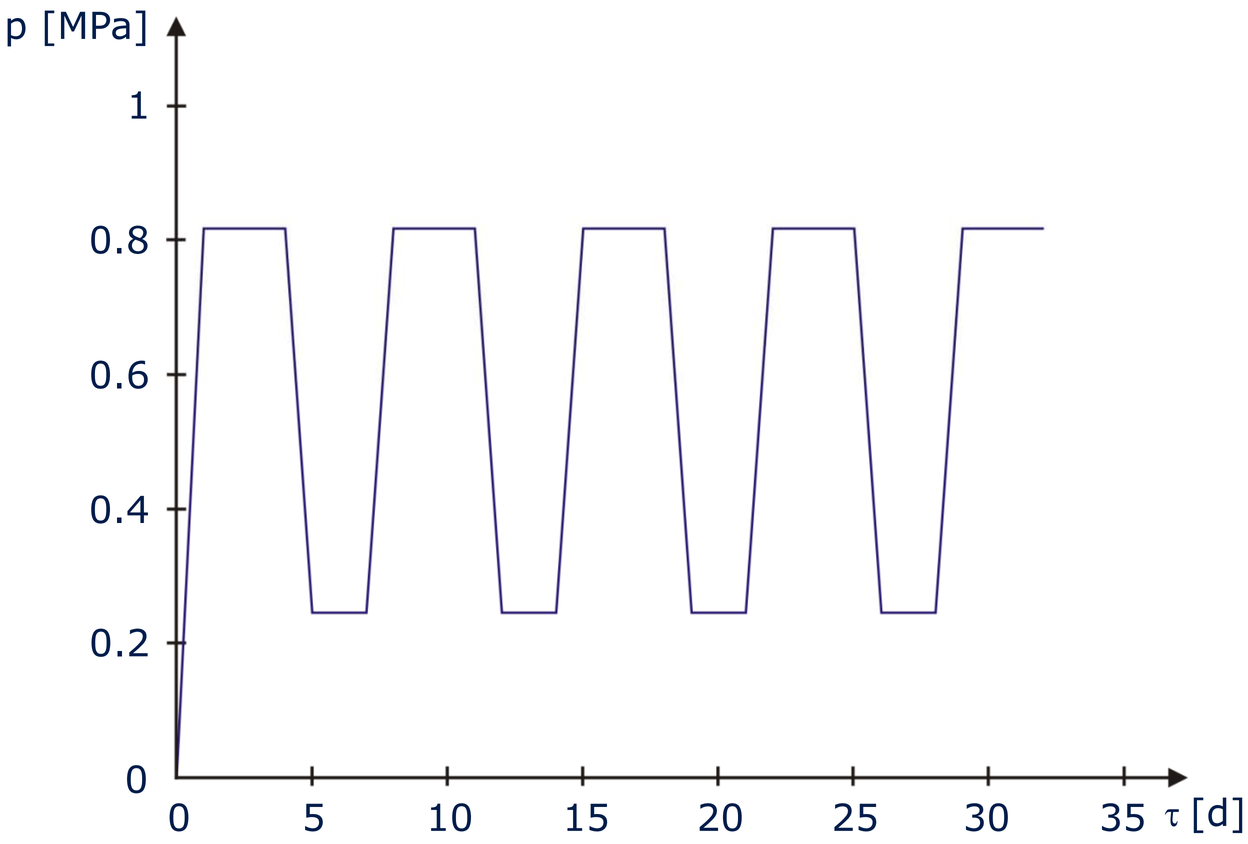


- FE model



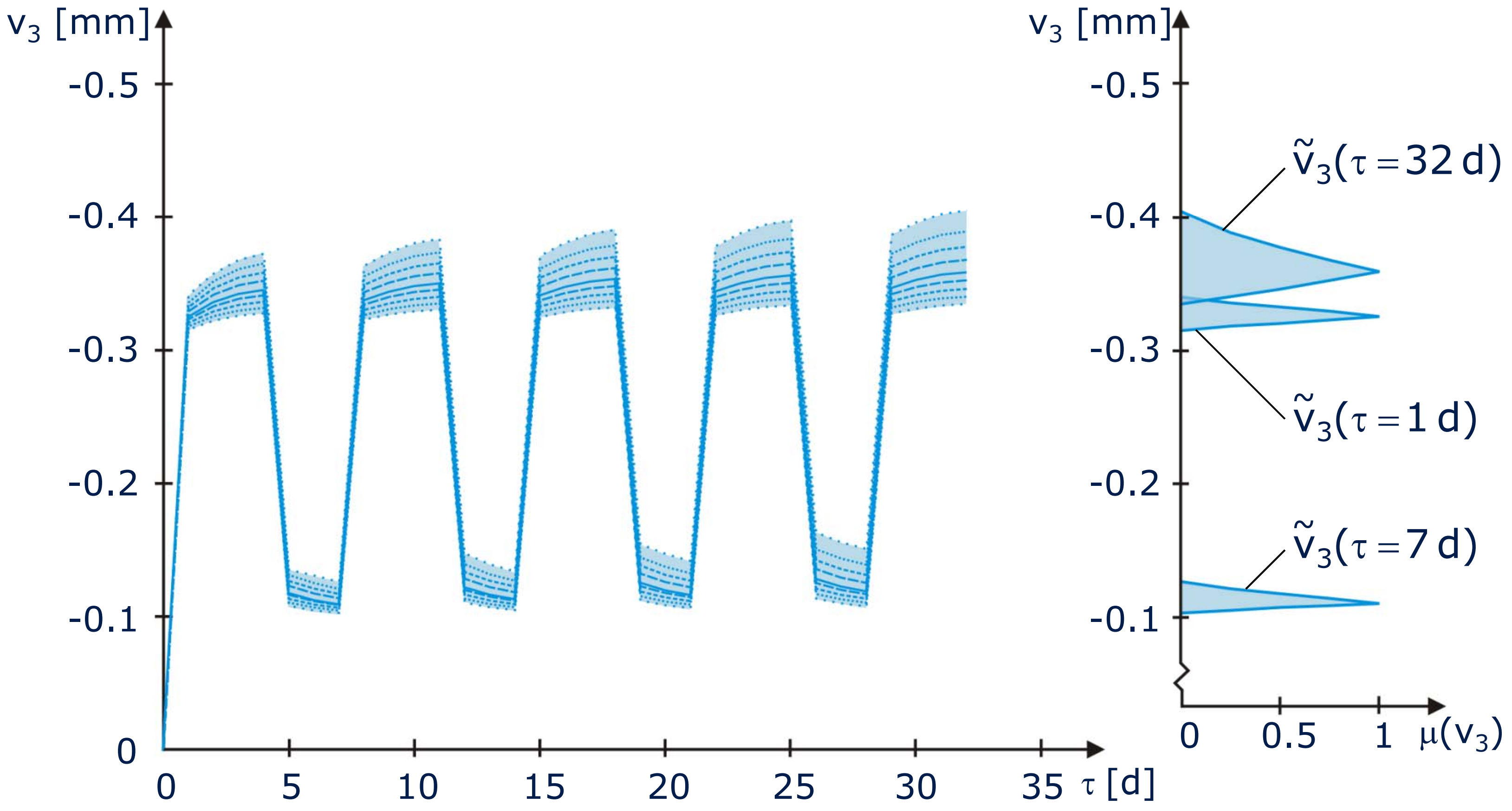
Pavement structure

- deterministic load process



Pavement structure

- fuzzy displacement process $\tilde{v}_3(\tau)$ of node 1074



Conclusion

- material description with artificial neural networks for interval or fuzzy data
- PSO for identification of uncertain network parameters
- application as material formulation in fuzzy or fuzzy stochastic FE analyses

Outlook

- indirect training with inhomogeneous stress and strain fields
- extension for coupled multi field simulations
- application for dynamical structural analyses