Gamma Process –Life cycle analysis of the Neumarkt Bridge, IT

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Abstract: In the maintenance management of infrastructures, stochastic processes as for example the Gamma process are gaining importance in methods for performance prediction. Gamma process approaches can be based on classical information gained from visual inspections, as for example crack formation, bending, but also the development of near surface strains (stresses), and enable the development of forecasting models as an effective decision making basis for the optimization of inspection intervals and maintenance measures. In this contribution, which is based on the material properties during and after the demolition of pre-stressed concrete bridges, we example the possibility of the Gamma process approach (in relation to visual inspections) to capture the internal mechanical changes, caused for example by pre-stressing steel corrosion processes. A pronounced correlation between the gamma process approach and the internal mechanical properties of structure are bases for (a) a quantitatively well ascertained remaining service life, (b) optimization of inspection periods, (c) identification of critical structural components for the overall condition and consequently (d) cost-efficient maintenance.

Keywords: Gamma process, Lifecycle analysis, aging of concrete structures

1. Introduction

Numerous highly developed industrial countries have pronouncedly different infrastructure management systems, which are there to ensure the safety levels demanded by the standards of their national economies. Every engineering structure is discrete and unique. The result is a deluge of data sets generated by these infrastructure management systems, which in turn leads to an overload of information that complicates the decision making process for the respective owners instead of facilitating it.

What is even more relevant, in the last few decades a wide range of modern monitoring technologies and numerical methods have been developed, ranging from novel sensor monitoring systems to routine video-imaging techniques. These developments make it possible to reform established infrastructure inspection methods and to bring them in line with a wide range of high-quality, site specific data based on physical and technical realities. Consequently these developments serve also to improve and adapt the inspection routines resulting in time-efficient and systematic optimization of those maintenance and management systems.

The present paper presents an approach for effective adaptive maintenance management employing additional information gained from new methods of inspection and analysis, as for example the simulation of deterioration process.

Generally speaking, the safety levels of all concrete structures decreases continuously as a result of aging processes caused by environment-induced mechanical and chemical loads. Today many countries prescribe certain specifications and norms (Schmidt and Sondermann, 2006; 2007; 2011) in order to control

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the progressive development of these processes and to plan efficient maintenance measures. These specifications and norms standardize the regular examination of structures with adequately defined inspection intervals and clear specifications of the scope of those inspections. In line with the current practice, most examination programs based on a visual evaluation of structural components and accessories are performed by qualified personnel, whose qualification admittedly corresponds to the subjective international grades (Schmidt, 2008). As a general rule, it can be said that they deliver only conditioned exploration about the actual bearing capacity and the serviceability of the structure. Further estimates of the remaining service life of a structure are usually based exclusively on experience, depending on the assessment of the concrete surface condition.

In Austria an inspection routine which is subdivided into inspection, continuing monitoring and examination is the norm. These performance examinations can be carried out either at definite time intervals or after the occurrence of specific circumstances and events. For the general evaluation of the structure, a separate examination will be assigned. Despite all the advantages of these historically developed inspection routines, the following flaws of the system are listed below to raise the awareness:

- The quality of the data gained from inspections may be (partly) faulty or unsuitable for a systematic filling.
- Only in rare cases are inspection data suitable for a quantitative analysis.
- Alternative inspection methods for lifecycle assessment need to be included.
- Quantitative information regarding the efficiency of various inspection techniques is not available in most cases.
- The transfer of the examination results into the optimization of maintenance and repair activities is at best be carried out by modeling random variables.
- The number of structure specific inspection data, crucial for a comprehensive evaluation is very limited.

This particular study focuses on the specific degradation process that was observed in a damaged bridge and simulated by the application of a Cellular Automata approach compared with the gamma process approach. Specifically, the gamma process approach examined structural behavior, like crack formation, bending, and surface strain (stress development), which can be captured by traditional inspection and/or monitoring method. By employing material analysis after the demolition of the examined Neumarkt-bridge in South Tyrol, we were able to examine the degradation process due to corrosion and carbonation of prestressing in very deep detail.

Consequently, this contribution introduces the possibility of the gamma process approach (in relation to visual inspections) as a means of capturing the internal mechanical changes, for example due to prestressing steel corrosion processes. A pronounced correlation between the gamma process approach and the internal mechanical properties of a structure provides the basis for (a) a quantitatively well ascertained remaining service life, (b) the optimization of inspection periods, (c) the identification of critical structural components for the overall condition and consequently (d) cost- efficient maintenance.

2. Gamma Process Approach

Degradation and aging processes of a structure can be described by non-negative and continuous functions. These functions can be characterized by non-negative increments with independent path and variable uncertainty. Furthermore the designated period of time until the occurrence of the observation of this undesirable incidence is mostly related to a considerable uncertainty and dependent on structural behavior.

Stochastic aging processes of mechanical components are frequently determined via the gamma process which is suitably applied in the analysis of civil engineering structures. Frangopol et al & Noortwijk recommended this approach for the assessment of deterioration processes and analyzed its suitability for engineering structures.

2.1. CHARACHTERISTICS OF A STOCHASTIC PROCESS

The reliability of assessment methods for engineering structures such as bridges, are usually based on assumptions of incomplete information on for instance, the material properties, the quality of the construction and the relevant loads. As a result of this insufficient information, the life time distributions which are derived from a very low failure rate are often contradictory and cannot be formulated accurately. Consequently, in structural engineering in general time dependent and highly uncertain properties/processes, such as an average deterioration per unit time are often considered as random quantities. For this, the suitably applicable process is the class of Markov processes as a class of stochastic processes which represents independent increments. Markov processes enable discrete and continuous processing (Noorwijk, 2009). For example Brownian motion with drift, Poisson Levy and Gamma process differ. The discrete stochastic model is, generally speaking, not suitable for determining the deterioration process in the field of engineering. Consequently, continuous gamma processes, as analyzed by Pandey et al are much better suited to this end. VanNoorwijk proved suitability of gamma processes illustrating the continuous stochastic process by which the temporal damage accumulation can be represented by small independent increments. In particular the essential positive increments are determined by a gamma distribution with identical scale parameter and a time dependent shape function. Consequently with this type of process, deteriorations such as wear, fatigue, creep, cracking corrosion etc. can be determined. Furthermore the gamma process offers the advantage of providing a relatively simple mathematical description.

In the gamma process modeling, we observe at the first step a random variable X with a Gamma distribution, which is characterized by the shape parameter $\alpha > 0$and the scale parameter $\beta > 0$.

From the above we note that sin $\theta = (x + y)z$ or:

$$Ga\left(x|\alpha,\beta\right) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \cdot x^{\alpha-1} \cdot \exp\left(-\beta \cdot x\right), \tag{1}$$

where

$$\Gamma\left(\alpha\right) = \int_{z=0}^{\infty} z^{\alpha-1} \cdot e^{-z} \cdot \mathrm{d}z \tag{2}$$

is the Gamma function for $\alpha > 0$

2.2. MODELING OF GAMMA PROCESSES

Gamma process distributions Ga are for different time variables are independent of each other. As a result it is possible to obtain conditional distribution of variable X only on the basis of current observations which are well suited to represent the degradation process of standard structures. Such types of deterioration prediction models of a given structure based on the brief time period of observation, should consider the current status and the past events that preceded the current state. In the prediction models, past deterioration profiles can be used to acquaint the engineer with relevant information. In other words, it should be possible to integrate historical deterioration profiles. However, their knowledge should not be a prerequisite.

The parameter $\alpha(t)$ is a non-decreasing, right continuous with left limits, real valued function for $t \geq 0$, with $\alpha(0) = 0$. The gamma process with shape function $\alpha(t) > 0$ and scale parameter $\beta > 0$ is a continuous-time stochastic process $\{X(t), t \geq 0\}$ with the following properties:

$$\mathrm{P}\big(X(0)=0\big)=1, \tag{3}$$

$$X(\eta)-X(t)=Ga\big(\alpha(\eta)-\alpha(t),\beta\big) \quad \text{for} \quad \eta > t \geq 0, \tag{4}$$

where $X(t)$ is characterized by independent increments. Thus the corresponding probability distribution function of $X(t)$, with the time variable t, in accordance with the characteristics of the gamma process is defined as follows:

$$f_{X(t)}(x)=Ga\big(x,\alpha(t),\beta\big). \tag{5}$$

The corresponding expected value:

$$E\big(X(t)\big)=\frac{\alpha(t)}{\beta} \tag{6}$$

and variance

$$Var\big(X(t)\big)=\frac{\alpha(t)}{\beta^2}. \tag{7}$$

Finally the time variable coefficient of variation is obtained from the ratio of the standard deviation and mean value as follows:

$$COV\big(X(t)\big)=\frac{\sqrt{Var\big(X(t)\big)}}{E\big(X(t)\big)}=\frac{1}{\sqrt{\alpha(t)}}. \tag{8}$$

2.3. GAMMA PROCESS MODELING OF DETERIORATION PROCESSES

The following power law formulation is often suitable for the shape function of degradation gamma processes

$$\alpha(t)=c\cdot t^b. \tag{9}$$

This is a standard formulation, which has a linear shape ($b = 1$) for the corrosion of concrete reinforcement, a parabolic ($b = 2$) for the sulphate attack, and a square root ($b = 0.5$) for the diffusion-controlled aging according to Elingwood and Mori. The deterioration rate $X(t)$ at the time t, with $t \geq 0$ can simply be described by the shape function $\alpha(t) = ct^b$ and the scale parameter β. In engineering, in most cases the shape of the expected deterioration b is known and can be taken as a constant, as pointed out by VanNoortwijk et al. However the random rate of degradation C and the scale parameter β are unknown and need to be determined by seeking experts' advice or applying statistical methods. The three most suited statistical methods are the Maximum Likelihood Method, the Method of Moments and Bayesian Statistics (VanNoortwijk, 2009). The determination of the population parameter by means of statistical sample moments is the simplest approach and provides in general a very good result for the first approximation. Provided that the main parameters, the expected value and the variance of the cumulative deterioration at time t are known, the non-stationary gamma process can be transformed to a stationary gamma process. This can be achieved by means of a monotonic transformation from the real time to the operational time as follows:

$$z(t) = t^b \cdot t(z) = z^{\frac{1}{b}}. \tag{10}$$

Thus the expected value is given as:

$$E\big(X\big(t(z)\big)\big) = \frac{c \cdot z}{\beta}, \tag{11}$$

and the equation of the variance as follows:

$$Var\big(X(t)\big) = \frac{c \cdot z}{\beta^2}. \tag{12}$$

Likewise, the transformation of the inspection times can be carried out, $z_i = t_i^b$, for $i = 1,\ldots,n$. The inspection interval between two inspection times is given as:

$$w_i = t_i^b - t_{i-1}^b \tag{13}$$

and

$$\gamma_i = X_i - X_{i-1} \tag{14}$$

as proposed by Van Noortwijk. The degradation increments γ_i have an approximate gamma distribution with a shape factor cwi and a scale parameter β for all $i = 1,2, \ldots, n$. The Method of Moments recommended by Van Noortwijk to estimate the parameters $\hat{c}$ and $\hat{\beta}$ is given by the following formulation:

$$\frac{\hat{c}}{\hat{\beta}} = \frac{\sum_{i=1}^{n} \gamma_i}{\sum_{i=1}^{n} w_i} = \frac{x_n}{t_n^b} \tag{15}$$

and with

$$\frac{\hat{c}}{\hat{\beta}} = \overline{\gamma} \cdot \frac{x_n}{\hat{\beta}} \cdot \left(1 - \frac{\sum_{i=1}^{n} w_i^{\,2}}{\left[\sum_{i=1}^{n} w_i\right]^2}\right) = \sum_{i=1}^{n} \left(\gamma_i - \overline{\gamma} \cdot w_i\right)^2. \tag{16}$$

The Method of Moments leads to a simplified formulation of parameter estimation and can be applied for the first estimation of the solutions of the probability equations. The intervals w_i can be the shorter or longer periods between the main inspections This can be of particular interest for the optimized selection of inspection techniques and inspection intervals.

3. Case Study: Neumarkt Bridge

Gamma processes, as it is mentioned in the previous sections, are very suitable for the characterization and capture of information from the visual inspections as well as from monitoring systems. There is substantial interest to use this information for an efficient analysis and assessment of the mechanical changes in the structure, wherever no real solution statements can be provided.

In the present case study, a precast element bridge in South Tyrol was evaluated following RVS guideline. As part of the study we examined concrete, prestressed and reinforcement steels for signs of corrosion, general degradation processes and their correlations, before and during the demolition of the bridge. The changes within the mechanical systems, as for example the changes in cross section of the reinforcement and/or presstressing, were monitored during the last decades of the structure using Cellular Automata analysis time dependent reliability analysis and for comparison, nonlinear model. The structural responses provided by the nonlinear model analysis were compared with the result the gamma process analysis.

3.1. Geomerty of the Neumarkt Bridge

The Neumarkt Bridge, a three-span bridge constructed from precast elements, crosses the A22, the Italian section of the Brenner highway, between the provincial towns of Neumarkt and Auer in South Tyrol. It exhibits features typical for the region's bridge design. The four V-shaped precast elements mounted side by side carry a thin concrete slab. In Figure 1 the most important dimensions of the bridge are illustrated with regard to elevation and transverse section. The main span of the bridge is 27.0 m long, and the outer spans are 9.14 m each. Each V-shaped girder of the main span was mounted with joints in the longitudinal axis as well as in the transverse axes. The girders were secured against side swaying by a 0.14 m strong concrete slab. As a result, the structure is identified as orthotropic with main parts of V-shaped girders. The bridge has two traffic lanes, each 3.75 m of wide and two sidewalks with a width of 1.0 m each.

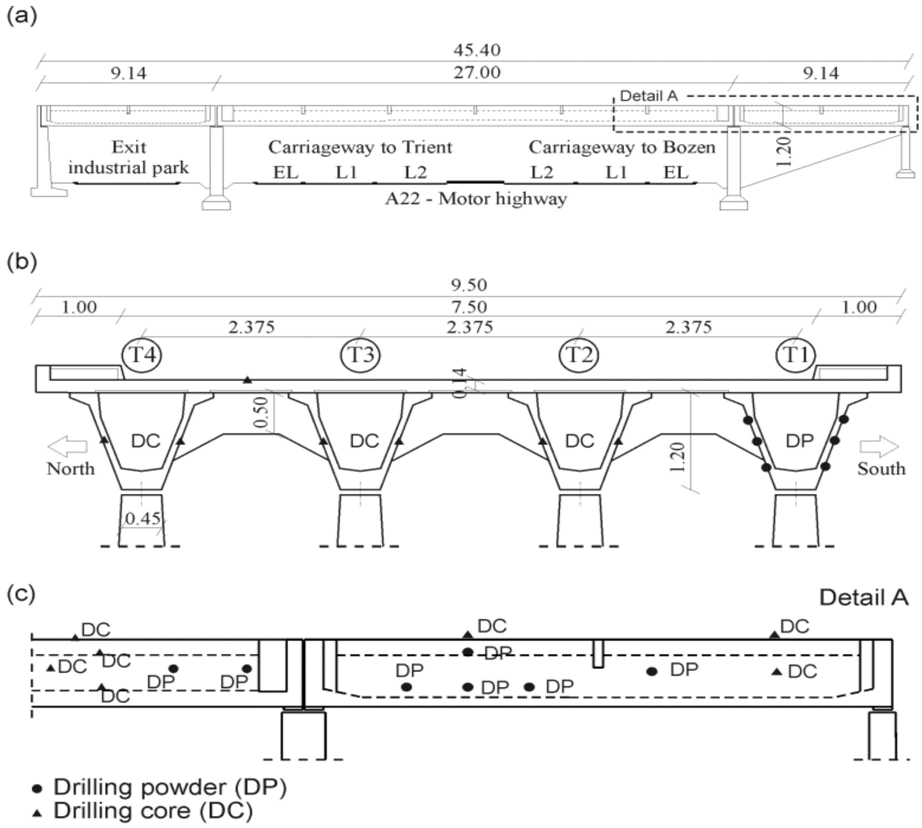

Figure 1. Geometry of the Neumarkt Bridge in South Tyrol: (a) longitudinal section, (b) section of the V-shaped girders of precast elements, and (c) sampling plan

Table I. Time variable statistical description values of pitting corrosion induced by prestressing

t_P 1) [Year]	Corrosion depth [mm] and/or A/A_o		Remaining cross sectional area of one of the lower prestressing wire, $A_r(t)$ [mm²] PDF	Mean	Hrs	COV
5 (3.5) 2)	0.35	0.95	LN	126.54	5.07	0.040
10 (7.0)	0.70		LN	125.98	5.07	0.040
15 (10.0)	1.05	0.80	LN	125.06	5.07	0.041
20 (13.5)	1.39		LN	123.83	5.06	0.041
25 (17.0)	1.74		LN	122.25	5.06	0.041
30 (20.5)	2.09	0.60	LN	120.35	5.06	0.042
35 (24.0)	2.44		LN	118.14	5.05	0.043
40 (27.5)	2.78		LN	115.74	5.04	0.044
45 (31.0)	3.13		LN	112.96	5.03	0.045
50 (34.5)	3.50		LN	109.75	5.02	0.046

1) propagation time after initiation of corrosion
2) propagation time due to corrosion in 3 layers

The condition evaluation of the Neumarkt Bridge, taking into consideration the structural degradation was divided into the discrete steps of data acquisition, analysis simulation of chloride contamination and estimation of time dependent steel corrosion. In addition, the computation of the structural response, taking into account the expected steel section reduction and finally the estimation of the resulting safety level and/or prediction of remaining service life were performed.

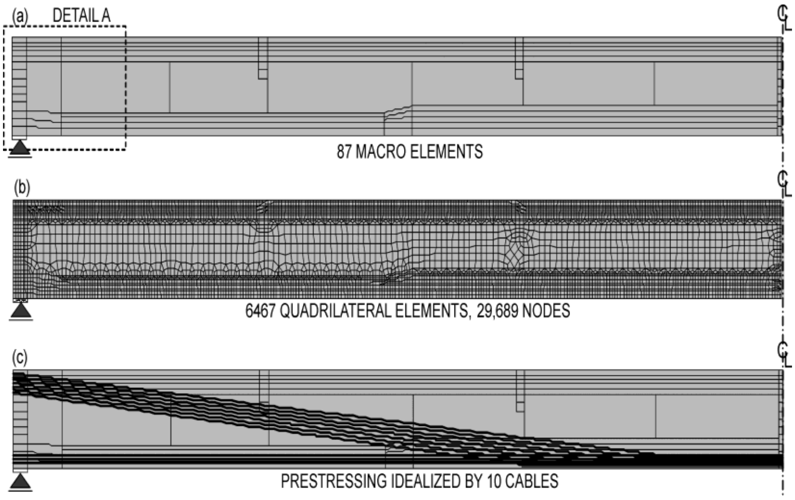

Figure 2. Finite element model to evaluate the SLS and ULS in the center of the beam for a cross section reduction of the prestressing wire in the center line (CL), (a) structuring of makro-elements, (b) FEM mesh generation, und (c) tendon layout of V- girders.

Table II. Limit states of the nonlinear model analysis

Limit state	Action S	Load Comb.	Barrier R	Unit
Material associated limit states				
G_1	Concrete comp. stress σ_c	QP	$0.45\,f_{ck}$	MPa
G_2	Concrete comp. stress σ_c	C	$0.60\,f_{ck}$	MPa
G_3	Mild steel stress σ_a	C	$0.80\,f_{yk}$	MPa
G_4	Mild steel stress σ_a	C	$1.00\,f_{yk}$	MPa
G_5	Pre-stressing steel stress σ_p	C	$0.75\,f_{pk}$	MPa
Deformation associated limit states				
G_6	Vertical deflection u	QP	$l/500$	mm
G_7	Vertical deflection u	FC	$l/250$	mm
G_8	Crack width w	QC	0.2	mm
G_9	Crack width w	C	0.3	mm

QP = Quasi permanent combination
C = Characteristic combination
FC = Frequent combination
f_{ck} = Characteristic concrete compressive strength
f_{yk} = Characteristic yield strength (reinforcement steel)
f_{pk} = Characteristic yield strength (prestressing steel)
l = Span width

The actual implementation of the steps outlined above was performed using the software package SARA, a program that allows the simulation of time dependent chloride ingression (CATES) and the FREET-D program (FREET-D, Teply et al., 2006) which was used to describe the degradation processes as for instance steel corrosion and carbonization induced by controlled inputs. The evaluation of the structural response for the degraded structure was performed via the nonlinear finite element software environment ATENA (Cervenka et al., 2011) on the basis of the fracture mechanical methods, where the generation of inputs and the evaluation of the limit state equation are accomplished by FREET (Novak et al., 2008).

The whole process predicting the chloride concentration up to reliability level for the discrete time t_i was carried out several times. Table I shows the corrosion progress from the time of corrosion occurrence in the lower prestressing position achieved by Cellular Automata Simulation. Accordingly a nonlinear reliability analysis was conducted for the limit states indicated in Table II in accordance with the FEM-Model shown in Figure 2.

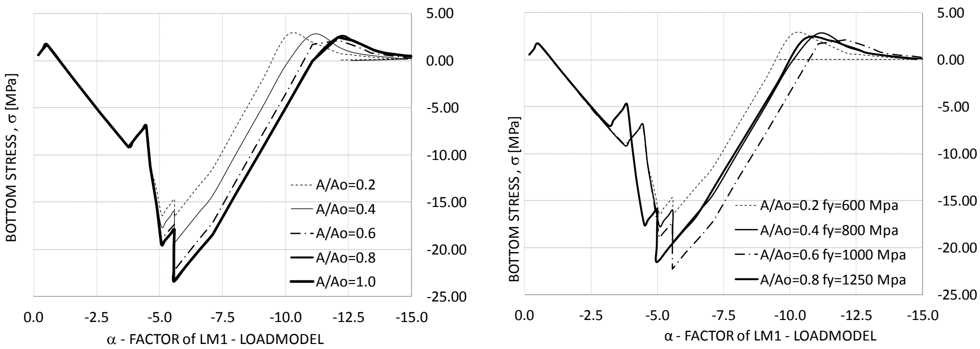

Figure 3. The course of the lower concrete stress- load model for the prestressing steel section lose A/A_0 in the center of the beam (a) without and (b) with the consideration of the yield strength f_y reduction

The nonlinear probabilistic FEM analysis delivers on one hand the continuous process of the structural response with regard to the load application for different degree of deterioration, as shown for example in Figure 3 where the process of monitored concrete stress on the V-girder lower side in the center of the beam is illustrated and in Figure 4 where the process of the monitored bending in the center of the beam is shown. On the other hand the statistical characteristics of the structural response and also the probability of failure with regard to the defined limit states for the current and also for the future conditions (see Table III) need to be taken into consideration. These statistical structural responses can be captured by means of visual inspection and/or monitoring system and are consequently a link between the gamma process based description of the deterioration process and the assessment of the structural mechanical changes.

3.2. GAMMA PROCESS BASED CONDITION ASSESSMENT

For the lifetime condition assessment and the illustration of the time dependent structural deterioration, the evolution or progression of deterioration over time is modeled by gamma processes. In the following structural life time response modeling in relation to deflection is conducted for 80% of the LM1 load model. Throughout the analysis the independent deterioration increments are characterized by gamma distribution function with different shape and scale parameters. As a result, the deterioration profile at different ages of the structure was defined and visualized. The gamma process computation was conducted for predictions that were based on inspections at the ages of 30 and 50. It may be worth mentioning that any estimation of parameters during the early years is best established by experts, as the Method of Moments provides unreliable results in the early lifetime of a structure. Table IV illustrates the gamma process prediction starting with the age of 30.

Table III. Statistical characteristics of the structural response and the corresponding safety levels for a) serviceability limit states and b) ultimate bearing capacity evaluated for 60%, 83% of the LM1 load model

Charact-eristica[1]	Time t_P[4] [years]	R Mean	R COV	S(60% LM1) Mean	S(60% LM1) COV	β[8]	S(83% LM1) Mean	S(83% LM1) COV	β[8]
Deflection, u_z [m][2]	0	0.11[5]		− 0.07	0.04	+	− 0.09	0.04	5.1
	30	0.11[5]		− 0.07	0.04	+	− 0.10	0.05	2.0
	50	0.11[5]		− 0.08	0.04	8.7	− 0.12	0.05	−
Crack width w [mm][2]	0	0.20[6]		0.07	0.17	+	0.06	0.11	+
	30	0.20[6]		0.05	0.20	+	0.07	0.14	+
	50	0.20[6]		0.01	0.11	+	0.02	0.08	+
Concrete stress[2] σ_{co}[MPa]	0	18.00[7]		− 14.12	0.01	+	− 19.02	0.01	−
	30	18.00[7]		− 15.34	0.01	+	− 20.25	0.01	−
	50	18.00[7]		− 17.10	0.01	7.5	− 21.83	0.01	−
Bearing Capacity [load step]	0	48.30	0.08	20.00		7.3	30.00		4.7
	30	45.10	0.01	20.00		5.6	30.00		3.3
	50	39.00	0.06	20.00		8.1	30.00		3.8

[1] all variables normal distribution
[2] serviceability limit state (SLS)
[3] load level interpolated for 100% of LM1
[4] time after corrosion initiation
[5] $_{z,limit} = l/250$ according to [38], 7.4.1
[6] $w_{,limit} = 0.2$ mm according to [38], 7.3.1
[7] $\sigma_{,limit} = 0.6 f_{ck}$ according to [38], 7.2
[8] $\beta > 10$ is indicated by "+", $\beta > 1$ by "− "

Table IV. Gamma process prediction of statistical characteristics of structural response; evaluated for 83% of the LM1 load model

	Time t_P[4] [years]	S (83% LM1) Mean	COV	β	C	$\alpha(t)$
Defle-ction, u_z [m][2]	0	0.05	0.04	-	-	
	30*	**0.07**	**0.04**	**13.12**	**0.044**	**1.312**
	50	0.09	0.04	**34.22**	**0.082**	**4.107**
	35	*0.08*	*0.08*	*1.764*	*0.006*	*0.206*
	40	*0.09*	*0.07*	*3.086*	*0.010*	*0.412*
	50	*0.12*	*0.06*	*5.967*	*0.020*	*0.995*
	60	*0.14*	*0.06*	*8.573*	*0.029*	*1.715*
	70	*0.16*	*0.05*	*10.87*	*0.036*	*2.538*
	80	*0.21*	*0.05*	*15.32*	*0.051*	*4.595*
	100	*0.23*	*0.05*	*17.49*	*0.058*	*5.830*

4. Conclusion

Within the current research project, three different testing methods were used to determine the material parameters of interest in experiment. For the assessment of the material characteristics of concrete, the compressive strength fc, the tensile strength fct and the fracture energy Gf were considered. Small deviations between the Vienna test results and the results of tests performed at TU Brno arose. Several of the stochastic concrete parameters, which were characterized by the Brno team, were modeled numerically based on data from three-point bending tests. In particular, the comparison of the fracture energy obtained from the three-point bending test with the result of the wedge splitting test revealed only minor divergences. These results allow the conclusion that all three test methods are reliable, comparable which each other, and provide consistent stochastic concrete methods. In addition to the verification of the test methods, the evaluation of the influence of concrete additives on the stochastic models were of interest.

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