A Multi-objective Optimization Approach with a View to Robustness Improvement

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Abstract. In this contribution, multi-objective optimization methods are applied together with uncertainty quantification approaches in order to provide a concept for a robust structural design. The concept enhances the utilization of numerical simulation methods (Finite Element analysis) and as such can be useful for the computer-aided engineering. In this study, the application of the approach for a design of tires is shown. The proposed methodology enables the consideration of fragmentary or dubious information within the design process, which leads to the introduction of fuzzy variables into the optimization task. The application of fuzzy set theory is motivated by the epistemic character of available uncertain data. The proposed concept enables the optimization of multiple objectives and simultaneously the uncertainty reduction in the optimization results, which leads to the robustness improvement. In order to increase the numerical efficiency of the proposed design approach, a response surface approximation based on artificial neural networks is applied.

Keywords: optimization, numerical simulations, uncertainty

1. Introduction

The intensive development of multi-objective optimization methods in the past decades, as well as the coupling of these methods with numerical simulation approaches, e.g. Finite Element method (FEM), enables currently the solution of complex design problems in numerous engineering fields. The main feature of these design tasks is that the considered multiple objectives are predominantly in conflict with each other. This means, that generally no ideal solution exists.

There are two well established optimization approaches, which yield a solution for these kind of problems. The first approach encloses the formulation of an aggregate objective function (AOF), which combines all considered objectives through the application of the weighted sum method. Within AOF, each single objective function is preserved with a weighting factor, which is chosen subjectively by the decision maker in order to express the preference of this objective. Thereby, the choice of weighting factors, which are gathered into a preference vector, strongly affects the optimization result.

In order to avoid the insertion of subjective decisions into the optimization process, some objective approaches are proposed. They enable the identification of a well distributed set of trade-off solutions (Pareto-optimal set), instead of finding one suboptimal solution (Pareto, 1971). The solutions in Pareto-optimal set P^* are satisfying the criterion of Pareto optimality. That means, that in the set of solutions P, the Pareto-optimal set P^* contains solutions, which are not dominated by any member of set P.

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After the completion of optimization, the decision maker can choose one solution from the Pareto-optimal set P^*, according to defined preferences. The main advantage of this approach, compared to the AOF method is that the expression of preferences occurs in the post-optimization step. A detailed description of the Pareto-optimality concept as well as the dominance concept is presented in Section 3.1.

The optimization approaches shortly described above are deterministic. Thereby, in the engineering design tasks, uncertainty has to be taken into account. Uncertainty is present in different forms: e.g. geometry parameters of structural parts can be regarded as uncertain, as well as the properties of materials utilized for the components. In some cases also the loads applied to the designed structure can be considered as uncertain. Thereby, the sources of uncertainty can be various, e.g. the uncertainty in geometry or material properties is caused by unstable production conditions of structural elements. The source of uncertainty in loading is vague information or variability, e.g. in the case of tire design, considered here, the vertical load applied to the tire changes in dependency on the car weight, which is different for diverse car models.

According to the uncertainty sources, it is distinguished between three characteristics of uncertainty: variability, imprecision and incompleteness. In this study, the focus is set on the incompleteness as in many engineering tasks we have to handle with vague information, leading to assumptions and expert evaluations. A suitable model for describing this kind of uncertainty is the uncertainty model fuzziness. In this paper, the application of fuzzy variables to the mentioned multi-objective optimization concepts will be studied.

If uncertainty is considered in the optimization process, a quantification of robustness can be accomplished subsequently, which is shown within the proposed optimization approach.

2. Modelling uncertain quantities

In order to properly consider the uncertainty within the design task, a suitable uncertainty model should be chosen, dependent on the type of available information. Commonly, the probabilistic uncertainty models are utilized (Benjamin and Cornell, 1970), which employ the random variables for the description of non-deterministic parameters. The application of probabilistic models is preconditioned by the availability of extensive statistical information. If this prerequisites are not met, other uncertainty models should be taken into account, especially enabling the consideration of subjective information.

These models account for the Bayesian methods (Bernardo and Smith, 1994) or approaches based on the fuzzy set theory. The uncertainty model fuzziness (Dubois and Prade, 1997) and (Zadeh, 1965) employs the fuzzy set theory for modelling the vague, incomplete or subjective information. Within fuzzy sets, the gradual membership of elements to the set is defined, which enable a subjective weightening of information inside the set. Alternatives for modelling with fuzzy sets represent the convex modelling (Elishakoff, 1995) and interval mathematics (Alefeld and Herzberger, 1983). Though, the last concepts are based on the assumption of crisp membership of the elements to the set and, therefore, provide limited modelling capabilities in comparison to fuzzy sets.

A generalized uncertainty model, enabling accounting for objective and subjective information simultaneously – fuzzy-randomness is described in (Kwakernaak, 1978) and (Möller and Beer, 2004). Fuzzy-randomness can be utilized for modelling of imprecise probabilities. Within this model, fuzziness and randomness might be considered as special cases. Another approach, which allows the consideration of uncertainty – the chaos theory (Kapitaniak, 2000) makes an attempt to describe the unpredictable behaviour of dynamical systems.

From the uncertainty models, mentioned above, the uncertainty model fuzziness is chosen for further investigation and applied within the multi-objective optimization approach. The choice of fuzziness is motivated by the capability of the model to describe vague, incomplete or subjective information, which is predominant in engineering design tasks.

The formulation of a fuzzy set refers to the definition of a crisp set. The membership to a crisp set $A \subseteq X$, where $X = \mathbb{R}^n$ can be defined in a binary way, an element either belongs to the set or not. Thereby, the membership $\mu_{\tilde{A}}$ to a fuzzy set $\tilde{A}$ is defined gradually, see Fig. 1. If $\mu_{\tilde{A}}$ takes values within the interval [0,1] and, at least, once the value 1 is achieved, than such a set is called a normalized fuzzy set $\tilde{A}$ or a fuzzy number on X

$$\mu_{\tilde{A}} : X \longrightarrow [0, \infty). \tag{1}$$

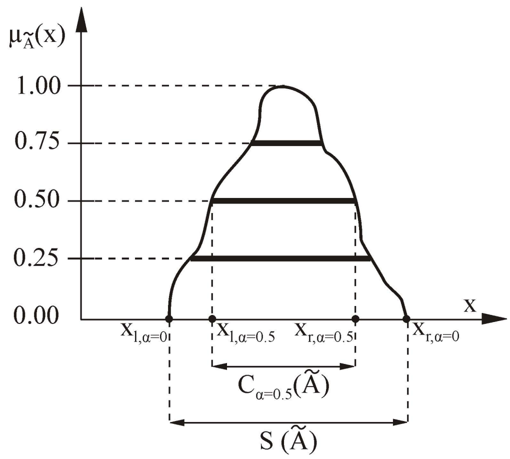

Figure 1. Fuzzy variable.

A fuzzy set is defined by its support $S(\tilde{A})$ and the membership function $\mu_{\tilde{A}}$. According to Fig. 1, $S(\tilde{A})$ is a crisp set, which contains elements

$$S(\tilde{A}) = \{x \in X, \mu_{\tilde{A}}(x) > 0\}. \tag{2}$$

In the optimization approach considered here, the fuzzy quantities are defined as normalized fuzzy sets. Thereby, the convexity of fuzzy sets is presumed. Convexity can be stated, if for every $x_1, x_2 \in X$ and $\lambda \in [0, 1]$

$$\mu_{\tilde{A}}(\lambda x_2 + (1 - \lambda)x_1) \geqslant min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)). \tag{3}$$

The numerical treatment of a fuzzy quantity $\tilde{A}$ occurs by means of the discretization of $\tilde{A}$ by numerous crisp sets $C_\alpha(\tilde{A})$ – so-called α-level sets

$$C_\alpha(\tilde{A}) = \{x \in X : \mu_{\tilde{A}} \geqslant \alpha\}, \tag{4}$$

$$\tilde{A} = (C_\alpha(\tilde{A}))_{\alpha \in (0,1]}. \tag{5}$$

Fuzzy quantities regarded in this study are n-dimensional. They are enclosed in the set of all fuzzy quantities $\mathcal{F}(\mathbb{R}^n)$.

3. Multi-objective optimization

3.1. Deterministic multi-objective optimization approaches

In the deterministic multi-objective optimization (MOO) task, we consider design variables, described in terms of design vectors $\underline{x}_d = (x_{d1}, x_{d2}, ..., x_{dn})$, which are defined in the design space $X_d = \mathbb{R}^n$. The design vectors $\underline{x}_d$ are mapped by means of the evaluation function $f : X_d \to Z$ onto the objective vectors $\underline{z}(z_1, z_2, ..., z_k)$ in the objective space $Z = \mathbb{R}^k$. Due to the fact, that the evaluation function f is vector-valued, the objective space is k-dimensional. The multi-objective optimization task with objective functions $f_i,\ i = 1, ..., k$, subjected to equality constraints $h(\underline{x}_d)$ and inequality constraints $g(\underline{x}_d)$ is formulated as

$$min\{f(\underline{x}_d) \mid h(\underline{x}_d) = 0,\ g(\underline{x}_d) \leq 0\}\ ,\ f(\underline{x}_d) = [f_1(\underline{x}_d), ..., f_k(\underline{x}_d)]^T. \tag{6}$$

Thereby, it should be pointed out, that if $k = 1$, a single objective optimization problem is to solve, or accordingly a multi-objective optimization problem reduced by means of an aggregate function to a single objective problem. If $k = 1$, a direct comparison of one-dimensional objective vectors is accomplished within the optimization process. For $k > 1$, the k-dimensional objective vectors shall be compared, which can be carried out only through the utilization of the dominance concept.

Dominance: an objective vector $\underline{z}^* = f(\underline{x}_d^*)$ dominates another objective vector $\underline{z}' = f(\underline{x}_d')$ if no component of $\underline{z}^*$ is greater than the corresponding component of $\underline{z}'$ and at least one component is smaller

$$\forall i \in \{1, ..., k\} : f_i(\underline{x}_d^*) \leq f_i(\underline{x}_d') \wedge \exists i \in \{1, ..., k\} : f_i(\underline{x}_d^*) < f_i(\underline{x}_d'). \tag{7}$$

The dominance is formulated as $\underline{z}^* \succ \underline{z}'$.

Pareto-optimal set: the goal of the multi-objective optimization approach is to find a set of solutions, which are not dominated with respect to each other (non-dominated set). According to the definition provided in Section 1, if P is the entire design space X_d, than the non-dominated set P^* (or X_d^*) is a Pareto-optimal set. The visualization of the Pareto-optimal set X_d^* in the objective space is the Pareto-front $Z^* = f(X_d^*) \subseteq Z$.

As mentioned in Section 1, the state of the art in the multi-objective optimization methods are approaches that either evaluate the aggregate objective function and refer to single objective optimization methods or approximate the Pareto-optimal set in different manners.

According to (Deb, 2002a), within the available multi-objective optimization methods it is distinguished between classical methods and evolutionary algorithms. The classical methods transform the multi-objective optimization problem into a single-objective optimization task by the application of different user-specified techniques. In this group, the weighted sum method, enabling the formulation of an aggregate function or the ϵ-constraint method, converting all objective functions, except of one, into constraints can be identified. These methods yield, after completing the optimization, one sub-optimal objective vector. An aggregate function, created, using the weighted sum method is formulated

$$f_{obj}(\underline{x}_d) = \sum_{i=1}^{k} w_i f_i(\underline{x}_d). \tag{8}$$

Thereby, w_i defines a user-specified weight vector. Some approaches are proposed, which presume, that through the utilization of the weighted sum method and an appropriate choice of the weight vector, a Pareto-optimal solution can be identified. A purposeful manipulation of the weight vector components in multiple optimization runs, could provide a whole Pareto-optimal set. Though, the evaluation of the optimization procedure numerous times, required for the identification of the whole Pareto-optimal set, is quite inefficient. Therefore, evolutionary algorithms, which enable providing the Pareto-optimal set in one optimization run, are commonly employed for the solution of multi-objective optimization problems.

The multi-objective evolutionary algorithms (MOEAs) provide not optimal trade-offs but an approximation of the Pareto-set. In general, MOEAs are expected to fulfil two tasks: guide the search through the Pareto-set and keep a diverse set of non-dominated solutions. The first goal is achieved by assigning the fitness to the population, based on the non-dominated sorting method, while the second goal by including the density information into the selection process. In 1990s several methods, e.g. the Nondominated Sorting Genetic Algorithm (NSGA) (Srinivas and Deb, 1995) or Multi-objective Genetic Algorithm (MOGA) (Fonseca and Fleming, 1993) were proposed, which were able to identify multiple diverse Pareto solutions. In further developments of MOEA, elitism was introduced in order to obtain a better convergence. Elitism enables the prevention of non-dominated solutions from being lost. Among the developed methods, which include elitism, three main approaches should be mentioned: NSGA-II (Deb, 2002b), the Strength-Pareto Evolutionary Algorithm (SPEA-2) and the Pareto-Archieved Evolution Strategy (PAES) (Knowles and Corne, 1999).

In the presented contribution, the application of fuzzy quantities within the weighted sum method (aggregate function) and to methods, providing the trade-off solutions will be studied.

3.2. Multi-objective optimization with consideration of uncertain quantities

3.2.1. *Problem formulation*

In the literature, several methods were proposed which evaluate uncertainty within the multi-objective optimization. Thereby, referring to (Das et al., 2009), different sources of uncertainty are taken into account, e.g. noisy data, objective function evaluation errors or user indecision concerning the prioritization of objective functions. Generally, within the available approaches it is distinguished between methods, which evaluate aleatory uncertainty and utilize probabilistic concepts and methods, which account for epistemic uncertainty and employ concepts based on the fuzzy set theory. Recently, ideas and algorithms for the simultaneous consideration of different types of uncertainty and different uncertainty models (polymorphic uncertainty) within the optimization task are developed.

The main difference between deterministic multi-objective optimization approaches and approaches considering uncertainty, is expressed within the formulation of dominance. In the probabilistic concepts, the dominance of vector $\underline{u}$ over the vector $\underline{v}$ is expressed by the probability of dominance $pr(\underline{u} \succ \underline{v})$. Exemplary, such approaches model the random error, present by the evaluation of objective functions. Among the epistemic procedures, the idea proposed by (Farina and Amato, 2004) for the introduction of a fuzzy measure for the comparison of two non-dominated solutions, is worth mentioning. The comparison is accomplished through the evaluation of the number of objectives, in which one solution dominates another one. Further developments enabled the comparison of two solutions, which do not have to be Pareto-optimal – like in the previous approach – by the application of the concept of a fuzzy dominance.

In this study, instead of employing the concept of fuzzy dominance, ideas for obtaining and evaluation of a Fuzzy-Pareto-Front will be discussed. A Fuzzy-Pareto-Front contains fuzzy objective vectors, gained from the optimization with the application of fuzzy variables. The method for obtaining fuzzy results will be first introduced within the multi-objective optimization, using the weighted sum method for the creation of the aggregate function. Subsequently, a method for evaluating fuzzy objective vectors within the Fuzzy-Pareto-Front will be discussed.

3.2.2. *Aggregate objective function evaluating fuzzy variables*

In the proposed methodology, a distinction is made within the definition of input quantities for the optimization task, between uncertain a-priori parameters $\underline{\tilde{p}}_a \in \mathcal{F}(\mathbb{R}^{np})$ and design variables $\underline{x}_d \in \mathbb{R}^n$. Uncertain a-priori parameters $\underline{\tilde{p}}_a$ are quantities, influencing the optimization task, which can not be modelled as crisp quantities due to information deficits. Therefore, they are considered as fuzzy numbers. Design variables $\underline{x}_d$ are defined within the user-specified ranges and can be arbitrarily chosen during the optimization.

The scheme of the optimization with fuzzy quantities is presented in Figure 2. The method is a three level approach. Optimization establishes the first level, that is the outer loop of the approach. Within the optimization loop, the fuzzy analysis is performed. The numerical realization encloses the execution of fuzzy analysis for every design vector $\underline{x}_d$. In consequence, if k objective functions are considered, k fuzzy result quantities are obtained for each design vector. These fuzzy result quantities are gathered into k-dimensional fuzzy objective vector $\underline{\tilde{z}} \in \mathcal{F}(\mathbb{R}^k)$.

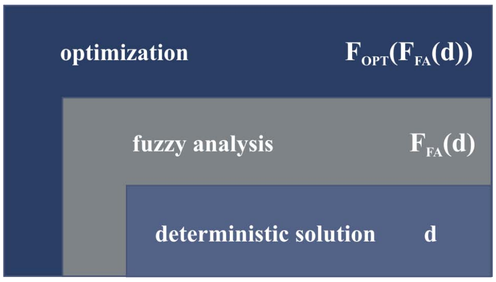

Figure 2. Optimization scheme.

Within the fuzzy analysis, the deterministic solution, that is a FE-solution or a response surface is evaluated numerous times. The fuzzy analysis uses the α-level optimization approach for the computation of membership functions of all fuzzy result quantities. For the α-level optimization, the modified evolution strategy is utilized, which was proposed in (Möller, Graf and Beer, 2000). In order to be able to evaluate the fuzzy result quantities, obtained for several designs, an aggregate objective function, enclosing the information reducing measures $\mathcal{M}_j$ is formulated

$$f_{obj} : \mathbb{R}^n \times \mathcal{F}(\mathbb{R}^{np}) \rightarrow \mathbb{R} \tag{9}$$

$$(\underline{x}_d, \underline{\tilde{p}}_a) \mapsto \sum_{i=1}^{k} \sum_{j=1}^{u} w_{ij}\, \mathcal{M}_j(f_i(\underline{x}_d, \underline{\tilde{p}}_a)). \tag{10}$$

In Eq. (10), w_{ij} defines the weighting factors, which enable the prioritization of chosen components (objectives). The information reducing measures $\mathcal{M}_j$ map the fuzzy result quantities onto real numbers and allow their quantification. As $\mathcal{M}_j$, uncertainty measures for fuzzy variables can be applied, e.g. the area (zeroth moment) of a fuzzy variable, variance or the Shannon's entropy. The uncertainty measure $\mathcal{M}_1$, based on the Shannon's entropy (Beer and Liebscher, 2008) and applied to the quantification of a one-dimensional fuzzy result quantity $\tilde{z}$, is formulated as

$$\mathcal{M}_1 = H_U = - \int_{z=-\infty}^{z=+\infty} \left[\mu(z) \cdot ln\mu(z) + (1 - \mu(z)) \cdot ln\,(1 - \mu(z)) \right] dz. \tag{11}$$

An uncertainty measure $\mathcal{M}_2$, evaluating the variance of a fuzzy variable $\tilde{z}$ is proposed in (Wu and Mendel, 2007)

$$\mathcal{M}_2 = V = \int_{z=-\infty}^{z=+\infty} (z - \overline{z})^2 \cdot \mu(z) dz \cdot \left(\int_{z=-\infty}^{z=+\infty} \mu(z) dz \right)^{-1}. \tag{12}$$

Another uncertainty measure $\mathcal{M}_3$, which assesses the area under the membership function $\mu(z)$ of a fuzzy variable is defined by

$$\mathcal{M}_3 = A = \int_{z=-\infty}^{z=+\infty} \mu(z) dz. \tag{13}$$

For the optimization approach, described in this study two information reducing measures are employed. The first measure is the uncertainty measure $\mathcal{M}_3$. The second information reducing measure $\mathcal{M}_4$ assesses the position of the first element of the fuzzy quantity support $S(\tilde{z})$ and can be regarded as a performance measure. Through the minimization of the smallest element of the support $S(\tilde{z})$, the minimization of the fuzzy result quantity $\tilde{z}$ is achieved.

In Fig. 3, the application of the information reducing measures for the comparison of two fuzzy result quantities $\tilde{z}_1$ and $\tilde{z}_2$, obtained for two different designs – A and B – is shown. Once the criteria for the comparison of uncertain quantities are formulated and included into the aggregate objective function, the optimization can succeed. The application of the method for single-objective optimization tasks is shown in (Pannier, 2011).

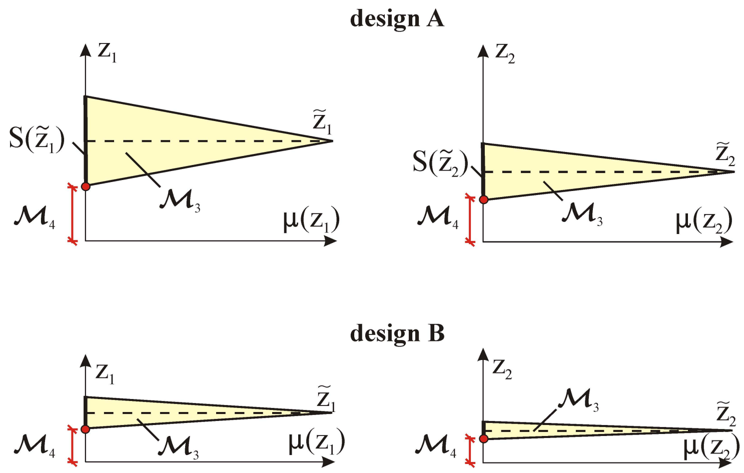

Figure 3. Comparison of fuzzy result quantities for design A and B.

3.2.3. *Fuzzy-Pareto solutions*

In Subsection 3.2.2., a method for handling fuzzy results within the optimization is presented, which focused on bringing together the fuzzy results from several objectives into one aggregate objective function, enclosing the information reducing measures $\mathcal{M}_j$. Another crucial issue, when regarding the objective space with numerous k-dimensional fuzzy objective vectors, is to find the Fuzzy-Pareto-Front. Fuzzy-Pareto-Front is the image of the Fuzzy-Pareto set in the objective space. In the context of the optimization approach with fuzzy variables, described above, the search for the best design should be coupled with the identification of the set of fuzzy result quantities, which are not dominated by any other fuzzy quantity from the set of all fuzzy quantities $\mathcal{F}(\mathbb{R}^k)$.

The objective space, containing k-dimensional fuzzy result vectors $\underline{\tilde{z}}^1 - \underline{\tilde{z}}^5$, obtained for corresponding design vectors $\underline{x}_{d1} - \underline{x}_{d5}$ is shown in Fig. 4 (here k = 2). Thereby, fuzzy result vectors $\underline{\tilde{z}}^1 - \underline{\tilde{z}}^3$ are not dominated by any other fuzzy result vector in the objective space.

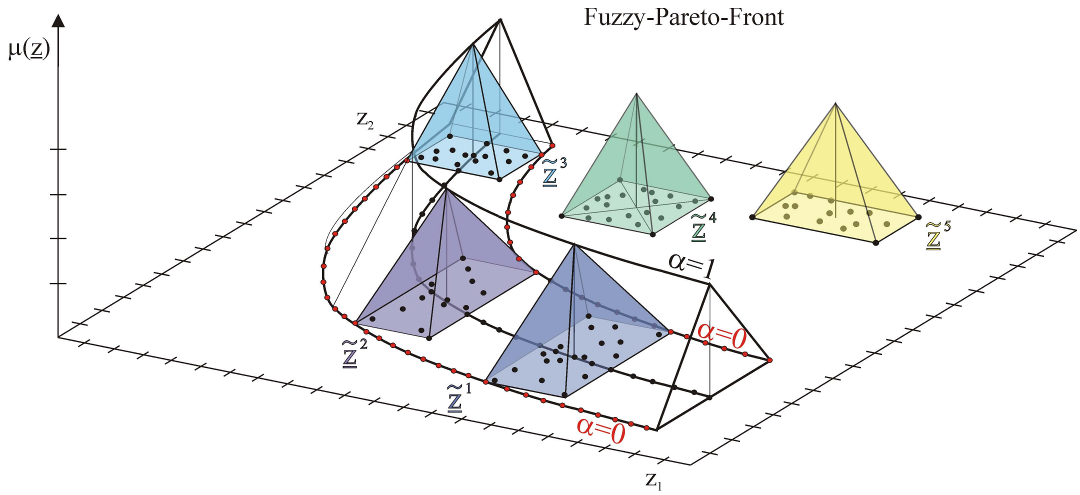

Figure 4. Fuzzy-Pareto-Front.

The k-dimensional fuzzy objective vector $\underline{\tilde{z}}^*$ is formulated

$$\underline{\tilde{z}}^* = \{(\underline{z}^* = (z_1, ..., z_k),\ \underline{\mu}^* = (\mu_1^*, ..., \mu_k^*)) \mid z_i \in \mathbb{R}^k\}. \tag{14}$$

The support $S(\underline{\tilde{z}}^*)$ is defined by

$$S(\underline{\tilde{z}}^*) = \{(\underline{z}^* = (z_1, ..., z_k),\ \underline{\mu}^* = (\mu_1^*, ..., \mu_k^*))\ \forall\, i \in [1, k],\ \mu_i^*(z_i) > 0\}. \tag{15}$$

The identification of the set of non-dominated k-dimensional fuzzy objective vectors prerequires the formulation of non-dominance criteria for fuzzy variables. A fuzzy objective vector $\underline{\tilde{z}}^*$ is non-dominated, if in the set of all fuzzy objective vectors $\mathcal{F}(\mathbb{R}^k)$, there exists no other fuzzy objective vector $\underline{\tilde{z}}'$, so that every element $\underline{z}' \in S(\underline{\tilde{z}}')$ dominates all elements $\underline{z}^* \in S(\underline{\tilde{z}}^*)$.

The proposed concept enables the check of dominance in the postcomputation step. Though, an approach is required, which would allow a dominance check for fuzzy quantities during the multi-objective optimization. In this way, the information concerning the dominance can influence the selection and variation step in the evolutionary optimization algorithm. First attempts are made to extend the non-dominated sorting procedure, applied within NSGA II for the consideration of the domination criteria for fuzzy variables.

3.3. Coupling of multi-objective optimization approaches with FE solution

According to Fig. 2, if FE simulation is applied as deterministic solution d, the evaluation function f formulated in Eq. (6) as well as the aggregate function f_{obj} defined in Eq. (10), depend on the solution of a mechanical system, especially depend on displacements $\varphi(\underline{x}_d, \tilde{\underline{p}}_a)$ gained from the FE analysis. $\varphi(\underline{x}_d, \tilde{\underline{p}}_a)$ are obtained from the evaluation of the nonlinear equation of motion. In the case of a stationary rolling body (tire analysis), the equation of motion is formulated as

$$(\underline{K} - \underline{W}) \cdot \Delta\underline{\varphi} = \underline{f} - \underline{f}_\sigma + \underline{f}_T. \tag{16}$$

In Eq. (16), $\underline{K}$ denotes the tangential stiffness matrix, $\underline{W}$ the Arbitrary Lagrangian Eulerian inertia matrix, $\underline{f}$ nodal forces of external loads, $\underline{f}_\sigma$ nodal forces resulting from internal stress state and $\underline{f}_T$ nodal forces due to inertia. The dependency of the aggregate function f_{obj} on displacements $\varphi(\underline{x}_d, \tilde{\underline{p}}_a)$ is defined as

$$f_{obj} : \mathbb{R}^n \times \mathcal{F}(\mathbb{R}^{np}) \rightarrow \mathbb{R}, \tag{17}$$

$$(\underline{x}_d, \tilde{\underline{p}}_a) \mapsto \sum_{i=1}^{k} \sum_{j=1}^{u} w_{ij}\ \mathcal{M}_j(f_i(\varphi(\underline{x}_d, \tilde{\underline{p}}_a))). \tag{18}$$

3.4. Implicit robustness quantification within the multi-objective optimization

Due to the consideration of one uncertainty measure – $\mathcal{M}_3$ – within the aggregate objective function f_{obj}, the minimization of f_{obj} will automatically cause the reduction of uncertainty in the fuzzy results. This concept refers to the robustness measure $R_{l,k}^{[p]}$, proposed in (Beer and Liebscher, 2008) and in (Graf et al., 2010), which quantifies the ratio of the uncertainty of input quantities versus the uncertainty of result quantities for each design. $R_{l,k}^{[p]}$ adapted for the application within the multi-objective optimization is defined

$$R_{l,k}^{[p]} = \frac{\sum\limits_{p=1}^{np} w_p \left(\mathcal{M}_j\left(\tilde{\underline{p}}_a\right)\right)}{\sum\limits_{q=1}^{l} w_q \sum\limits_{i=1}^{k} w_i \left(\mathcal{M}_j\left(f_i\left(\underline{x}_d, \tilde{\underline{p}}_a\right)\right)\right)}. \tag{19}$$

In Eq. (19), $R_{l,k}^{[p]}$ denotes the robustness measure of $[p]$-th structural design under consideration of l load cases and k objective functions. $\mathcal{M}_j$ indicates the $[j]$-th uncertainty measure (here $j = 3$) and w_p, w_q, w_i the weighting factors. The configuration of uncertain a-priori parameters does not change during the optimization task. Therefore, the numerator of the fraction in Eq. (19) remains constant and can be neglected. Only the expression in the denominator of the fraction is considered within the aggregate objective function f_{obj}.

The proposed design approach enables beside of optimization of numerous objectives also the uncertainty reduction within the fuzzy results, which contributes to the robustness improvement. The application of the developed method to the structural tire design is shown by the way of an example.

4. Example

The goal of this study is the optimization and the robustness improvement of a passenger car tire 195/60 R15. Generally, within the tire structure the tread layer, made of rubber, several reinforcement layers – capplies and belts – as well as the bead, consisting of numerous steel cords can be identified, see Fig. 5.

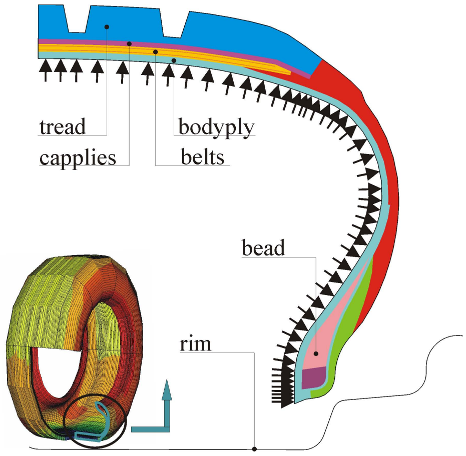

Figure 5. Tire cross-section.

Through the modifications of these structural parts, two objective functions f_1 and f_2 will be improved. The first objective function f_1 aims at providing regular tire wear, whereas f_2 at improving the fatigue performance. Regular tire wear is obtained, if a uniform contact pressure distribution in the tire-road contact zone occurs. Therefore, a ratio of the contact pressure in the shoulder region versus the contact pressure in the central region of the tire cross-section will be optimized. Within f_2, a fatigue criterion based on the evaluation of the strain energy density at the critical area – the belt edge – is applied.

The optimization of mentioned objectives is accomplished by the consideration of three design variables – the belt angle x_{d1}, the thickness of the tread layer x_{d2} and the number of capplies x_{d3} as well as three uncertain a-priori parameters – the tire inner pressure $\tilde{p}_{a1}$, the fiber spacing in bodyply $\tilde{p}_{a2}$ and the stiffness of the tread compound $\tilde{p}_{a3}$. The following ranges for the design variables are specified: x_{d1}: $\langle 18°; 30° \rangle$, x_{d2}: $\langle -1.5; 1.5 \rangle$mm and x_{d3}: $\langle 0; 2 \rangle$. The uncertain a-priori parameters are defined as fuzzy numbers – $\tilde{p}_{a1}$: $\langle 0.23; 0.25; 0.27 \rangle$N/mm^2, $\tilde{p}_{a2}$: $\langle 1.17; 1.304; 1.44 \rangle$mm and $\tilde{p}_{a3}$: $\langle 0.875; 0.976; 1.075 \rangle$N/mm^2.

In the first analysis step, the design of experiments (DOE) is performed. For each of the sampling points a 3D Finite Element tire model is evaluated in steady state rolling situation. The steady state rolling analysis is executed in terms of the Arbitrary Lagrangian Eulerian approach (Kaliske et al., 2003) and (Nackenhorst, 2004). Due to the high computational cost of the rolling tire analysis, the FE solution is substituted by a neural network based response surface approximation. The training of the feedforward neural networks occurs for the sampling points, evaluated within DOE. The obtained response surface can be applied as the deterministic solution within the optimization approach, according to the scheme in Fig. 2.

After the execution of the coupled approach of optimization and fuzzy analysis, which uses the aggregate objective function, one design vector is identified as the optimal solution. For this design, the FE analysis is performed in the 'post processing' step in order to validate the neural network outputs. The results, obtained for the optimal solution are compared with results gained for reference designs. In Fig. 6, the contact pressure distributions in the optimal design – Fig. 6a), 6b) – and in the reference design – Fig. 6c), 6d) – are depicted. For each design, distributions obtained for the most advantageous – Fig. 6a), 6c) – and most disadvantageous – Fig. 6b), 6d) – combination of a-priori parameters are shown. These most advantageous and most disadvantageous combinations are derived from the definition of the contact pressure ratio as a fuzzy quantity, obtained for the regarded design.

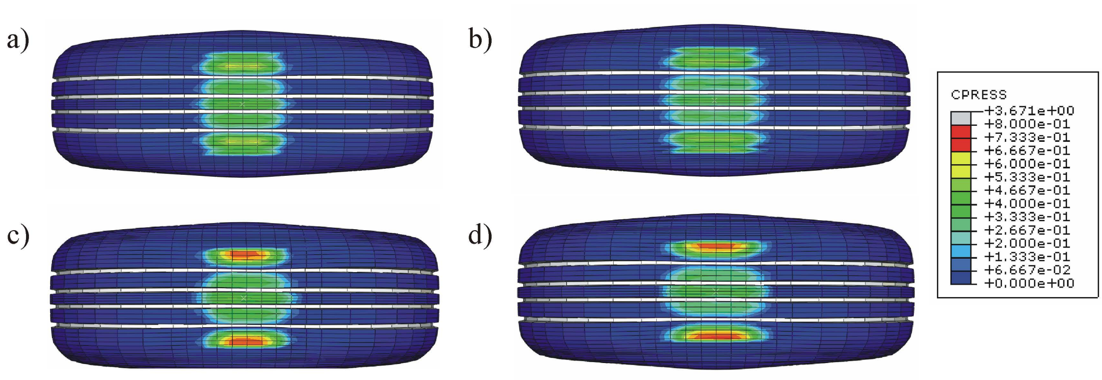

Figure 6. Contact pressure distribution for the optimal design: a) best case, b) worst case and reference design: c) best case and d) worst case.

It can be stated, that for the optimal design a uniform contact pressure distribution is obtained in the best case as well as in the worst case. This fact is confirmed by the according contact pressure ratios $p_{coeff} = 1.23$ and $p_{coeff} = 1.27$, see Fig. 7a). Therefore, the occurrence of a regular wear is expected for the optimal design. Within the reference design, a non-uniform contact pressure distribution occurs, resulting in $p_{coeff} = 1.68$ (best case) and $p_{coeff} = 1.97$ (worst case). Thus, not only the minimization of the contact pressure ratio but also the uncertainty reduction (small support) is achieved for the optimal design.

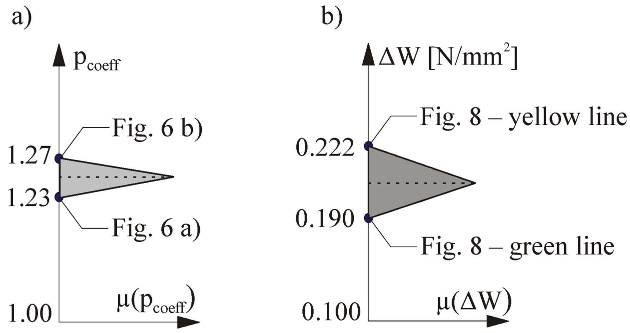

Figure 7. Contact pressure ratio and strain energy density amplitude obtained for the optimal design.

The second design task presumed the minimization of the strain energy density delta, which leads to the improvement of the tire resistance to fatigue. In Fig. 8, the strain energy density evaluated over the circumference of a tire is shown. According to the strain energy density as a fuzzy result quantity, four curves are shown – signifying the worse and best case for the optimal design and for the reference design as well. The fuzzy result quantity, obtained for the optimal design is shown in Fig. 7b). From the comparison of the curves in Fig. 8, it can be stated that, peaks of curves obtained for the optimal design lie below the peaks of curves obtained for the reference design. The improvement is confirmed also by the strain energy density amplitudes: for the optimal design $\Delta W = 0.190\text{N/mm}^2$ (best case) and $\Delta W = 0.222\text{N/mm}^2$ (worst case), whereas for the reference design $\Delta W = 0.309\text{N/mm}^2$ and analogically $\Delta W = 0.321\text{N/mm}^2$.

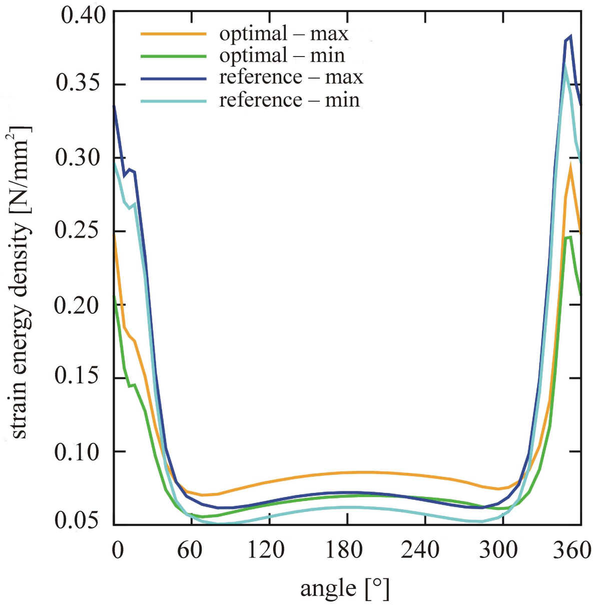

Figure 8. Strain energy density versus the circumference of a tire.

In the example, the capability of the approach to optimize the objectives and to reduce the uncertainty of fuzzy results is shown.

5. Conclusions

In this contribution, concepts for the consideration of fuzzy variables within the multi-objective optimization approaches are discussed. Fuzziness is chosen as an appropriate model for the description of parameters, which can not be defined as crisp quantities due to limited data and information deficits. Therefore, procedures are proposed, which enable handling of these uncertain a-priori parameters next to the design variables within the optimization task. The consideration of fuzzy variables within an aggregate objective function, formulated by means of the weighted sum method is studied. The application of the approach to the optimization and robustness improvement of a passenger car tire is shown. Additionally, concepts for the identification of the set of non-dominated fuzzy quantities are proposed in the context of Pareto-optimality. The developed approaches can be coupled with a FE simulation or a response surface approximation and therefore are suitable for the solution of engineering design tasks.

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Dynamic Response of Beams to Interval Load

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Abstract. Parameters of mathematical models are most often represented by real numbers, while in practice it is impossible or at least very difficult to get reliable information about their exact values. Hence, it is unreasonable to take point data for that may lead to incorrect results, which is not welcome especially when inaccuracy cannot be neglected. Depending on available information, one can use different ways of modelling of uncertainty. Interval computing plays an important role in this field, because very often the only available information are lower and upper bounds on a physical quantity. This paper focuses on a transient dynamic analysis of a beam with uncertain parameters. Finite difference and finite element methods are used to solve partial differential equation which represents the model for the motion of a straight elastic beam. In order to compute the time-history response of the beam under uncertainty, interval dynamic beam equations are solved using Search method, Gradient method, Taylor method, adaptive Taylor method, direct optimisation and Direct method for solving parametric interval linear systems. The applicability, i.e. effectiveness and accuracy, of those methods is illustrated through solution of beams with interval value of modulus of elasticity and mass density and subjected to interval dynamic loading.

Keywords: Euler-Bernoulli beam, Dynamic response, Interval arithmetic, Search method, Gradient method, Taylor method, adaptive Taylor method, Direct method, Direct optimisation.

1. Introduction

Airplane wings, high-rise buildings and suspension bridges are just some of the mechanical and structural examples where vibration analysis of beams is essential for the safe design. Safety issues are the greatest concern of structural engineering as the design and construction of secure and safe structures can prevent disasters like the collapse of Tacoma Narrow Bridge November 7, 1940, just few month after it was finished. This was probably the most dramatic failure in bridge engineering history. Safety studies in structural engineering are supposed to prevent failure during the lifetime of a structure.

Constantly increasing computational capabilities allow for detailed numerical models of structural systems. However, those models are built, inter alia, on a number of model parameters subject to uncertainty.

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The use of models that include the uncertainty, which is central to reliability/risk analysis of engineering systems, is of great importance for a design engineer.

Uncertainty of structural parameters is mainly due to the scarcity or lack of data which may be resulted from manufacturing/construction tolerances or caused by progressive deterioration of concrete and corrosion of steel. In engineering applications, uncertainty also exists in determining external loads. To make a decision based on an inexact data say some parameter $\tilde{p}$, a measurement error $\Delta p = |\tilde{p} - p|$ must known at least. Very often, the only available information about the error is its upper bound $\Delta p \leqslant \Delta$. In this case, once the measurement $\tilde{p}$ is obtained, one can conclude that the possible values $\tilde{p} + \Delta p$ form an interval $\boldsymbol{p} = [\tilde{p} - \Delta, \tilde{p} + \Delta]$ which is guaranteed to contain the exact value p of the parameter. Once interval quantities are introduced, they must be handled appropriately to obtain the result which is guaranteed to contain the exact solution.

Though interval arithmetic was introduced by Moore (Moore, 1966) already in 1966, the application of interval concepts to structural analysis is more recent. Some important advances on reliability-based design and modelling of uncertainty when data is limited were made during last years. Structural analysis using interval variables has been used by several researchers to incorporate uncertainty into structural analysis ((Köylüoglu et al., 1995), (Nakagiri and Yoshikawa, 1996), (Rao and Sawyer, 1995), (Rao and Berke, 1997), (Rao and Chen, 1998), (Mullen and Muhanna, 2001), (Neumaier and Pownuk, 2007), (Skalna, Pownuk and Rama Rao, 2008)).

In this paper, the problem of vibrations of an Euler-Bernoulli beam with interval material properties subjected to interval load is considered. Two different approaches are employed to obtain beam deflection in time. In the first approach, the Euler-Bernoulli equation governing the behaviour of the beam is descretized in space and time. The beam bending in the respective time step is obtained by solving a system of equations with coefficient depending on interval parameters. Several methods are used for this purpose. Search method, Gradient method ((Skalna, Pownuk and Rama Rao, 2008)), Taylor method and adaptive Taylor method (Pownuk, 2011) utilise the fact that in many structural engineering problems relation between the solution and uncertain parameters is monotone. In such a case, the extreme values of a solution are attained at respective endpoints of given intervals. Monotonicity can be verified by using Taylor series or an interval method (Hansen, 1992). Methods exploiting monotonicity tests are useful for solving large scale problems, but they may underestimate. When monotonicity is not assumed, the solution can be obtained using methods for solving parametric interval linear systems (Skalna, 2010)). Those methods give guaranteed enclosures, but their usage is limited e.g. by the amount of uncertainty. In the second approach, the Finite Element Method is a starting point for considerations. The Wilson-θ method and optimisation approach are used for the solution of the problem (Rama Rao, Pownuk and Vandewalle, 2010).

The paper has the following structure. In Sections 3 and 6, the considered problem is described in terms of the mathematical theory. Section 4 describes the discretization of the problem in time and space. Section 5 and 6.1 are devoted to the methods for solving interval linear systems obtained from the discretization of the Euler-Bernoulli equation. Numerical examples are given in Section 7. The paper ends with concluding remarks.