Simultaneous loads in structural design

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Abstract: The current codes include three load combination methods: Permanent loads are always combined dependently. A permanent and a variable load are combined dependently or independently and two variable loads semi-dependently or dependently.

However, if the loads are simultaneous, i.e. the loads are active at the same time with one on the other, they are combined dependently. If the loads are not active at the same time, the distributions can be altered in a way the loads are simultaneous i.e. loads are always combined dependently in the structural design.

The dependent load combination results in higher safety factors, γ_G, γ_Q, γ_M, and combination factors, ψ_0, than the ones obtained semi-dependently and independently.

The dependent load combination is reliable, it is simple and it requires little calculation work.

Keywords: load combination, code, design

1. Introduction

The load combination is one of the key issues of the structural design and the design codes, (EN 1990, 2002; ISO 2394, 1996). A uniform theory of the load combination is missing. The dominant hypothesis is that the loads are combined independently if the loads are independent and dependently if the loads are dependent. However, the permanent loads are independent, but combined always in current codes dependently. The permanent load and the variable load are often considered independent and combined sometimes independently and sometimes dependently but these loads are dependent during the normal service time, 50 years, and must therefore be combined dependently. The one-year loads are independent but simultaneous and must be combined dependently i.e. by accumulation, too. The variable loads are combined semi-dependently but these loads should be combined dependently after the distributions are altered in a way the loads are simultaneous. This paper explains that loads are always combined dependently in the structural design which results in higher safety, γ_G, γ_Q, γ_M, and combination factors, ψ_0, than the ones in the current codes.

1.1. Symbols

Symbols in this paper are mainly the same as used in the eurocode:

- G Permanent load
- Q Variable load
- γ Safety factor
- μ Mean
- σ Deviation

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β Reliability index, $P_f = \Phi(-\beta)$, Φ is standardised cumulative normal function
P_f Failure probability, in the eurocode $P_f = 1/15400$, $\beta = 3.826$
V Coefficient of variation $V = \sigma / \mu$
dp Design point value, the cumulative distribution value at the design point
d_s Duration of part time load, e.g. snow load
d_w Duration of full time load, e.g. wind or imposed load

1.2. TERMS

Some essential terms in this paper are:

Simultaneous – semi-simultaneous – non-simultaneous loads

Two loads are simultaneous during a reference time, if the loads are active during this time with one on the other. These loads may be dependent or independent but combined always dependently without combination or reduction factors which would result in a load vanish. However, the combination of two variable loads may include a combination factor induced by a distribution conversion.

The loads are non-simultaneous if the loads are not active at the reference time and semi-simultaneous if not simultaneous or non-simultaneous.

Permanent loads with each other and a permanent load and a variable load are always simultaneous. Variable loads are normally semi-simultaneous but the distributions can be altered in a way these loads are simultaneous.

Dependent – semi-dependent – independent loads

Two loads are dependent if the loads at the same fractiles occur at the same time.

If one load of the two loads to be combined is a variable load, the loads may be equally dependent – semi-dependent – independent at the same time. The reason is that the load distribution of the variable load is defined to be the maximum load during one year. When time increases, several distributions and loads become available for the combination. At the infinite time there are infinite combination options and all dependent options (dependent – semi-dependent – independent) are equally possible.

Two individual simultaneous loads are independent but when time increases and/or number of loads increase, the loads become dependent.

When the distributions of the loads are fixed to the active time of both loads with one on the other and when time or number of loads increase while one load is selected the other load becomes automatically defined, too, as the loads occur at the same fractile at the same time. Due to this relation, the loads are dependent. A permanent and a variable load are dependent during one year only at low fractiles, 0.02 or less, i.e. the loads are semi-dependent and virtually independent, but these loads become more dependent when time increases, e.g. they are dependent up to fractile ca 0.98 in 50 years i.e. these loads are virtually fully dependent during the normal service time of structures, 50 years.

Two variable loads are similarly dependent, too, when the distributions are altered in a way both loads are active at the same time with one on the other.

The dependence of the simultaneous loads may be explicit and physical i.e. the actual loads occur at the same time, e.g. the permanent and the variable load during 50 years.

The dependence of the simultaneous loads may also be implicit i.e. the individual loads are independent but a group of loads include at least one dependent load pair and the group acts dependently, e.g. imposed and permanent loads of a multi storey house.

The semi-dependent loads are normally dependent at low fractiles.

Two loads are independent if the loads do not occur at the same time at the same fractile.

Dependent – semi-dependent – independent load combination

Two loads are combined dependently by adding up the distributions by fractiles i.e. a load X with an item x_i in fractile i and a load Y with an item y_i in fractile i is combined dependently to obtain the combination load XY with an item xy_i in fractile i by adding up x_i and y_i, i.e. $xy_i = x_i + y_i$ (Poutanen, 2011). If the Monte Carlo simulation is used to combine the loads, in the dependent combination one seed number is used. If the convolution equation is used to combine the loads, the deviation of the combination load is fixed in a way the combination distribution crosses the crossing point of the partial distributions (Poutanen, 2011). In the dependent combination, the action of a new load in the combination is independent of other loads in the load combination.

The semi-dependent combination is an imprecise abstraction. Several semi-dependent combination methods exist, e.g. Turkstra's method where one load has the maximum deterministic value corresponding to the target reliability and the other load has a random value. The semi-dependent combination should lie between the dependent and independent combination. This is normally true at least at high fractiles.

In the independent combination, the loads are combined randomly e.g. by using the convolution equation or by using the Ferry Borges – Castanheta's method or by using the Monte Carlo simulation and two seed numbers. In the independent combination, the action of the new combination load depends on the earlier loads of the combination.

The current terms dependent and independent combination are misleading as in the independent combination the partial loads are independent but the combination load is dependent of the partial loads and in the dependent combination vice versa. Therefore it would be clearer to use terms random and accumulation summation.

The rule of the maximum load combination

A basic rule of the structural design is that the loads must be combined to obtain the maximum load. According to this rule, all loads should be combined dependently as the dependent combination results in the highest load. However, this rule is currently applied only to load combination alternatives with equal occurrence probability. Therefore this rule is not always applied as the independent or the semi-dependent combinations are considered more probable.

The permanent load and the variable load may be combined dependently, semi-dependently or independently. A new finding is that all these combination options are virtually equally possible during 50 years and therefore the maximum load combination rule must be applied i.e. these loads must be combined dependently. A further argument for the dependent combination is that even the

independent one year loads must be combined by accumulation. Variable loads are dependent, too if the distributions apply to the same time, and these loads are combined dependently.
Two permanent loads are independent but combined dependently.

1.3. ASSUMPTIONS, LIMITATIONS

The assumptions of the eurocode (EN 1990, 2002) are applied except for the load combination.

The variability and error induced by the code uncertainty, robustness, design, execution, use, degradation and wear out are excluded here.

2. Load combination in current codes

Loads are combined in the current codes in three ways:

- Permanent loads are always combined dependently.
- A permanent and a variable load are combined in the failure state sometimes dependently and sometimes independently but in the serviceability state always dependently.
- Two variable loads are always combined semi-dependently if these loads are the first and the second load in the load combination, but always dependently if the loads are third, fourth etc. load in the load combination.

The eurocode (EN 1990, 2002) includes three options to combine the permanent and the variable load, 6.10, 6.10a,b and 6.10a,mod. The first one is dependent and the others are independent. The Finnish eurocode is based on 6.10a,mod.

3. Load vanish

In the dependent load combination, no load vanishes. The loads are added up as such without any reductions or combination factors which would result in a load vanish.

In the semi-dependent and in the independent load combination a part of the load disappears in the combination. When two variable loads are combined semi-dependently a combination factor $\psi_0 \approx 0.6\ldots0.8$ is applied, which results in a load vanish of ca 0…20 %.

When a permanent and a variable load is combined independently, a load vanish of ca 0…10 % occurs, which is realized in the material safety factor γ_M.

We may deduce the independent load combinations wrong due to the load vanish: Assume a material (or a structure) has the survival probability S and the resistance 1 for the permanent load G alone and the variable load Q load alone. Now, if the material is loaded by $0.5G$ and $0.5Q$ and the loads are combined independently, the material has the resistance of ca 1.1[1] and if combined dependently, the resistance is 1. The independent combination is not viable. It is impossible that the effect of one load decreases if the other

[1] In the eurocode more precisely 1.0646, $V_{G,normal} = 0.09147$, $V_{Q,gumbel} = 0.4$, $\beta = 3.826$.

load is present as there is no link between the loads. The semi-dependent load combination is wrong for the same reason.

4. Simultaneous loads are combined dependently

Next examples are presented which show that simultaneous loads are combined dependently. The target survival probability is 0.98[2], $\gamma_G = 1$, $\gamma_Q = 1$ and the material is ideal with no variation $V_M = 0$, $\gamma_M = 1$.

4.1. COMBINATION OF PERMANENT LOADS

Permanent loads are simultaneous with each other. Permanent loads are independent, but these loads are combined dependently in all codes. This is correct as the independent and the semi-dependent load combinations are unrealistic: Assume a multi-storey house with the total design permanent load 1 on n floors. Now, if the number of floors n increases and the total permanent load remains and the total design load must remain, but the independent and the semi-dependent combination result in a decreased load.

4.2. COMBINATION OF PERMANENT AND VARIABLE LOADS

Structural design codes include a permanent load distribution G which defines the probability for the load not to reach the design point value. Accordingly, the variable load distribution Q defines an analogous probability for the variable load *during one year*. These distributions are independent and if combined, the combination distribution applies the random combination of the loads. The loads are combined in the structural design definitely, i.e. by accumulation and dependently.

A further reason for the dependent combination is that the permanent and the variable loads are dependent during the normal service time of structures: The probability of the variable load not to reach the design point value in one year is a low probability P_{f1} (0.02 in the eurocode). When time increases this probability increases, e.g. in t years it is $1 - (1 - P_{f1})^t$. We find that each fractile value of the Q distribution is associated to a fixed time. When the time is long, G and Q are fully dependent as all fractile values of G distribution occur at the same time as the corresponding fractile values of Q distribution. *Due to this relation, G and Q must be combined dependently, i.e. by the accumulation summation.*

4.3. COMBINATION OF VARIABLE LOADS

Two variable loads are combined almost analogously to the combination of the permanent and the variable load i.e. dependently when the distributions are first converted to the same time and simultaneous. In this combination, assumptions must be made about the basic characteristics of the variable loads. In my article (Poutanen, 2012) I assume that two kinds of variable loads exist: full time and part time loads, Figure 1. Each variable load has its characteristic duration d_w and d_s.

This variable load combination model is approximate: It is assumed that the load has a constant value through its duration i.e. the gradual increase and decrease of the load in the beginning and at the end is

[2] In this example the characteristic permanent load is the 0.98-value for the permanent load, in the eurocode it is the 0.5-value.

ignored. Further, appropriate load duration data, d_s and d_w, is lacking. However, this model allows us to make applicable conclusions by setting a range in the load durations.

When the wind load and the snow load is combined according to this model, the wind load distribution is converted to the time of the snow load and thereafter the loads are added up as such without any reductions, combination factors etc. In this combination, the snow load has no combination factor $\psi_0 = 1$, as the snow load distribution defined to one year is equal as the distribution defined to the winter. The wind load has a combination factor ψ_0 which is caused by the distribution conversion. Individual wind and snow loads are independent but combined dependently. These loads become dependent when time and number of loads increase. Ca 50 snow-wind-load pairs are virtually fully dependent during 50 years.

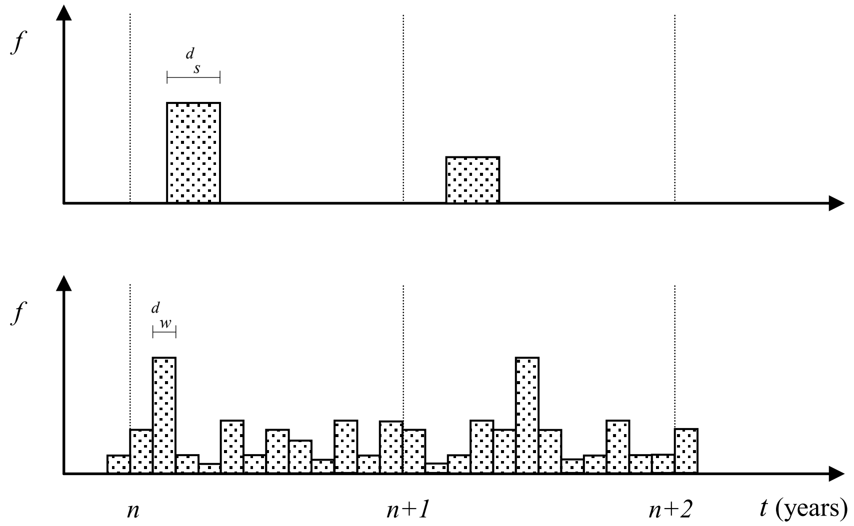

Figure 1. The variable load model, the part time load, upper Figure and full time load, lower Figure. The loads *f* are gumbel distributed and have a constant value within periods d_s and d_w.

We can change a distribution assigned to one year to *n* years by using an equation (EN 1990, 2002):

$$\Phi(\beta_n) = (\Phi(\beta_1))^n \tag{1}$$

where

β_1 reliability during one year, survival probability and the cumulative distribution value at the design point, in the eurocode $\beta_1 = 4.7$

β_n reliability at time *n*

n time (in years)

In the eurocode the variable load distribution $FG(x, \mu, \sigma)$ is gumbel and the target 50-year reliability value β_{50} is 3.826, where the parameters, μ, σ, can be solved $\mu = 0.2613$, $\sigma = 0.1045$. When the distribution is changed to time *n*, in years, the distribution must be multiplied by factor k_{Qn}, which can be solved from equation:

$$FG\left(1, \mu \cdot k_{Qn}, \sigma \cdot k_{Qn}\right) = \Phi\left(\beta_{50}\right)^{\frac{n}{50}} \tag{2}$$

Table 1 includes k_{Qn} – values for certain times n calculated from equation 2. The k_{Qn} – value is the load combination factor ψ_0, which is applied in the loads. Depending on the character of the loads the combination factor is applied in a fixed load, or in the lesser load.

Table 1. k_{Qn} –values calculated for certain times n.

Time n (load duration)	k_{Qn}
0.01 second	0.32
0.1 second	0.34
1 second	0.37
1 minute	0.41
10 minutes	0.45
1 hour	0.49
10 hours	0.53
1 day	0.56
1 week	0.61
2 weeks	0.63
1 month	0.66
2 months	0.68
3 months	0.70
6 months	0.73
1 year	1

According to this load combination model, the snow load never has a combination factor, $\psi_0 = 1$. A load or a sum of loads, combined to the snow load has a theoretical combination factor $\psi_0 = 0.63...0.73$ if the snow load lasts for 2 weeks...6 months. The code factor should be little higher to take into account the uncertainty of the model, ca $\psi_0 = 0.8$. If we assume that the live load and the imposed load lasts for 10 minutes...10 hours, these loads combined to each other have a theoretical combination factor $\psi_0 = 0.45...0.53$ and a code factor ca $\psi_0 = 0.6$ assigned in the lesser load.

When imposed loads are combined to each other a combination factor is not applied $\psi_0 = 1$ regardless of the duration of these loads as the imposed loads are proportions of the total imposed load in a house.

5. Conclusions

The simultaneous loads are always combined dependently in the structural design. If the loads are not simultaneous, the load distributions can be converted in a way the loads are simultaneous, i.e. the loads are always combined dependently.

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