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**Abstract:** The paper presents a complex methodology for statistical and reliability analyses of concrete structures. It describes the virtual simulation concept and tools used on the way from assessment of experimental results to reliability analysis. The methodology is represented by consequent steps starting by material parameters identification based on artificial neural networks and finite element modelling. The aim is finally to perform advanced reliability assessment using appropriate stochastic finite element model. Software tools are briefly described. Selected examples of application illustrate the applicability of the approach.

**Keywords:** Reliability, concrete, inverse analysis, artificial neural networks, nonlinear analysis, fracture mechanics, simulation.

## 1. Introduction

Reliable computing for reliability assessment requires combination of advanced techniques to treat both nonlinearity and uncertainty. A large number of efficient stochastic analysis methods have been developed during last years. The common feature of all methods is the fact that they require a repetitive evaluation (simulations) of the response or limit state functions. The development of reliability methods is from the historical perspective certainly a struggle to decrease an excessive number of simulations. In spite of the increasing capabilities of computer hardware using a large number of simulations is still a problem when dealing with computationally demanding tasks and small-sample simulation is needed.

The objective of the contribution is to present methods and software for efficient statistical, sensitivity and reliability assessment implemented in FReET software (Novák et al. 2011). The attention is given to those techniques that are developed for analyses of computationally intensive problems like nonlinear FEM. Sensitivity analysis is based on nonparametric rank-order correlation. Statistical correlation is imposed by the simulated annealing. As software development is performed in a complex project and system for reliability assessment of concrete structures SIMSOFT, the full role of software FReET will be also shortly described – including degradation module FReET-D and methodology for inverse analysis and identification.

The paper presents briefly a complex methodology for statistical, reliability and risk analyses of concrete structures. But the methodology is valid generally, not only for concrete structures. It describes the virtual simulation concept and tool used on the way from assessment of experimental results to reliability analysis. The whole approach is based on small-sample randomization of nonlinear fracture mechanics finite element analysis of reinforced concrete structures. Efficient techniques of both nonlinear numerical analysis of concrete structures and stochastic simulation methods have been combined in order to offer an

advanced tool for assessment of realistic behaviour of concrete structures from reliability and risk points of view.

# 2. Methodology

The stochastic response requires repeated analyses of the structure with stochastic input parameters, which reflects randomness and uncertainties in the input values. The system uses the nonlinear computer simulation for realistic prediction of structural response and its resistance. Nonlinear fracture mechanics simulation utilizes state of art techniques including: damage mechanics, fracture mechanics and plasticity material models, smeared crack approach - fictitious crack, crack band method, softening of concrete in both tension and compression, combination of nonlinear concrete behavior with discrete and smeared reinforcement in reinforced concrete and pre-stressed structures. As the nonlinear structural analysis is computationally very demanding, a suitable technique of statistical sampling should be utilized, which allows relatively small number of simulations. Final results are: statistical characteristics of response (stresses, deflections, crack width etc.), information on dominating and non-dominating variables (sensitivity analysis) and estimation of reliability using reliability index and theoretical failure probability). In order to use appropriate parameters of material laws in the computational model, an inverse analysis based on experiments in a laboratory or in situ has to be performed. A suitable technique for the inverse analysis is the stratified sampling scheme for the modeling of uncertain model parameters combined with artificial neural networks.

The procedure can be outlined as follows:

- experiment (laboratory, in situ);
- development of a deterministic computational model to capture the experiment;
- inverse analysis to obtain parameters of the computational model;
- deterministic computational model of a structure;
- stochastic model of a structure;
- statistical, sensitivity and reliability analyses of a structure.

## 3. Key soft computing methods

## **3.1. MATERIAL PARAMETERS IDENTIFICATION**

The basic step for efficient nonlinear FEM modeling is to solve the inverse problem: "Which material model parameters should be used to capture the experiment well?" The recently proposed identification strategy is based on a coupling of the stratified sampling in the nonlinear fracture mechanics analysis and in the artificial neural network (Novák & Lehký 2006). The fundamental scheme of the approach is shown in Fig. 1; the neural network is trained by the values of the load-deflection curve and the values of identified parameters (considered to be random variables) in a repeated stochastic way – the preparation of a training set for a neural network uses stratified simulation. A multiple calculation of a deterministic computational

model using random realizations of model parameters is performed resulting in a "bundle" of loaddeflection curves (usually overlapping the experimental curve). Realizations of the load-deflection curves serve as a basis for the training of an appropriate artificial neural network. Such training can be called stochastic training due to the stochastic origin of the load-deflection curves. After the training procedure, the neural network is ready for the key task: to select the material model parameters which can capture the experimental load-deflection curve as closely as possible.



Figure 1. Scheme of inverse analysis.

## **3.2.** UNCERTAINTIES SIMULATION

For time-intensive calculations such as those involving nonlinear fracture mechanics of concrete, smallsample simulation techniques based on stratified sampling of the Monte Carlo type represent a rational compromise between feasibility and accuracy. Therefore, Latin hypercube sampling (LHS) was selected as a key fundamental technique.

The method belongs to the category of stratified simulation methods (e.g. Mc Kay & Conover 1979, Novák et. al 1998). It is a special type of Monte Carlo simulation which uses the stratification of the theoretical probability distribution function of input random variables. It requires a relatively small (tens or hundreds) number of simulations (repetitive calculations of the structural response) to estimate the requested statistics of the response.

The basic feature of LHS is that the probability distribution functions for all random variables are divided into  $N_{Sim}$  equivalent intervals ( $N_{Sim}$  is the number of simulations); the values from the intervals are then used in the simulation process (random selection, middle of interval or mean value). This means that the range of the probability distribution function of each random variable is divided into intervals of equal probability. The samples are chosen directly from the distribution function based on an inverse transformation of the distribution function.

It has been proved that the best LHS strategy, which simulates the means and variances very well, is the approach suggested e.g. by Keramat & Kielbasa (1997) or Huntington & Lyrintzis (1998). The sample of each interval is chosen as the mean (Fig. 2):

$$x_{i,k} = \frac{\int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x) \, dx}{\int_{y_{i,k-1}}^{y_{i,k}} f_i(x) \, dx} = N_{Sim} \cdot \int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x) \, dx$$
(1)

where  $f_i$  is the PDF of variable  $X_i$ , and the integration limits are:

$$y_{i,k} = F_i^{-1} \left( \frac{k}{N_{Sim}} \right)$$
(2)

A robust technique for imposing statistical correlation based on the stochastic method of optimization, called simulated annealing, has been proposed recently by Vořechovský & Novák (2009). The imposition of the prescribed correlation matrix into the sampling scheme can be understood as an optimization problem.



Figure 2. Illustration of sampling.

#### **3.3. RELIABILITY ANALYSIS**

In cases when we are constrained by a small number of simulations (tens, hundreds) it can be difficult to estimate the failure probability. The following approaches are therefore utilized here; they are

approximately ordered from elementary (an extremely small number of simulations, inaccurate) to more advanced techniques:

- Cornell's reliability index the calculation of a reliability index from the estimation of the statistical characteristics of the safety margin;
- curve-fitting approaches based on the selection of the most suitable probability distribution of the safety margin;
- FORM approximation (Hasofer-Lind's index);
- importance sampling techniques;
- response surface methods.

These approaches are well known in reliability literature, and also the provision of all details is beyond the aim of this paper. In spite of the fact that the calculation of the failure probability (and/or reliability index) using some of these techniques does not always belong to the category of very accurate reliability techniques (the first three in the list), they represent a feasible alternative in many practical cases.

# 4. Software tools

## 4.1. SARA - COMPLEX SOLUTION

The authors combined efficient techniques of both nonlinear numerical analysis of engineering structures and stochastic methods to offer an advanced tool for the reliability assessment of concrete structures. Within the framework of this complex system attention is also paid to the modeling of degradation phenomena, such as carbonation of concrete, corrosion of reinforcement, chloride attack, etc. The combination of all parts (structural analysis, reliability assessment, inverse analysis and degradation modeling) is presented together as the SARA software. The recently developed version of the SARA software is called RLACS or SARA Science; its structure is similar to the SARA system, but it incorporates an extended version of the ATENA NLFEM software: ATENA Science.

A representation of the program combination within SARA software is presented in Fig. 3. It includes: SARA (Bergmeister et al. 2004, Pukl et al. 2003a,b; Strauss et al. 2008; Novák et al. 2005) – a software shell which controls the communication between following individual programs: ATENA (Červenka et al. 2007) – FEM nonlinear analysis of concrete structures; FReET (Novák et al. 2011) – the probabilistic engine based on LHS simulation; DLNNET (Lehký 2011; Novák & Lehký 2006) – artificial neural network software; FReET-D (Teplý et al. 2011) – degradation module based on FReET. The fundamental version of ATENA is called ATENA Engineering; its native GUI is directly integrated into the SARA system as shown in Fig. 3. Recent development of ATENA represents ATENA Science package. Within the reliability analysis it is controlled by RLACS Studio through special commands in ATENA input files.

## 4.2. FREET – UNCERTAINTIES SIMULATION

The probabilistic software FReET (Novák et al. 2011) allows simulations of uncertainties of the analyzed problem basically at random variables level (typically in civil/mechanical engineering – material properties,



Figure 3. The program combination within SARA software.

loading, geometrical imperfections, environment). The attention is given to those techniques that are developed for analyses of computationally intensive problems; nonlinear FEM analysis being a typical example. Stratified simulation technique Latin hypercube sampling (LHS) is used in order to keep the number of required simulations at an acceptable level (Novák et al. 1998). This technique can be used for both random variables' and random fields' levels.

Statistical correlation is efficiently imposed by the stochastic optimization technique – the simulated annealing (Vořechovský & Novák 2009). Sensitivity analysis is based on nonparametric rank-order correlation coefficients and may serve e.g. for model reduction in subsequent analyses. State-of-the-art probabilistic algorithms are implemented to compute the probabilistic response and reliability generally, including durability limit states.

# 4.3. FREET-D – DEGRADATION SIMULATION

There are many predictive computational models for degradation modelling mainly carbonation of concrete, chloride ingress and corrosion of reinforcement at different sophistication levels. Frequently, heuristic models are employed using more or less simplified approaches and data. Common feature of all these models is that input data are very uncertain. There is a software implementation where all relatively well-known models are summarized within the framework of unified software environment. It is called FReET-D where a combination of analytical models and simulation techniques has been amalgamated to form specialized software for assessing the potential degradation of newly designed as well as existing concrete structures (Teplý et al. 2011, 2012; Veselý et al. 2010). Models implemented (mainly simple-to-use "point-in-space" probabilistic models) for carbonation, chloride ingress, corrosion of reinforcement and others which may serve directly in the durability assessment of concrete structures in the form of a durability limit states, i.e. the assessment of service life and the level of the relevant reliability measure. Several features are offered including parametric studies and Bayesian updating. Altogether, 32 models are implemented as pre-defined dynamic-link library functions. FReET-D actually represents a specialized module of FReET software (Novák et al. 2011), mentioned above.

## 4.4. ATENA - NONLINEAR SIMULATION

The ATENA software (Červenka et al. 2002, 2007) was developed for realistic simulation of reinforced concrete structures. It is based on the finite element method with non-linear material models, and utilized for analysis of beams and girders, plates and shells, bridges, tunnels, dams, composite structures, strengthening, structural details, fastenings, fibre reinforced structures and masonry structures etc.

The ATENA software consists of calculating core ensuring the non-linear numerical analysis, and a user-friendly graphical interface for an efficient communication between end-user and program core. The numerical core covers the finite element technology, non-linear material models and non-linear solution. The non-linear material models are based on the orthotropic damage theory and special concrete-related theory of plasticity. As one of the main features the non-linear fracture mechanics is employed for concrete cracking in tension. Based on the fracture energy approach the tensile cracks are modeled as smeared material damage which enables utilization of the continuum mechanics even for the damaged material. Objectivity of the solution is ensured using crack band method. The material law exhibits softening after reaching the tensile strength. The behavior of concrete in compression is defined by special theory of plasticity (three-parameter model), with non-associated plastic flow rule and softening. This material model for concrete can successfully reproduce also other important effect, such as volume change under plastic compression or compressive confinement. The native graphical user-interface supports all the specifics of reinforced concrete, e.g. input of discrete reinforcing bars, or evaluation of crack patterns in the damaged structural model.

The new ATENA software class ATENA Science (www.cervenka.cz) enables time-dependent (dynamic, fatigue) and temperature-dependent (fire resistance) nonlinear analysis of complex concrete structures. In the last version the ATENA Science is equipped with a new user friendly interface shell called ATENA Studio.

## 5. Examples of application

The complex methodology and software has been applied mainly for reliability analysis of concrete bridges, e.g. Pukl et al. 2003ab, Lehký et al. 2010, Podroužek et al. 2010, Strauss et al. 2008. Detailed description of particular application for deteriorated bridge structure can be found in fib bulletin 62 Structural concrete, section 9.19., that example can be regarded as the most elaborated one.

The interesting application is analysis of facade panels made of alternative FRC-material. As it represents a new facade system utilizing a new composite material which exhibits a large variability, computational analysis was desirable to address reliability issues connected with this special structure (Keršner et al. 1997).

A 3D FEM computational model has been developed using ATENA 3D Engineering nonlinear fracture mechanics software (Červenka et al. 2007). Wind intake was simulated by continuous loading. The Newton-Raphson solution method with a loading increment step of 1 kN/m<sup>2</sup> provided a non-linear solution to obtain ultimate load, cracks at final stage are depicted in Fig. 4. A 3D cementitious material model was used with material parameters identified by identification technique based on artificial neural networks. All of the input basic random variables involved and the particular set of their statistical parameters (mean

value, COV, probability distribution function (PDF)) are summarized in Table I for both the reference panel (R) and the degraded panel (D).

Statistical simulations were performed using parameters from Table 1, the resulting bundle of l-d curves is shown in Fig. 5 and ultimate load statistics were evaluated. Action of load – wind intake was considered deterministic at several levels up to 20 kN/m<sup>2</sup>. The theoretical failure probability – the probability that the panel will not resist the load (wind intake) was calculated using mathematical model of a PDF. The results of this reliability study are shown in Fig. 6.



Figure 4. Cracks at failure of panel.

Table I. Basic faildoill variables of hole-feililoiced collefete.					
Variable	Unit	Mean		COV	PDF
Modulus of elasticity	GPa	10.1	R	0.195	Rayleigh
		7.8	D	0.199	Weibull min
Compressive	MPa	53.5	R	0.250	Log-normal
strength		31.5	D	0.250	Log-normal
Tensile	MPa	6.50	R	0.250	Weibull min
strength		3.81	D	0.250	Weibull min
Fracture	J/m <sup>2</sup>	816.2	R	0.383	Weibull max
energy		195.8	D	0.418	Log-normal

Table I. Basic random variables of fibre-reinforced concrete.



Figure 5. Random *l*-*d* curves – the reference facade panel.



Figure 6. Theoretical failure probabilities for different levels of load - wind intake.

# 6. Conclusions

Virtual simulation concept and tools used on the way from assessment of experimental results to reliability analysis are briefly presented. The advanced methods for nonlinear, stochastic, reliability and degradation analysis were integrated into software package usable for complex reliability assessment of engineering

structures, which enables realistic simulation of the structural behaviour, damage and failure accounting uncertainties of input parameters, nonlinear material response and material deterioration. The presented tools and methods have been used in numerous practical applications of analysis, design, and life time assessment of concrete bridges, buildings, tunnels, power plants and other civil engineering structures.

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