Robust Design Optimization in industrial virtual product development

Thomas Most & Johannes Will
Dynardo – Dynamic Software and Engineering GmbH, Weimar, Germany
thomas.most@dynardo.de, johannes.will@dynardo.de

Abstract. In this paper different strategies to search for a robust design are presented and investigated with respect to their efficiency and applicability to time consuming numerical models. After starting with deterministic optimization we introduce different measures to define the robustness of a design. An iterative Robust Design Optimization (RDO) is proposed where deterministic optimization is combined with variance-based robustness analysis and final reliability proof. The iterative procedure is compared to coupled RDO approaches, where the robustness or reliability measures are calculated for each optimization design. For such a procedure often global approximation models are used in order to enable the application for more complex problems.

Keywords: Robustness, reliability, optimization, sensitivity analysis

1. Introduction

Due to target-oriented, automatic optimization of virtual products new design possibilities are explored. However, highly optimized designs lead to high imperfection sensitivities and tend to loose robustness. Often the deterministic optimum is pushed to the boundaries of the feasible design space. As a result the optimized design, which was found by assuming deterministic model properties, may not be realizable in a production process. For this reason it is necessary to investigate, how the optimized design is affected by scattering model input variables, which could be e.g. geometry and material parameters, boundary conditions and loads. The scattering inputs are modeled int this paper by means of scalar random variables having a certain dependence between each other. Random variables have the advantage compared to other uncertainty models, that efficient methods of the well developed probability theory can be applied.

In this paper different strategies to search for a robust design are presented and investigated with respect to their efficiency and applicability to time consuming numerical models. After starting with deterministic optimization we introduce different measures to define the robustness of a design. An iterative Robust Design Optimization (RDO) is proposed where deterministic optimization is combined with variance-based robustness analysis and final reliability proof. This procedure is state-of-the-art in Dynardo's supported RDO projects (Roos and Hoffmann2008),(Roos et al.2009). The iterative procedure is compared to coupled RDO approaches, where the robustness or reliability measures are calculated for each optimization design. For such a procedure often global approximation models are used in order to enable the application for more complex problems. All presented methods are available in Dynardo's optiSLang software package (optiSLang2011), which supports a wide range of CAE solvers in order to perform a reliable optimization, sensitivity, robustness and reliability analysis as well as Robust Design Optimization.

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2. Deterministic optimization

For deterministic single-objective optimization problems the optimization task can be formulated by a single scalar-valued objective function

$$f\left(x_1, x_2, \ldots, x_k\right) \rightarrow \min, \tag{1}$$

which is often an implicit function of the design variables. The design variables can be defined as continuous variables with a lower and upper bound or as discrete variables which assume several discrete values. In unconstrained optimization problems only the bounds or values of the design variables limit the optimization space. The optimizer searches between these limits for the minimum value of the objective function $f(\mathbf{x})$.

In engineering problems often additional restrictions have to be fulfilled by the optimal design. With help of equality and inequality constraints

$$g_i(x_1, x_2, \ldots, x_k) = 0,\ i = 1 \ldots m_e, \quad h_j(x_1, x_2, \ldots, x_k) \geq 0,\ j = 1 \ldots m_u, \tag{2}$$

such restrictions can be formulated.

As optimization pre-processing a global sensitivity analysis may help to understand or to formulate the optimization problem and to possibly reduce the number of optimization variables, which enables the application of more efficient optimization strategies. In our analysis we perform variance based sensitivity analysis (Saltelli et al.2008). By representing continuous optimization variables by uniform distributions, variance based sensitivity analysis quantifies the contribution of each optimization variable to a possible improvement of the model responses. In contrast to local derivative based sensitivity methods, the variance based approach quantifies the contribution with respect to the defined variable ranges. Using the results of the sensitivity analysis the number of optimization variables may be reduced and suitable start points can be found for a following optimization.

Unfortunately, sufficiently accurate variance based methods require huge numerical effort due to the large number of simulation runs. Therefore, often meta-models are used to represent the model responses surrogate functions in terms of the model inputs. However, many meta-model approaches exist and it is often not clear which one is most suitable for which problem (Roos et al.2007). Another disadvantage of meta-modeling is its limitation to a small number of input variables. Due to the curse of dimensionality the approximation quality decreases for all meta-model types dramatically with increasing dimension. As a result, an enormous number of samples is necessary to represent high-dimensional problems with sufficient accuracy. In order to overcome these problems, Dynardo developed the Metamodel of Optimal Prognosis (Most and Will2008),(Most and Will2011). In this approach the optimal input variable subspace together with the optimal meta-model approach are determined with help of an objective and model independent quality measure, the Coefficient of Prognosis.

In Figure 1 the recommended flow of single-objective optimization procedure is shown: after the definition of the design variables and objective and constraint functions the design space is explored by sensitivity analysis. The obtained variable sensitivities may help to reduce the number of design variables. The best designs found in the sensitivity analysis could be used as start designs for the following optimization procedure which finally will determine an optimal design.

Figure 1. Flowchart of deterministic single-objective optimization

3. Robust Design Optimization

In Robust Design optimization the optimization task is formulated under the consideration of uncertainties. For this purpose we model the uncertainties with scalar random variables with a given distribution type and a possible correlations. In the RDO framework the optimization variables itself (e.g. geometry parameters of a structure) and even additional variables (e.g. material properties) may be assumed as random. This may result in pure optimization, pure stochastic and mixed optimization-stochastic variables. Additionally to the deterministic objective and constraint functions the robustness of a design is considered within the RDO procedure.

A robust design may be characterized intuitively in that way, that its performance is largely unaffected by random perturbations of the model inputs. A possible measure is the variance indicator, where the relative variations of the critical model responses are compared to the relative variation of the input variables. If certain model responses are limited with respect to an undesired performance, the safety margin can be quantified as the interval between the mean value of the model response and the limit. This is shown in

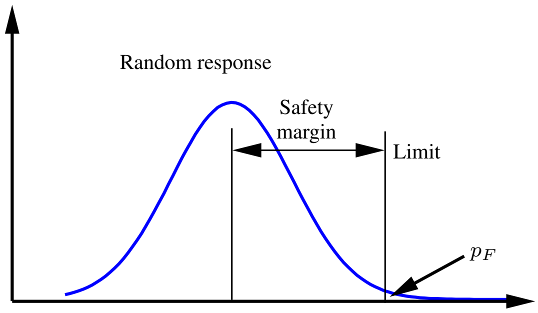


Figure 2. Random model response with given limit value and corresponding safety margin and failure probability p_F

Figure 2. The safety margin can be formulated in terms of the standard deviation of the model response. In the variance-based robustness analysis a specific safety margin $\alpha\sigma_Y$, which has to be defined by the designer, has to be proven for all critical responses

$$\|y_{limit} - \bar{Y}\| \leq \alpha\sigma_Y. \tag{3}$$

Alternatively the probability that a certain limit is exceeded can be quantified and proven to be less than an acceptable value. This probability indicator can be evaluated by the probability-based robustness analysis, which is called reliability analysis.

3.1. Variance-based robustness analysis

In variance-based robustness analysis the variations of the critical model responses are investigated. In our study random sampling methods are used to generate discrete samples of the joined probability density function of the given random variables. Based on these samples the statistical properties of the model responses as mean value, standard deviation, quantiles and higher order stochastic moments are estimated. In order to obtain a sufficient quality of these estimates, it is required, that the sampling scheme represents the marginal distributions of the single random variables as well as the defined correlations between each other with high accuracy. Some very basic stochastic methods to generate sample sets are variants of the Monte-Carlo method. The simplest version is the so-called plain Monte-Carlo method (PMC). With this methods the natural scatter can be modeled quite well, but the statistical uncertainty is fairly large if the sample size is small. Therefore we utilize Latin Hypercube Sampling (LHS) with minimized correlation errors (Hungtington and Lyrintzis1998), where the marginal distributions and the predefined input correlations are represented with a small number of samples.

Based on the estimates of the mean value and the standard deviation the safety margin can be estimated by using Eq. (3) for the responses where a performance limit is given. However, by using variance-based robustness analysis only safety margins up to two sigma can be proven with a small number of samples. For larger safety margins (e.g. six sigma) the true failure rate may be heavily vary for different distribution types of the output. Since the distribution of the output is not exactly known, an estimate of low failure probabilities by variance-based measures may be very inaccurate. Therefore, we recommend to prove safety margins larger than three sigma by reliability analysis.

3.2. Reliability analysis

In reliability analysis the limit state function divides the random variable space into a safe domain $S = \{\mathbf{x} : g(\mathbf{x}) > 0\}$ and a failure domain $F = \{\mathbf{x} : g(\mathbf{x}) \leq 0\}$. The vector $\mathbf{x}$ denotes a position in the space spanned by the random variable vector $\mathbf{X} = [X_1, X_2, \ldots, X_m]$. The failure probability p_F is defined as the integral of the joint probability density function $f_{\mathbf{X}}(\mathbf{x})$ of the random variables with respect to the failure domain

$$p_F = P\left[\mathbf{X} : g(\mathbf{X}) \leq 0\right] = \int_{g(\mathbf{X})\leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}. \tag{4}$$

The computational challenge in determining the integral of Eq. (4) lies in the evaluation of the limit state function $g(\mathbf{x})$ at a specific position $\mathbf{x}$. In CAE-based analyses the limit state function is generally an implicit function of the input variables.

The most simple and robust method for the evaluation of Eq. (4) is the Monte Carlo Simulation (MCS) where the estimated failure probability is obtained from a set of n samples $\mathbf{x}_i$ as

$$\hat{p}_F^{MCS} = \frac{1}{n} \sum_{i=1}^{n} I\left(g(\mathbf{x}_i)\right), \tag{5}$$

where the indicator function $I\left(g(\mathbf{x}_i)\right)$ is one if $g(\mathbf{x}_i)$ is negative or zero and zero else. MCS can represent arbitrary types of LSFs including discontinuities and multiple design points. The disadvantage of this method is the large number of required samples, which increases dramatically with decreasing failure probability.

Thus for engineering problems, where we deal with small probabilities of failure, this method may be very inefficient.

Further well-known methods are directional sampling (Bjerager1988), which can be applied for small failure probabilities but is limited to a small number of random variables, and the First Order Reliability Method (FORM) (Hasofer and Lind1974), which may be very efficient with respect to the number of solver evaluations. However, FORM is limited to only one dominant failure region and to a smooth limit state function, if gradient-based methods are used for the design point search.

In our study we investigate a global approximation technique, the Metamodel of Optimal Prognosis (Most and Will2008), where a global or local polynomial is constructed on the samples obtained in the variance-based robustness analysis and an adaptive approximation technique (Roos and Adam2006), (optiSLang2011) where the regions around possible design points are adapted with new support points for the approximation. The adaptive method has been proven to be very efficient for industrial problems with nonlinear limit state functions and multiple design points (Roos and Hoffmann2008),(Roos et al.2009).

3.3. Iterative Robust Design Optimization

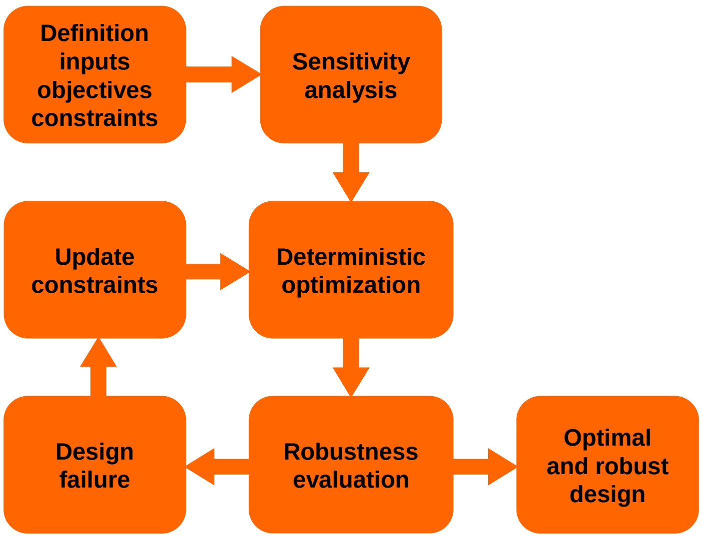


Figure 3. Flowchart of the iterative Robust Design Optimization

In iterative RDO procedure deterministic optimization is utilized by considering safety factors within the constraint conditions. These safety factors should be chosen in that way, that the robustness requirements are fulfilled. Generally the safety factors are not known a priori. In this case a suitable initial guess is specified and the initial deterministic optimization is performed. Additionally the robustness criteria are evaluated at the optimal design found by the optimizers. If the robustness requirements are not fulfilled, the optimization constraints are adjusted in a next step and the deterministic optimization procedure and the corresponding robustness analysis are performed again. This procedure is repeated until the robustness requirements are fulfilled. In Figure 3 the flowchart of the iterative RDO procedure is shown. If a small probability of failure is required a final reliability analysis is performed.

In order to fulfill the required failure probability a suitable safety margin has to been chosen. Since the distribution of the investigated responses is unknown, this choice may be critical for the success of the iterative RDO procedure. In Table I the safety margin is given for different distribution types at different failure probabilities. The table indicates for $p_F = 10^{-2}$ a much smaller deviation between the different distributions as for $p_F = 10^{-6}$. This means that for small failure probabilities the safety margin used in the iterative RDO procedure should be taken by assuming a non-normal distribution. For example for $p_F = 10^{-6}$ a safety margin of 6σ may be a good choice.

Table I. Required safety margins to assure a given failure probability p_F

Distribution type	Required safety margin		
	$p_F = 10^{-2}$	$p_F = 10^{-3}$	$p_F = 10^{-6}$
Normal	2.32	3.09	4.75
Log-normal	3.37	5.70	14.90
Rayleigh	2.72	3.76	6.11
Weibull	2.67	3.66	5.88

3.4. Coupled Robust Design Optimization

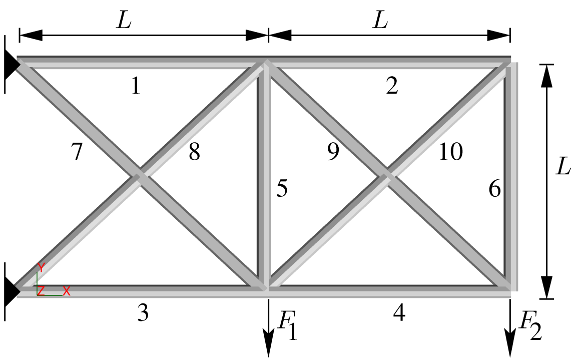
In the coupled RDO procedure an optimization is performed by considering robustness and reliability constraints directly. This means that the robustness and/or reliability measures have to been evaluated at every optimization (nominal) design. This leads to a nested double loop with the pure optimization procedure in the outer loop and the robustness analysis in the inner loop. This procedure may require a very large number of solver runs, especially if the optimization is coupled with reliability analysis. Such a strategy would limit the coupled procedure to simple and fast models. For more complex problems an improvement with respect to the number of solver runs is necessary.

One possibility could be to use a global approximation of the model responses with respect to the optimization and stochastic variables. For this purpose support points have to been generated in the mixed optimization-stochastic space which cover the possible variable values sufficiently. However, since the approximation is not exact a final robustness or reliability proof of the obtained design should be performed.

Another possibility to reduced the numerical effort of a coupled RDO procedure is to use an estimate of the safety margin, similar to the iterative approach, but with a reduced number of samples for the calculation of the mean values and standard deviations. However, in this case statistical errors may be significant and the corresponding objective and constraint function may contain additional noise. Therefore only optimization strategies should be applied which can handle such distortions. Again a more accurate robustness or reliability proof should by performed for the detected optimal design.

4. Application example

In the following example the proposed methodology is applied exemplary. For this purpose the ten-bar-truss structure shown in Figure 4 is investigated. This structure has been investigated e.g. in (Haftka and Gürdal1992). The optimization task is to minimize the mass of the truss structure. The absolute stresses of the each of the trusses should not exceed 30000 psi. Under the consideration of scattering cross sections (normally distributed with coefficient of variation of 5%), scattering material properties (Young's modulus is log-normally distributed with 5% variation) and scattering loads (normally distributed with 10% variation) the total probability of exceeding the stress limit in one of the trusses should be below 10^{-6}. The cross section areas a_i are taken as continuous optimization variables with the bounds given in Figure 4.



$$
\begin{aligned}
F_1 &= F_2 = 100000 \text{ lb}_f \\
E &= 10^7 \text{ psi} \\
\rho &= 0.1 \text{ lbs/in}^3 \\
L &= 360 \text{ in} \\
0.1 \text{ in}^2 &\leq a_i \leq 20 \text{ in}^2
\end{aligned}
$$

Figure 4. Investigated initial truss structure

4.1. Deterministic optimization

In a first step a pure deterministic optimization is performed in order to find a suitable truss topology. In the next section this optimal topology is optimized under the consideration of uncertainties. The limit of the maximum stress is reduced by a global safety factor of 1.2 to 25000 psi. Before starting the optimization task a sensitivity analysis is done. For this purpose 100 Latin Hypercube samples are generated uniformly distributed in the space of the optimization variables. Each design is evaluated by a finite element solver using geometrically linear truss elements. Using the MOP approach for sensitivity analysis the variable importance is quantified. The MOP approach indicates highly nonlinear dependencies between the optimization variables and the truss stresses as indicated in Figure 5. The number of optimization variables can not be reduced in this example since each cross section is the most important variable with respect to the belonging stress value.

The best design of the sensitivity analysis which fulfills the constraints is taken as start point for a gradient-based optimization. The mass of this start design is 3369.4 lbs. After 13 iterations with total 143 solver calls the optimizer found the optimal parameter set indicated in Figure 6. The parameter values agree excellent with the solution given in (Haftka and Gürdal1992). The results indicate, that the trusses 2,5,6 and 10 are set to its minimum value since they are not needed to carry the loads. The total mass of the optimized structure is 1593.2 lbs which is less than 50% of the start design.

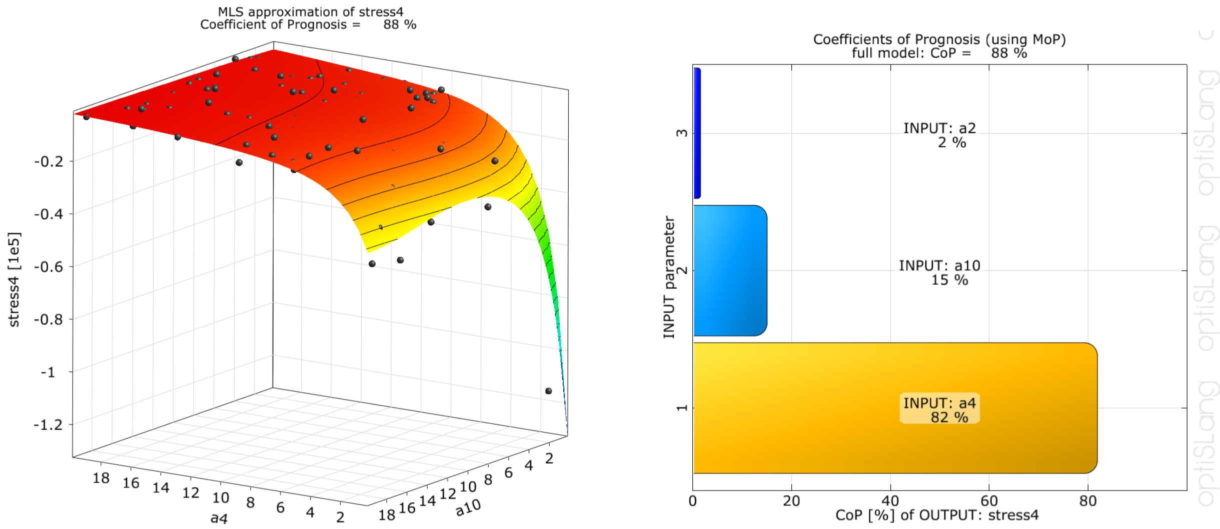


Figure 5. Approximation function and variable sensitivities obtained with the MOP approach using 100 Latin Hypercube samples as design exploration

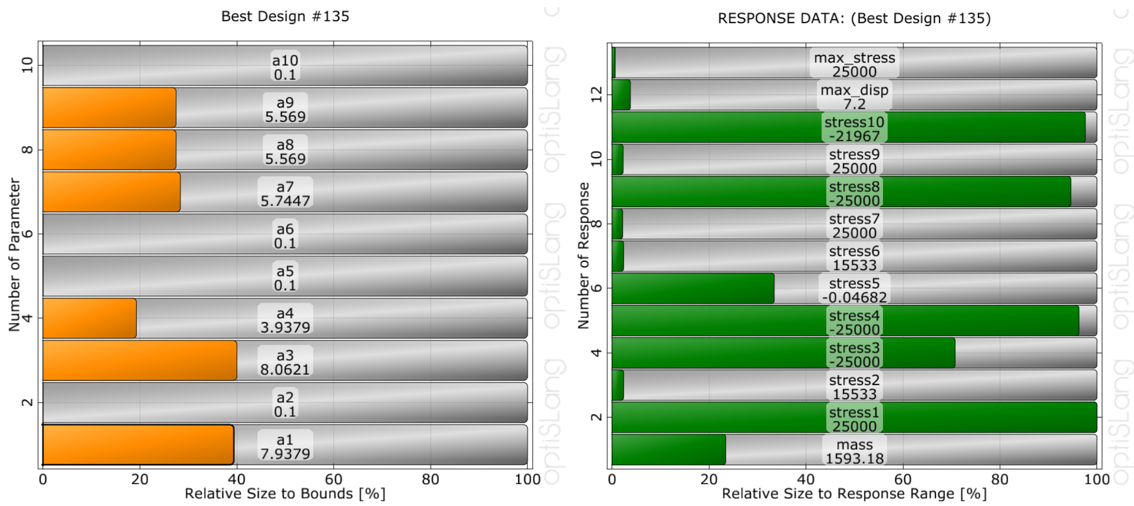
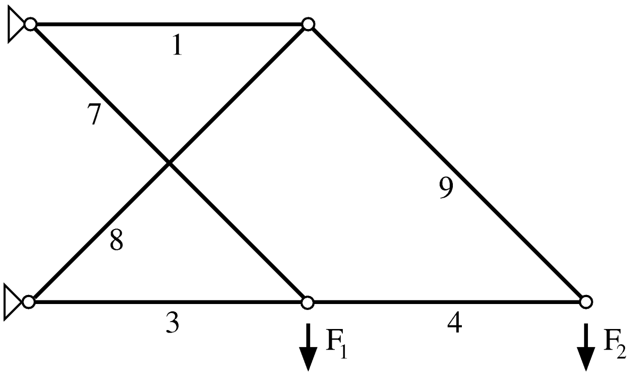


Figure 6. Results of deterministic gradient-based optimization of the full truss structure

The ten-bar-truss can be reduced by removing the unimportant trusses 2,5,6 and 10 from the structure. The stresses of the reduced structure can be simply calculated by using equilibrium equation of the forces at each of the truss nodes. The resulting stresses are given in Figure 7. Since the structure is statically determined the stresses in the trusses are independent of the Young's modulus. Thus it is not necessary to consider the Young's modulus in the optimization or robustness analysis anymore. In Figure 8 the results of a gradient-based optimization of the reduced truss structure are given. The figure indicates a slightly lower mass of the structure. Furthermore the stress limit is reached in all trusses.



$$
\begin{aligned}
stress1 &= \frac{2F_2}{a_1} & stress3 &= -\frac{F_1 + F_2}{a_3} \\
stress4 &= -\frac{F_2}{a_4} & stress7 &= \sqrt{2}\frac{F_1}{a_7} \\
stress8 &= -\sqrt{2}\frac{F_2}{a_8} & stress9 &= \sqrt{2}\frac{F_2}{a_9}
\end{aligned}
$$

Figure 7. Reduced truss structure with analytical stress values

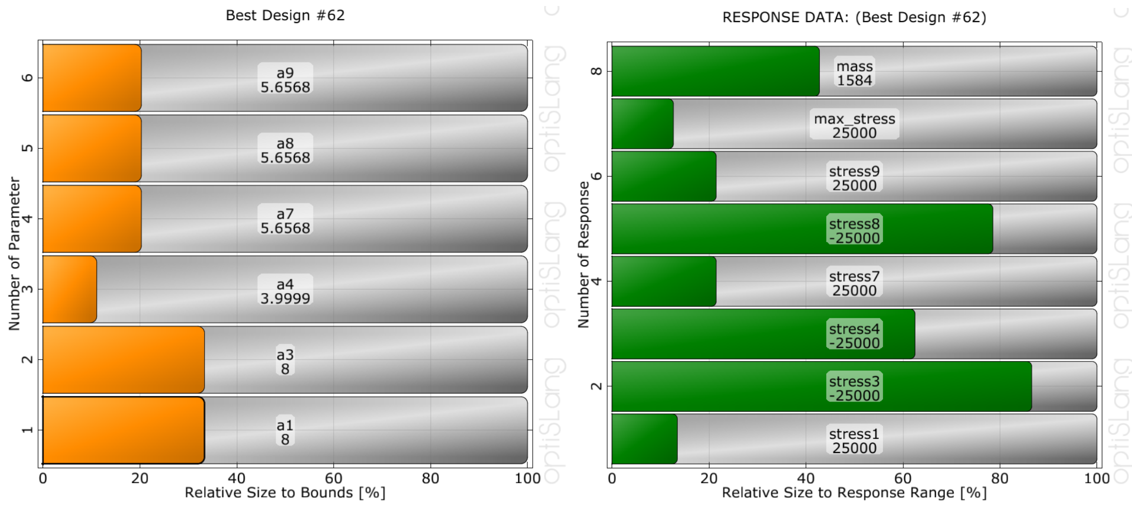


Figure 8. Results of deterministic gradient-based optimization of the reduced truss structure

4.2. Iterative Robust Design Optimization

The iterative Robust Design Optimization is performed by combining deterministic optimization using safety factors for the constraint conditions with variance-based robustness analysis. If the robustness analysis indicates a robust design the required failure probability is proven by reliability analysis. For this purpose the optimized reduced truss structure shown in Figure 7 is investigated by variance-based robustness analysis. The assumed statistical properties and distribution types of the scattering variables are used to generate 100 Latin Hypercube samples. Based on the solver evaluations the statistical properties of the truss stresses can be obtained. In Table II the results are given for the first optimization step including following robustness analysis. The table indicates for five of the six trusses a safety margin of about 1.75σ. Since we want to obtain a structure with a failure probability below 10^{-6} a safety margin of 6σ seems to be necessary in order to consider non-normal distributions of the output (see Table I). If we assume that the coefficient of variation of each stress values is constant, if the mean value is changed, we can estimate the required constraint for

Table II. Iterative Robust Design Optimization and final reliability proof of the reduced truss structure

Optimization Step 1	68 designs	Constraint	a_1	a_3	a_4	a_7	a_8	a_9	mass
		25000	8.000	8.000	4.000	5.657	5.657	5.657	1584.0
Robustness Step 1	100 samples	max. stress	stress1	stress3	stress4	stress7	stress8	stress9	
Mean value		27540	25070	-25060	-25060	25060	-25060	25060	
Standard deviation		2425	2853	2105	2825	2765	2788	2827	
Cov. of variation		0.088	0.114	0.084	0.113	0.110	0.111	0.113	
Safety margin		1.01σ	1.73σ	2.35σ	1.75σ	1.79σ	1.77σ	1.75σ	

Optimization Step 2	35 designs	Constraint	a_1	a_3	a_4	a_7	a_8	a_9	mass
		18000	11.111	11.111	5.555	7.857	7.857	7.857	2200.0
Robustness Step 2	100 samples	max. stress	stress1	stress3	stress4	stress7	stress8	stress9	
Mean value		19810	18050	-18050	-18040	18040	-18050	18040	
Standard deviation		1772	2044	1552	2000	1988	2060	1991	
Cov. of variation		0.089	0.113	0.086	0.111	0.110	0.114	0.110	
Safety margin		5.75σ	5.85σ	7.70σ	5.98σ	6.02σ	5.80σ	6.01σ	

Reliability analysis	Number of solver runs	Failure probability	Reliability index
Directional sampling	3674	$3.19 \cdot 10^{-7}$	4.98
FORM	225	-	9.70
MOP+DS	100 (from robustness)	$5.05 \cdot 10^{-7}$	4.89
ARSM+DS	101	$5.75 \cdot 10^{-7}$	4.86

the second iteration step by an extrapolation of the mean stress value

$$\begin{aligned} constraint_{step2} + 6 \cdot CV_{step1} \cdot constraint_{step2} &\leq 30000, \\ constraint_{step2} &= 30000/(1 + 6 \cdot CV_{step1}), \end{aligned}$$

which leads to a value of about 18000. The deterministic optimization is repeated with the new constraint value and the robustness of the optimized structure is assessed again by 100 Latin Hypercube samples. Table II indicates that the optimized structure of step 2 almost fulfills a safety margin of 6σ for all truss stresses.

Finally the failure probability is estimated more accurately by reliability analysis. For this purpose we investigate different methods with respect to their efficiency and accuracy. As reference solution directional sampling is used. First the First Order Reliability Method (FORM) is applied which converges to a minor important design point with a very low failure probability. Due to the individual stress limits in the six trusses the combined limit state function has several kinks and design points which lead to the wrong convergence point of FORM.

As second procedure we use a global approximation with the robustness samples as support points. For this purpose the Metamodel of Optimal Prognosis is built with these samples and an almost linear

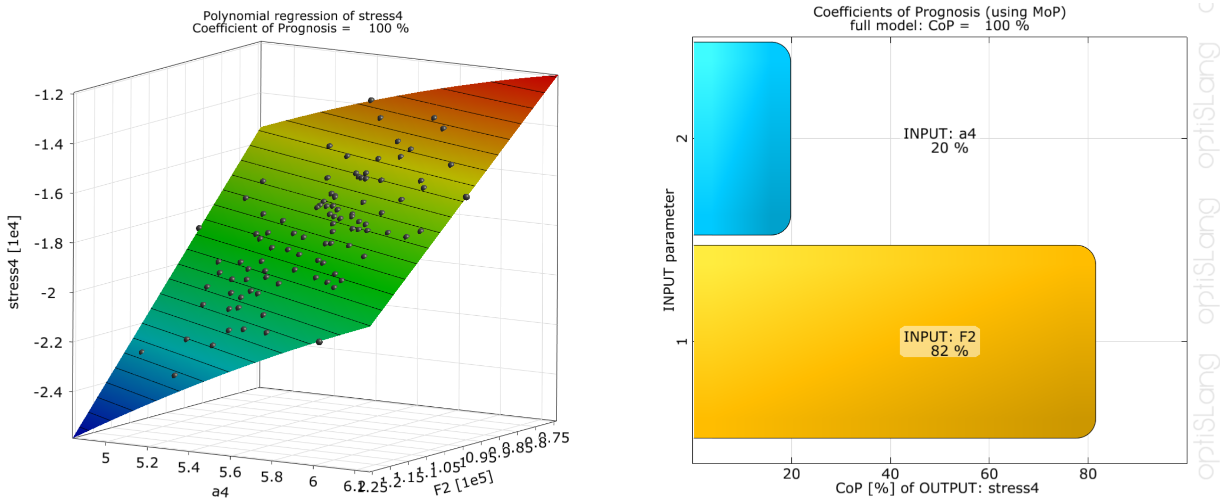


Figure 9. Approximation function and variables sensitivities obtained with the MOP approach using the 100 robustness samples of step 2 (CoP=100.00%)

dependence is indicated as shown in Figure 9. The individual stress values can be represented with very excellent approximation quality indicated by a Coefficient of Prognosis of 100.00%. However, since only 100 robustness samples are generated with the original distributions functions, the estimation of a very small failure probability requires an extrapolation of the approximation model, which may lead to a wrong estimate of the failure probability. Nevertheless, the calculated failure probability given in Table II shows good agreement with the reference from direction sampling. Since the robustness samples are available anyhow, this procedure requires no additional solver runs and should be investigated if the MOP indicates a good approximation quality.

The third investigated procedure is the Adaptive Response Surface Method according to (Roos and Adam2006). This methods uses an initial sampling scheme as support points which is stretched by factor three in order to cover a larger domain. With two additional adaption steps, where new sampling schemes are placed around the detected important regions, the method converges to a failure probability close to the reference solution. Since the number of solver evaluations is very small and since no extrapolation is used in the approximation, from our viewpoint this ARSM approach is the method of choice for reliability analysis of complex engineering problems. The proposed iterative Robust Design Optimization procedure including the ARSM reliability proof has been successfully applied to real industrial problems in (Roos and Hoffmann2008),(Roos et al.2009).

4.3. Coupled Robust Design Optimization

In a further analysis the coupled Robust Design Optimization approach is applied. For this purpose in a first step a global approximation model is used and in a second step direct solver runs are evaluated. The global approximation model is built by using a uniform distribution for all optimization and stochastic variables, where the lower and upper bounds are taken for the cross section areas as $2 \text{ in}^2 \leq a_i \leq 20 \text{ in}^2$ and for the pure stochastic forces the bounds are taken as mean value +/- 5σ. 500 Latin Hypercube samples

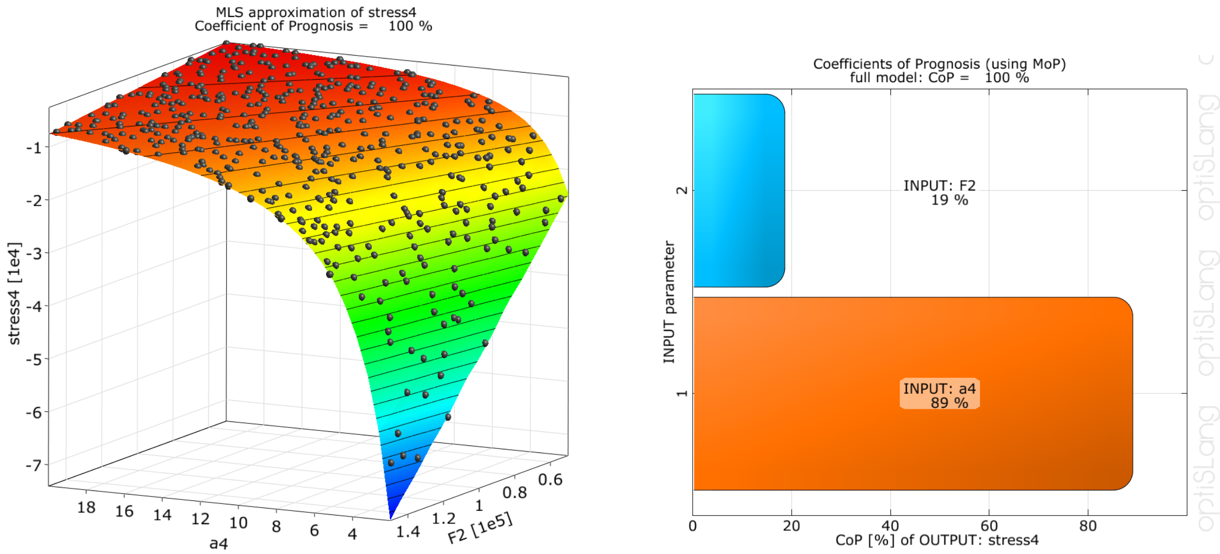


Figure 10. Approximation function and variables sensitivities obtained with the MOP approach using the 500 samples in the mixed design-stochastic space (CoP=99.99%)

are generated within this mixed optimization-stochastic space and the Metamodel of Optimal Prognosis is built. For all individual stress values and the mass an approximation quality better or equal 99.99% could be reached by the quadratic Moving Least Squares approximation, which is included in the MOP approach. In Figure 10 the approximation function and the variable sensitivities are shown exemplary. If we compare the variable sensitivities of Figure 10 with these of Figure 9, we notice, that the pure stochastic force variation is dominant in the local robustness problem but minor important in the mixed space. In many other applications we observed similar results, that the pure stochastic variables are minor dominant with respect to the optimization variables in the mixed space due to their smaller variation. This fact may lead to an inaccurate representation of the influence of the stochastic variables in an approximation model. As a consequence the estimated robustness measures may be inaccurate as well.

For our example we use an Evolutionary Algorithm (EA) running with the approximation model, where for each optimization design a variance-based robustness analysis is performed by using 100 Latin hypercube samples. The mass is taken as deterministic objective function and the stress constraints are formulated with respect to the statistical measures of the robustness analysis

$$mean\_stress_i + 6 \cdot sigma\_stress_i \leq 30000. \quad (6)$$

The results of this optimization are given in Table III. The obtained mass is almost similar to the mass obtained by the iterative procedure, but some cross section areas are different. The final optimum is investigated by a robustness analysis with direct solver calls which results in an estimated safety margin slightly larger as 6σ for all stress values. The reliability proof reports an failure probability below the required 10^{-6}. Again the ARSM approach is very efficient. In the investigated example the coupled RDO approach using a global approximation model gives satisfactory results with a relatively small number of solver runs. However, in cases where the approximation quality is not as excellent as in our example the global approximation may fail. In such cases the iterative approach should give the most efficient solution.

Table III. Coupled Robust Design Optimization using a global response surface with final robustness and reliability proof

RDO on global MOP	500 support points	a_1	a_3	a_4	a_7	a_8	a_9	mass
EA on MOP	591 nominal designs (100 robustness samples each)	11.306	10.303	5.640	8.049	8.131	7.980	2211.0

Robustness analysis	100 samples	stress1	stress3	stress4	stress7	stress8	stress9
Mean value		17730	-19470	-17770	17610	-17470	17790
Standard deviation		1961	1683	1964	1977	1958	1993
Cov. of variation		0.111	0.086	0.111	0.110	0.112	0.112
Safety margin		6.26σ	6.26σ	6.23σ	6.27σ	6.40σ	6.13σ

Reliability analysis	Number of solver runs	Failure probability	Reliability index
Directional sampling	3366	$2.26 \cdot 10^{-7}$	5.05
ARSM+DS	84	$1.24 \cdot 10^{-7}$	5.16

Table IV. Coupled Robust Design Optimization using the direct solver with rough robustness estimates and final robustness and reliability proof

RDO	3822 solver calls	a_1	a_3	a_4	a_7	a_8	a_9	mass
ARSM	(182 nominal designs with 20 robustness samples each)	11.035	10.024	5.562	7.697	7.931	7.848	2153.5

Robustness analysis	100 samples	stress1	stress3	stress4	stress7	stress8	stress9
Mean value		18170	-20000	-18030	18420	-17880	18070
Standard deviation		2018	1716	2021	2010	2003	2020
Cov. of variation		0.111	0.086	0.112	0.109	0.112	0.112
Safety margin		5.86σ	5.83σ	5.92σ	5.76σ	6.05σ	5.91σ

Reliability analysis	Number of solver runs	Failure probability	Reliability index
Directional sampling	4444	$1.57 \cdot 10^{-6}$	4.66
ARSM+DS	121	$1.77 \cdot 10^{-6}$	4.64

Finally the coupled RDO procedure is performed by direct solver runs. In order to limit the number of solver evaluations the robustness analysis inside the RDO is performed using only 20 samples to estimate the statistical properties. The final results are again verified by a more accurate robustness analysis and a reliability proof. In Table IV the results of the direct RDO procedure are given. As optimizer an adaptive polynomial response surface approach is used, which can handle noisy model responses (optiSLang2011). Due to the small number of robustness samples the estimated mean values and standard deviations contain statistical errors which may lead to noticeable solver noise. The direct RDO approach leads to a truss structure having a smaller mass and safety margins for all stress values close to 6σ. The results of the reliability analysis indicate a small violation of the reliability requirement. Additionally, the number of necessary solver runs is much larger as by using the iterative procedure or the global approximation by MOP. For this reason an application of the direct RDO approach for industrial problems is not very attractive.

5. Conclusions

In this paper Robust Design Optimization concepts have been proposed, which are applicable for complex engineering problems, where the underlying structural model is often a very time consuming numerical simulation model. By means of an investigated truss structure different procedures have been analyzed. First, an iterative RDO procedure has been proposed. In this approach after each deterministic optimization the required safety margin is checked by variance-based robustness analysis. If the safety margin is not sufficient the deterministic optimization constraints are adapted. For a satisfactory safety margin the required failure probability is proven finally by efficient reliability methods.

The iterative method was compared with a global response surface method built up in the mixed optimization-stochastic space. If the approximation has very high accuracy, which was checked by the Coefficient of Prognosis, a coupled RDO procedure applied on the response surface may lead to sufficient results. However, since the global approximation has often low accuracy and since the numerical effort with respect to the number of solver runs is similar to the iterative procedure, we recommend the iterative procedure for practical applications. The iterative procedure has been proven to be very efficient and accurate for real product development in (Roos and Hoffmann2008),(Roos et al.2009).

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