# Determination of statistical material parameters of concrete using fracture test and inverse analysis based on FraMePID-3PB tool

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**Abstract:** The knowledge of fracture mechanical parameters values is fundamental for virtual failure modeling of elements and structures made of concrete. A key parameter of nonlinear fracture mechanics modeling is certainly specific fracture energy of concrete and its variability. Within this paper experimental results from three-point bending tests on notched-beam specimens are analyzed. Two basic approaches are applied to determine fracture mechanical parameters from these tests: (i) effective crack model / work-of-fracture method, (ii) inverse analysis using artificial neural networks and virtual stochastic simulations. In order to automate the whole time consuming process of inverse analysis a FraMePID-3PB software tool has been developed. The paper is focused on the determination of statistical fracture-mechanical parameters values of four different concrete types. It is a part of complex methodology for statistical and reliability analyses of concrete structures.

**Keywords:** Fracture parameters, concrete, inverse analysis, artificial neural networks, nonlinear analysis, fracture mechanics, FraMePID-3PB software

## 1. Introduction

The stochastic nonlinear computational mechanics faces in real-world application problems a fundamental obstacle – the lack of the knowledge of basic random variables involved in the problem. The direct experimental testing, often performed as compression test on cubic specimens, provides incomplete information about mechanical and fracture parameters and the lack of information is often substituted by an engineering judgment or by the information from literature. One possibility is to get parameters of computational model indirectly – based on combination of fracture test with inverse analysis. The paper describes a methodology to get such parameters using experimental data of three-point bending tests used in inverse analysis based on combination of artificial neural networks and stochastic analysis (Novák and Lehký, 2006). Since the whole procedure of inverse analysis is time consuming and complicated from data handling and artificial neural network training point of view a software tool FraMePID-3PB has been developed to automate fully the whole task.

A key parameter of nonlinear fracture mechanics modeling is certainly specific fracture energy of concrete and its variability, which is a subject of research of many authors, e.g. Bažant and Planas (1998). Other important parameters of concrete are modulus of elasticity, tensile and compressive strength. Crack propagation resistivity is described by e.g. effective crack elongation, effective fracture toughness etc. (Karihaloo, 1995). Determination of parameters values was done using two techniques – (i) direct

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evaluation from experimental load–deflection diagram by effective crack model and work-of-fracture method; (ii) inverse analysis using artificial neural network based method.

Depending on sample size of statistical set, statistical characteristics of material parameters being identified can be determined using two approaches: (i) “One by one” approach – parameters of each specimen are identified separately and final statistics are calculated from the set of all values for each parameter. (ii) “Direct approach” – in case of larger statistical set it is more efficient not to identify each specimen one by one but to identify the whole statistical set for all specimens together based on random response of fracture tests (Lehký and Novák, 2011). The first approach was used in this paper.

## 2. Laboratory tests

Laboratory experiments and evaluation of fracture-mechanical parameters were performed using four sets of specimens of different concrete types: I (C30/37 H), II (C25/30 B3), III (C25/30 XC1 GK16), and IV (C20/25 XC1 GK16) prepared and casted in co-operation with Bautechnische Prüf- und Versuchsanstalt GmbH and University of Natural Resources and Life Sciences in Vienna, Austria. Specimens were tested in laboratory at Faculty of Civil Engineering, Brno University of Technology in Brno, Czech Republic in following ages: 91 days (set I), 87 days (set II), 67 days (set III) and 66 days (set IV). Each set consists of 9 specimens except of set IV which consists of 8 specimens. Nominal sizes of specimens were 100×100×400 mm. In the center of the beam the edge notch of the depth about 1/3 of the depth of the specimen was cut using diamond blade saw. Specimens were tested in three-point bending (3PB) configuration. Loading span was equal to 300 mm. Example of the tested specimen is in Figure 1.

Testing was performed using mechanical press Heckert FPZ 100/1. Loading of specimen was applied continuously with constant increment of displacement 0.1 mm/min in the middle of the span (300 mm). Midspan deflections were recorded using inductive sensor with accuracy of 0.001 mm. Result of measurement is diagram load vs. midspan deflection (*l–d* diagram).

For enlarging the set of material parameters with compressive strength which is not obtained from 3PB test the compression tests were performed too. It was carried out using two broken parts obtained after each three-point bending test. Broken parts were cut to nominal size 100×100×100 mm using diamond blade saw (Figure 2).

## 3. Evaluation of material parameters

### 3.1. Effective crack approach, work-of-fracture method

Recorded *l–d* diagram serves as a basis for evaluation of effective crack elongation and effective fracture toughness (or effective toughness) using models of equivalent elastic crack (Karihaloo, 1995). Then, using work-of-fracture method, a fracture work or specific fracture energy are assessed. As was already mentioned specific fracture energy is basic parameter of cohesive crack models which are used for prediction of fracture behavior of structures made of quasi-brittle materials (Stibor, 2004; Veselý, 2004).

a) b)

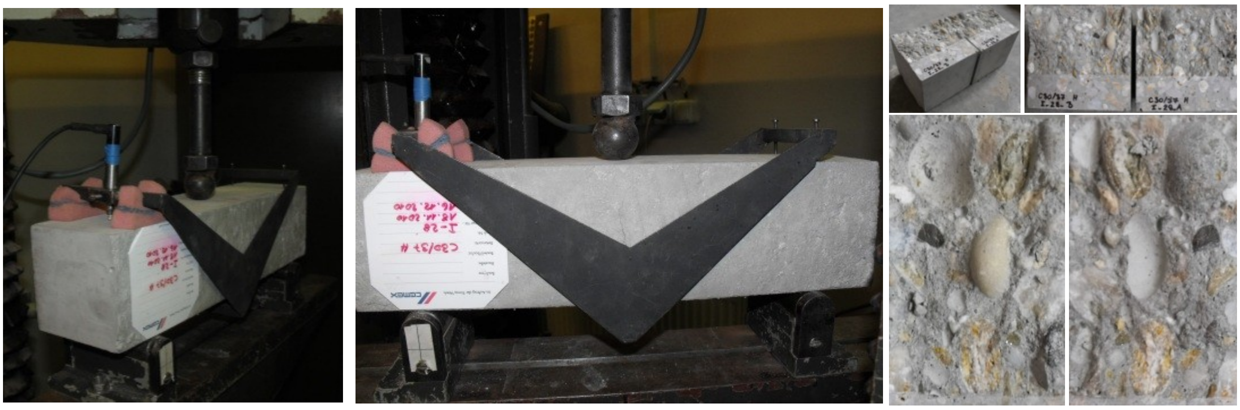

*Figure 1*. a) Selected specimen tested in three-point bending configuration and b) the fracture parts/surfaces after test [photo: B. Kucharczyková].

a) b)

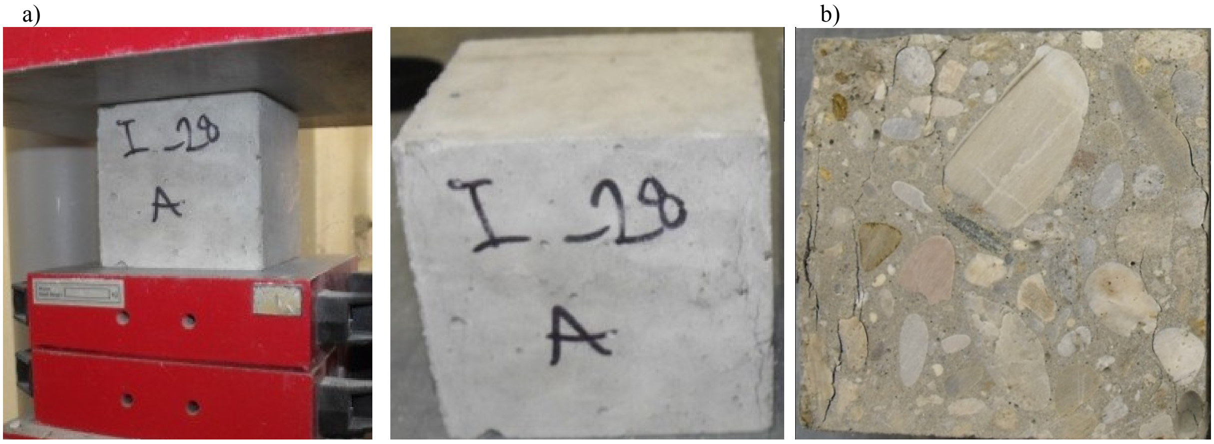


*Figure 2*. a) Compression test on broken parts of specimens and b) the shape after destruction [photo: B. Kucharczyková].

Important step before parameters evaluation from obtained *l–d* diagram is to recognize the origination of “catastrophe” in measured data (Frantík and Keršner, 2006). In time series of deflections in loading point an irregularity of loading speed and sudden increase of displacement can occur. Then, the time derivative of deflection is a useful criterion to detect the origin and range of the so-called fold catastrophe. The catastrophe is recognizable as extreme values of loading speed. The corrected *l–d* diagram and fold catastrophe have such properties which can help to discover the probable development of the diagram in the catastrophic part.

### 3.2. INVERSE ANALYSIS

Along with classical fracture mechanical parameters evaluation from fracture tests, parameters identification using artificial neural network based inverse method was carried out; see details in Novák and

Lehký (2006). The basis of inverse analysis is finite element method (FEM) model which is used for numerical simulation of three-point bending fracture test (Figure 3). FEM model was created in ATENA software (Červenka et al., 2007); material model 3D Nonlinear Cementitious 2 with rotational cracks was used. Subject of identification were following three parameters of concrete: modulus of elasticity, tensile strength and fracture energy. Other parameters of material model mentioned above, e.g. compressive strength, were omitted from identification based on sensitivity analysis. Here, Spearman's nonparametric rank-order correlation coefficient was used (Novák et al., 1993).

The material model parameters are considered as random variables described by a probability distribution, rectangular distribution is a "natural choice" as the lower and upper limits represent the bounded range of physical existence. The variables are then simulated randomly based on the Monte Carlo type simulation; the small-sample simulation Latin Hypercube Sampling (LHS) is recommended (McKay et al., 1979). A multiple calculation of deterministic computational model using random realizations of material model parameters is performed and a statistical set of the virtual structural response is obtained. Random realizations and the corresponding responses from the computational model serve as the basis for the training of an appropriate neural network (Cichocki and Unbehauen, 1993). After the training the neural network is ready to solve the main task: To provide the best material parameters in order the numerical simulation will result in the best agreement with experiment. This is performed by means of the simulation of network using measured response as an input. It results in a set of identified material parameters. The last step is results verification – calculation of computational model using identified parameters. A comparison with experiment will show to what extend the inverse analysis was successful. More details about structure of artificial neural network, training set, etc. are described in section 4.

To obtain statistical characteristics of material parameters inverse analysis is performed for each specimen (*l–d* diagram) individually. The set of identified values is obtained as the result of individual identification and can be assessed statistically as it is usually done for experiments.

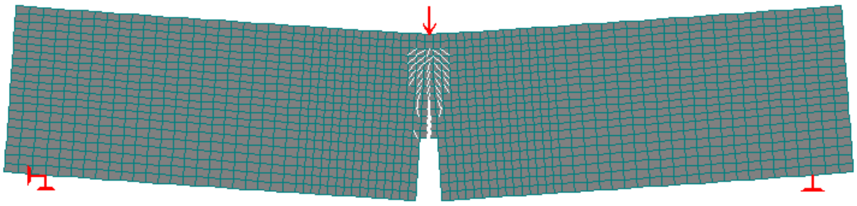

*Figure 3*. Scheme of nonlinear FEM computational model of three-point bending test.

## 4. FraMePID-3PB software tool

The methodology of artificial neural network based inverse analysis is general and can be used for any inverse task, which is its advantage. On the other hand it is very time consuming. In order to automate the whole difficult process of material parameters identification a FraMePID-3PB software tool has been developed. The whole system is based on standardized fracture test of beam with central edge notch in three-point bending configuration described in section 2. Finite element computational model implemented in FraMePID-3PB is created in ATENA software (Červenka et al., 2007). 3D Nonlinear Cementitious 2 material model for concrete is used. Softening of concrete is modeled using model according to Hordijk (1991).

Previous identifications using various types of concrete mixtures and ages showed that the structure of artificial neural network used for identification in this testing configuration is in all cases almost the same. Thank to that and using standardized test the time needed for inverse analysis can be significantly reduced because only one neural network is created, trained, tested and implemented within FraMePID-3PB system. Therefore, time consuming training set preparation using stochastic nonlinear analysis and training of the network using suitable optimization technique is performed only once. Structure of neural network implemented within FraMePID-3PB system is as follows (see Figure 5): 1 hidden layer with 5 nonlinear neurons (hyperbolic tangent transfer function), output layer with 3 linear neurons (linear transfer function) and 3 inputs of the network. Three output neurons correspond to three material parameters which are being identified (modulus of elasticity, tensile strength and specific fracture energy), three inputs correspond to three parameters extracted from *l–d* diagram.

During training set preparation for artificial neural network material parameters are randomized. Here, purposely large variability was used in order to create rather general network which will be able to identify parameters of concretes of various strengths and ages. Mean values were 40 GPa for modulus of elasticity, 4.5 MPa for tensile strength and 200 J/m$^2$ for fracture energy. Coefficients of variation were 0.2 for modulus of elasticity, 0.25 for tensile strength and 0.4 for fracture energy. Training set was generated using 100 simulations of Latin Hypercube Sampling method. Training of the network was carried out using Levenberg–Marquardt (Singh et al., 2007) and genetic algorithms (Schwefel, 1991) optimization methods.

Procedure of material parameters identification using FraMePID-3PB tool can be itemized as follows:

1. *L–d* diagram obtained from experiment is loaded into FraMePID-3PB. Curve is analyzed and inputs of inverse analysis are extracted and prepared for neural network (Figure 4).
2. Input signal is transmitted through the neural network and from the output layer of the network the best set of material parameters is obtained. This step is possible because neural network is trained in advance and remains the same for parameters identification of various concretes (Figure 5). Emphasize, that there is no new nonlinear fracture mechanics calculations to train network – the network is ready to use and implemented in FraMePID-3PB.
3. Verification of identification is performed. Obtained material parameters are used in the computational model and numerical analysis is carried out. Here, ATENA software is linked to FraMePID-3PB for data transfer. Resulting *l–d* diagram is compared with experimental one which will show to what extent the inverse analysis was successful (Figure 6).

At present, FraMePID-3PB software operates with "basic" configuration of experiment and model as was mentioned above. But, it was designed more generally with respect to next future extension for other configurations, e.g. specimens with various notch depths, other softening models of concrete (linear, multilinear, etc.), additional testing configurations (compressive test, wedge splitting test), etc. This will help with routine material parameters identification for various research and practical tasks.

## 5. Results

Values of selected parameters of all 35 specimens of four sets obtained using both above mentioned methods were statistically evaluated and their mean values and coefficients of variation (COV) can be found in Table 1. There is a significant advantage in case of using inverse analysis – value of tensile strength of concrete can also be determined.

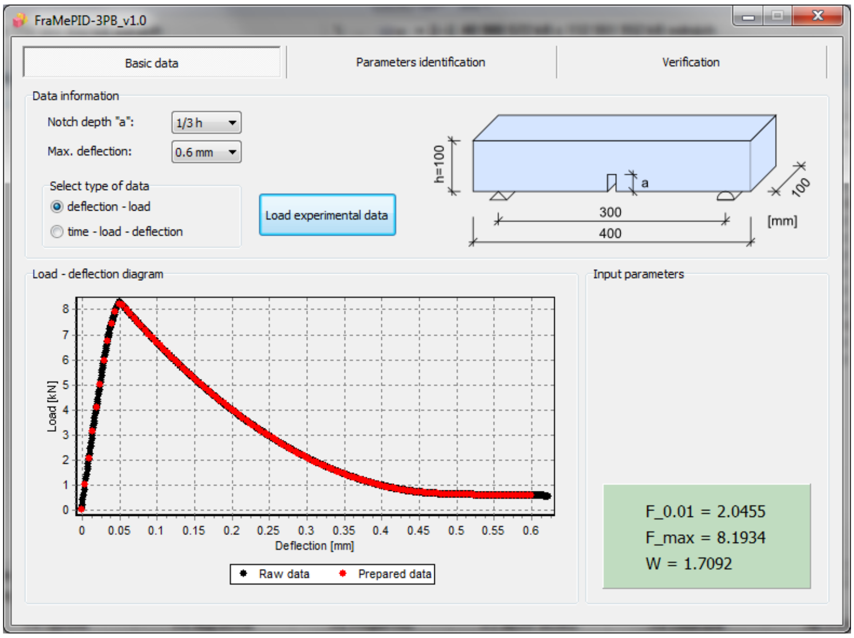


*Figure 4*. FraMePID-3PB tool panel – experimental data loading and preparation of input signal for neural network.

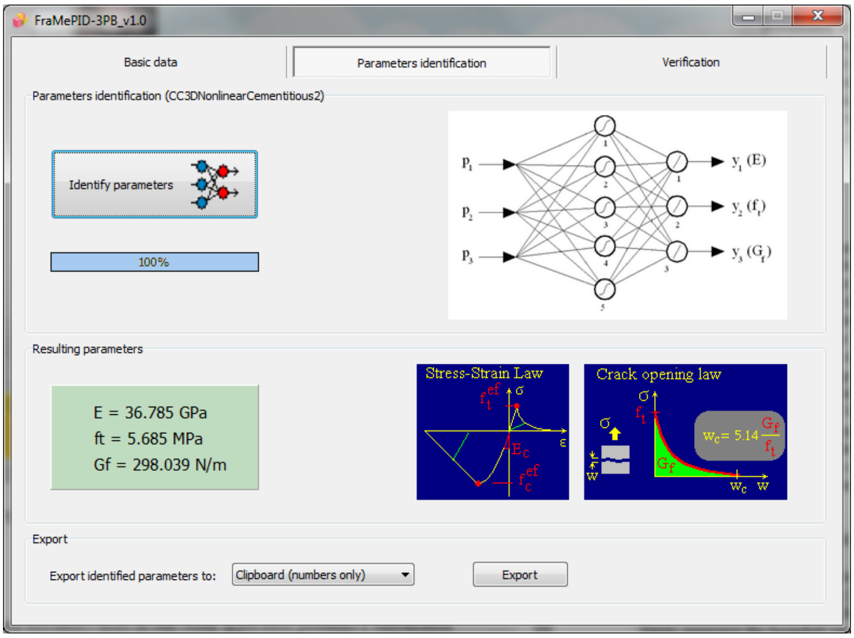


*Figure 5*. FraMePID-3PB tool panel – structure of neural network and material parameters identification.

Table I. Selected statistical parameters of set I (C30/37 H) obtained from experiment and inverse analysis.

| Parameter of concrete / model | Experiment | | Inverse analysis | | Ratio (inv. analysis / experiment) |
|---|---|---|---|---|---|
| | Mean value | COV [%] | Mean value | COV [%] | |
| Modulus of elasticity [GPa] | 35.5 | 7.4 | 40.3 | 16.6 | 1.14 |
| Tensile strength [MPa] | – | – | 5.0 | 14.3 | – |
| Compressive strength [MPa] | 58.5 | 8.0 | – | – | – |
| Specific fracture energy [$J/m^2$] | 235.9 | 18.6 | 281.5 | 19.5 | 1.19 |
| Effective crack elongation [mm] | 9.5 | 21.9 | – | – | – |
| Effective fracture toughness [$MPa.m^{1/2}$] | 1.489 | 9.9 | – | – | – |
| Effective toughness [J/m2] | 62.3 | 13.7 | – | – | – |
| Volume density [$kg/m^3$] | 2341.8 | 0.7 | – | – | – |

Table II. Selected statistical parameters of set II (C25/30 B3) obtained from experiment and inverse analysis.

| Parameter of concrete / model | Experiment | | Inverse analysis | | Ratio (inv. analysis / experiment) |
|---|---|---|---|---|---|
| | Mean value | COV [%] | Mean value | COV [%] | |
| Modulus of elasticity [GPa] | 30.8 | 8.6 | 35.0 | 8.2 | 1.14 |
| Tensile strength [MPa] | – | – | 4.1 | 17.2 | – |
| Compressive strength [MPa] | 47.3 | 5.4 | – | – | – |
| Specific fracture energy [$J/m^2$] | 188.9 | 11.5 | 211.8 | 18.1 | 1.12 |
| Effective crack elongation [mm] | 12.5 | 23.5 | – | – | – |
| Effective fracture toughness [$MPa.m^{1/2}$] | 1.406 | 8.0 | – | – | – |
| Effective toughness [J/m2] | 65.2 | 21.8 | – | – | – |
| Volume density [$kg/m^3$] | 2286.2 | 1.5 | – | – | – |

Table III. Selected statistical parameters of set III (C25/30 XC1 GK16) obtained from experiment and inverse analysis.

| Parameter of concrete / model | Experiment | | Inverse analysis | | Ratio (inv. analysis / experiment) |
|---|---|---|---|---|---|
| | Mean value | COV [%] | Mean value | COV [%] | |
| Modulus of elasticity [GPa] | 35.4 | 5.6 | 40.4 | 9.5 | 1.14 |
| Tensile strength [MPa] | – | – | 4.2 | 12.1 | – |
| Compressive strength [MPa] | 53.4 | 5.2 | – | – | – |
| Specific fracture energy [J/m$^2$] | 183.3 | 5.5 | 214.0 | 6.0 | 1.17 |
| Effective crack elongation [mm] | 12.4 | 22.7 | – | – | – |
| Effective fracture toughness [MPa.m$^{1/2}$] | 1.405 | 9.0 | – | – | – |
| Effective toughness [J/m2] | 56.3 | 19.9 | – | – | – |
| Volume density [kg/m$^3$] | 2326.9 | 0.9 | – | – | – |

Table IV. Selected statistical parameters of set IV (C20/25 XC1 GK16) obtained from experiment and inverse analysis.

| Parameter of concrete / model | Experiment | | Inverse analysis | | Ratio (inv. analysis / experiment) |
|---|---|---|---|---|---|
| | Mean value | COV [%] | Mean value | COV [%] | |
| Modulus of elasticity [GPa] | 31.2 | 4.3 | 34.8 | 5.3 | 1.12 |
| Tensile strength [MPa] | – | – | 3.1 | 15.6 | – |
| Compressive strength [MPa] | 39.8 | 5.4 | – | – | – |
| Specific fracture energy [J/m$^2$] | 146.2 | 13.3 | 166.8 | 15.4 | 1.14 |
| Effective crack elongation [mm] | 13.0 | 14.3 | – | – | – |
| Effective fracture toughness [MPa.m$^{1/2}$] | 1.131 | 10.6 | – | – | – |
| Effective toughness [J/m2] | 41.4 | 20.7 | – | – | – |
| Volume density [kg/m$^3$] | 2292.2 | 0.6 | – | – | – |

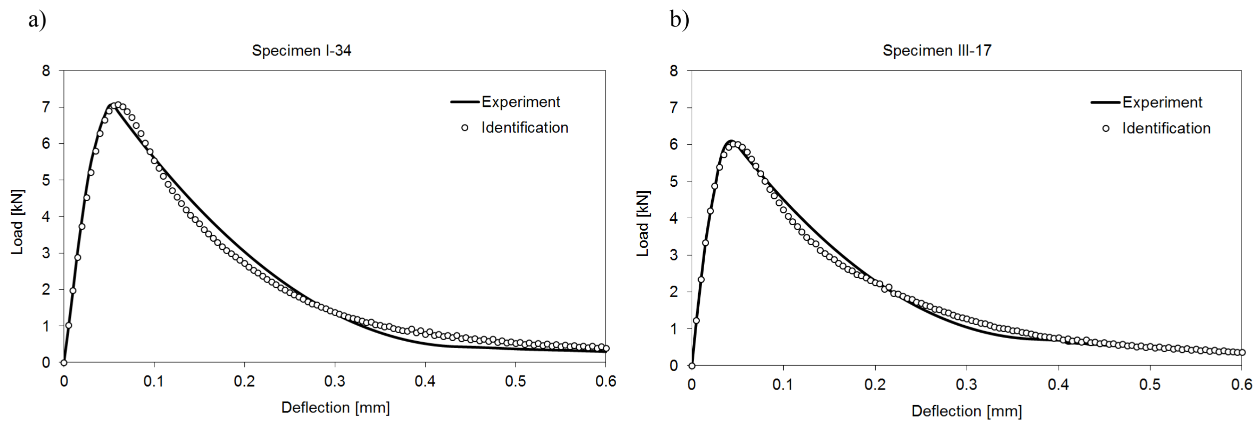


*Figure 6*. Comparison of selected experimental and numerically simulated load–deflection diagrams with material parameters obtained from identification: a) specimen I-34 (C30/37 H) and b) specimen III-17 (C25/30 XC1 GK16).

From the presented results it is possible to conclude recommended values of mechanical–fracture parameters for deterministic and stochastic nonlinear FEM analyses of beam/structures made of all four analyzed concretes, see Tables I–IV. Two-parametric lognormal probability distribution function is suggested for all three identified parameters (modulus of elasticity, tensile strength and fracture energy) and all four tested concrete types based on curve fitting tests carried out using FReET software (Novák et al., 2011) and JCSS Probabilistic Model Code recommendations (2001). Detailed results of all parameters for every single specimen and comparison of experimental and numerical *l–d* diagrams can be found in Keršner et al. (2011).

## 6. Conclusions

The proposed paper describes fracture tests and consequent evaluation of fracture mechanical parameters of specimens made of four different concrete types. Determination of values of these parameters was done using two techniques – (i) direct evaluation of parameters from experimental *l–d* diagram by effective crack model and work-of-fracture method; (ii) inverse analysis using artificial neural network based method. Results were compared; both techniques provided results which are close to each other including basic information on variability (COV). The inverse analysis technique provided additionally values of tensile strength of concretes. *L–d* diagrams from numerical simulations of all six specimens with identified parameters shows very good agreement with experimental ones. Results can serve efficiently as input data for stochastic nonlinear simulation of studied concretes.

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