Accuracy of Concrete Creep Predictions Based on Extrapolation of Short-Time Data

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Abstract. The paper evaluates the accuracy of predictions obtained with various creep models after updating of their parameters based on short-time data. The models considered in this comparative study include relatively simple formulae recommended by major design codes (ACI, *fib*) as well as more sophisticated models developed by researchers (B3, GL). Appropriate error measures are defined and two updating strategies are examined. Predictions of the models are checked against basic creep data from a comprehensive database. The dependence of the error on the load duration after which the update is performed is described. Finally, preliminary conclusions and recommendations regarding the choice of the model and updating strategy are formulated.

Keywords: concrete, creep, updating

1. Introduction

Concrete exhibits creep already at low stress levels and normal temperatures, and long-time measurements on laboratory samples as well as on concrete structures indicate that the growth of strain at constant stress continues even after many decades, see e.g. (Brooks, 2005) and (Bažant et al., 2010). Problems with excessive deflections caused by creep have been reported for many large-span prestressed concrete bridges, and comparative numerical simulations based on design codes and advanced models have revealed the essential role played by a good predictive creep model (Bažant et al., 2010; Bažant et al., 2011). Unfortunately, empirical formulae for determination of creep model parameters based exclusively on the fundamental properties (such as compressive strength, concrete mix composition, size and shape of the member, environmental conditions and curing) have a very limited accuracy and often lead to gross errors. It is essential to update the model parameters based on laboratory tests or measurements of the early response of the real structure.

The present paper compares updated predictions obtained with the following creep models:

- the ACI model, recommended by the permanent committee TC 209 of the American Concrete Institute;
- the *fib* model, recommended in the Model Code 2010 of the International Federation for Structural Concrete;
- the B3 model, developed at Northwestern University by Bažant and coworkers;
- the GL2000 model, developed at the University of Ottawa by Gardner and coworkers.

5th International Conference on Reliable Engineering Computing (REC 2012)
Edited by M. Vořechovský, V. Sadílek, S. Seitl, V. Veselý, R. L. Muhanna and R. L. Mullen
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ISBN 978-80-214-4507-9. Published by Ing. Vladislav Pokorný − LITERA

Two updating strategies are considered: the standard one is based on simple least-square fitting, while its modification introduces weight factors that emphasize the influence of measured data directly preceding the interval of extrapolation. Absolute and relative error measures are defined and accuracy of the initial "blind" as well as updated predictions is checked against data from a comprehensive creep database (Bažant and Li, 2008), with focus on the dependence of the error on the load duration after which the update is performed.

2. Creep Models

Concrete creep at low and moderate stress levels is usually handled within the framework of aging linear viscoelasticity. Based on the superposition principle, the strain history corresponding to a given continuous and differentiable stress history can be computed using the integral formula

$$\varepsilon(t) = \int_{t_0}^{t} J(t,t')\dot{\sigma}(t')\,\mathrm{d}t' \tag{1}$$

For discontinuous stress histories, additional terms that reflect the influence of stress jumps can be added. In formula (1), ε is the strain, σ is the stress, t_0 is the time at the onset of loading, t is the current time, and J is the compliance function that can be determined from a creep test at constant stress. The value of $J(t,t')$ corresponds to the strain at time t in a creep test started at time t', divided by the stress level at which the test takes place. The time is measured from the set of concrete, i.e., it corresponds to the age of the material. For a non-aging viscoelastic material, the compliance function would depend only on the elapsed time $t-t'$, but for an aging material such as concrete it depends on t and t' separately. In all the models presented here, the values of time variables are supposed to be substituted in days.

For simplicity, the stress-strain relation (1) has been presented in a scalar format, valid for uniaxial stress. In a general extension to multiaxial stress, the volumetric and deviatoric parts of the response could be treated separately. In the absence of more precise data, it is usually assumed that all compliance coefficients are proportional to one single compliance function, which is equivalent to the assumption that the Poisson ratio remains constant and is not affected by creep.

2.1. ACI 209 Model

The model recommended by the permanent committee TC 209 Creep and Shrinkage in Concrete of the American Concrete Institute (ACI) was first adopted in 1971. Its most recent version, labeled as 209R-92, was published in 1992 (ACI, 1992) and again reapproved in 2008. The compliance function has the form

$$J(t,t') = \frac{1}{E_C}\sqrt{b+\frac{a}{t'}}\left[1+\frac{2.35\gamma}{(t')^m}\frac{(t-t')^{0.6}}{10+(t-t')^{0.6}}\right] \tag{2}$$

with time variables t and t' substituted in days. Parameter E_C is the conventional elastic modulus of concrete, measured at age 28 days. Parameters a, b and m depend on the type of cement and type of curing. For moist-cured concrete and cement of type I, their recommended values are $a = 4$, $b = 0.85$ and $m = 0.118$. Parameter γ is the product of six partial factors that depend on the type of curing, environmental humidity, volume-surface ratio of the concrete member, slump, mass fractions of fine and total aggregate and on the air content.

2.2. *fib* MODEL CODE

The *fib* Model Code 2010 (*fib*, 2010), accepted in 2011 by the International Federation for Structural Concrete (in French “fédération internationale du béton, *fib*”), is a successor of CEB Model Codes 1990 and 1999, developed by the Euro-International Committee for Concrete (CEB). The compliance function has the form

$$J(t,t') = \frac{1}{E_C} \exp\left(-\frac{s}{2}\left[1-\sqrt{\frac{28}{t'}}\right]\right) + \frac{\phi_{RH}\beta_f}{E_C}\frac{1}{0.1+t'^{0.2}}\left(\frac{t-t'}{\beta_H\beta_T+t-t'}\right)^{0.3} \tag{3}$$

Parameter E_C is the conventional elastic modulus, parameter s depends on the strength class of cement and hardening characteristics (e.g., $s = 0.25$ for normal cement of strength class 42.5 or for rapidly hardening cement of strength class 32.5), parameters ϕ_{RH} and β_f express the influence of environmental humidity and mean compressive strength, parameter β_H depends on humidity and strength as well as on the notional member size, and parameter β_T reflects the influence of temperature and is equal to 1 at room temperature.

2.3. B3 MODEL

Model B3 (Bažant and Baweja, 1995; Bažant and Baweja, 2000) covers creep and shrinkage of concrete, including their coupling. The compliance function has the general form

$$J(t,t') = q_1 + q_2 Q(t,t') + q_3 \ln[1+(t-t')^n] + q_4 \ln\left(\frac{t}{t'}\right) + J_{\mathrm{d}}(t,t') \tag{4}$$

where $n = 0.1$, q_1 is the inverse of the asymptotic elastic modulus, the terms containing parameters q_2, q_3 and q_4 represent the aging viscoelastic compliance, non-aging viscoelastic compliance and flow compliance, respectively, and $J_{\mathrm{d}}(t,t')$ is the additional compliance due to drying. Here we consider only basic creep, i.e., creep of sealed specimens, not affected by drying, and thus $J_{\mathrm{d}}(t,t')$ can be omitted. Function Q is not available in a closed form and is defined by the integral formula

$$Q(t,t') = \int_{t'}^{t} \frac{ns^{-m}}{(s-t')+(s-t')^{1-n}}\,\mathrm{d}s \tag{5}$$

where $m = 0.5$. Its specific values can be obtained by numerical integration or approximated using an explicit formula given in (Bažant and Baweja, 1995) and (Bažant and Baweja, 2000). Parameters q_i, $i = 1,2,3,4$, can be estimated based on composition of the concrete mix and mean compressive strength of concrete using empirical formulae.

2.4. GL MODEL

The model proposed by (Gardner and Lockman, 2001) and denoted as the GL2000 Model is a modification of the earlier Atlanta97 Model (or GZ Model) of (Gardner and Zhao, 1993). The compliance function has the form

$$J(t,t') = \frac{1}{3.5 + (E_C - 3.5)\exp\left(\frac{s}{2}\left[1 - \sqrt{\frac{28}{t'}}\right]\right)} + \qquad (6)$$
$$+\frac{\Phi}{E_C}\left[\frac{2(t-t')^{0.3}}{(t-t')^{0.3}+14} + \sqrt{\frac{7(t-t')}{t'(t-t'+7)}} + c_h\sqrt{\frac{t-t'}{t-t'+0.12(V/S)^2}}\right]$$

Parameter E_C is the conventional elastic modulus, parameter s depends on the type of cement, parameter Φ is different from 1 only if the first loading is preceded by drying and, if this is the case, depends on the drying time before loading and on the volume-surface ratio V/S, and parameter c_h depends on the environmental humidity.

3. Updating of Model Parameters

Parameters of creep models presented in the previous section can be estimated from the basic characteristics of the concrete mix, curing procedure and environmental conditions. In this sense, the models can be considered as predictive and used already in the design stage. However, the dependence of model parameters on the basic characteristics described by empirical equations has a limited accuracy. To get a better agreement between the model and the real behavior, it would be advisable to perform creep tests of samples made of the specific concrete intended for the designed structure. Due to the long-term nature of the creep process, it is impossible to run the complete tests before construction. A compromise consists in continuous updating of the model parameters from measurements on the real structure or on companion specimens kept under the same environmental conditions. In the design stage, the parameters can be estimated from composition and corrected based on short-term tests. During construction and even after completion of the structure, the parameters can be continuously updated as more and more measured data become available. For this purpose, it is essential to know how the accuracy of predictions of the future behavior of the structure evolves depending on the growing amount of available information describing the past behavior (of the structure or of a specimen made of the same concrete).

In the present preliminary study, we restrict attention to basic creep, so that the effect of environmental humidity on the compliance function is eliminated. Measured values of the compliance function are taken from a comprehensive creep database assembled at Northwestern University (Bažant and Li, 2008). The database contains a wide range of creep tests run in the past in many laboratories around the globe under a variety of conditions. For our purpose, only sufficiently long tests (at least 1000 days of loading) performed under sealed conditions are considered. Furthermore, tests at extremely high or low temperatures (below 5°C or above 50°C) are excluded. The study is limited to concrete with mean compressive strength at 28 days lower than 82 MPa, loaded at stress levels not exceeding 45% of the strength. By applying these criteria, 40 tests from 12 laboratories have been extracted from the database.

In principle, the updating procedure could be applied to all parameters of each model. However, this would result into complicated problems of nonlinear regression, with multiple local minima of the error function and with a danger of extremely high sensitivity to the unavoidable scatter of experimental data, especially during early stages of the response when only a few measured values are available. For this reason, it seems preferable to consider the updated compliance function as the original compliance function transformed in a linear fashion, i.e., by vertical scaling and shifting. Mathematically, we can write

$$J_u(t,t') = p_1 + p_2 J_o(t,t') \tag{7}$$

where J_o is the original compliance function with parameters estimated from composition and J_u is the updated compliance function, adjusted such that the early part of the measured response up to the updating time t_m is reproduced with the minimum possible error. Coefficients p_1 and p_2 are obtained by minimizing the function

$$F(p_1,p_2) = \sum_{i=1}^{m} \left[p_1 + p_2 J_o(t_i,t_0) - J_e(t_i,t_0)\right]^2 \tag{8}$$

where J_e denotes the experimentally determined compliance function, t_0 is the age of concrete at load application, and $t_1 < t_2 < \ldots < t_m$ are the times at which individual measurements were taken, up to the selected updating time t_m. For each specific test, t_0 is fixed but t_m can have an arbitrary value between t_1 and the age at the end of the test, $t_{\max}$. Therefore, coefficients p_1 and p_2 and the resulting updated function J_u depend on the time t_m at which the updating is performed.

Function F defined in (8) is quadratic in terms of the variables p_1 and p_2, and the stationarity conditions lead to two linear equations,

$$p_1 m + p_2 \sum_{i=1}^{m} J_o(t_i,t_0) = \sum_{i=1}^{m} J_e(t_i,t_0) \tag{9}$$

$$p_1 \sum_{i=1}^{m} J_o(t_i,t_0) + p_2 \sum_{i=1}^{m} J_o^2(t_i,t_0) = \sum_{i=1}^{m} J_e(t_i,t_0) J_o(t_i,t_0) \tag{10}$$

from which the optimal values of p_1 and p_2 are easily computed.

As an example, consider the data on Water Tower Place concrete (Russell and Burg, 1996). The concrete mix consisted of $c = 501.7$ kg/m^3 of cement, $w = 195.7$ kg/m^3 of water and $a = 1676$ kg/m^3 of aggregates, and the mean compressive strength at 28 days was $\bar{f}_c = 63$ MPa. From these data, parameters of the B3 model can be estimated as follows:

$$q_1 = 126.77\, \bar{f}_c^{-0.5} = 15.97 \quad [10^{-6}/\text{MPa}] \tag{11}$$
$$q_2 = 185.4\, c^{0.5} \bar{f}_c^{-0.9} = 99.75 \quad [10^{-6}/\text{MPa}] \tag{12}$$
$$q_3 = 0.29(w/c)^4 q_2 = 0.669 \quad [10^{-6}/\text{MPa}] \tag{13}$$
$$q_4 = 20.3(a/c)^{-0.7} = 8.727 \quad [10^{-6}/\text{MPa}] \tag{14}$$

The cement was of type R (rapid hardening) according to the CEB classification, and the mix also contained 11.8 kg/m^3 of fly ash. The experiments were performed on standard 6-inch cylinders (152 mm in diameter and 305 mm in height) at room temperature (23°C) and stress level 15.5 MPa (i.e., 25% of the mean strength). The specific test considered here (test C_078_05 from the database) started at age $t_1 = 28$ days and was run under sealed conditions for 6768 days, i.e., 18.5 years.

The "blind" prediction based on the parameter values (11)–(14) is plotted as the dash-dotted curve in Figure 1a. If the data measured during the first 143 days of loading are taken into account, equations (9)–(10) lead to $p_1 = -16.77 \times 10^{-6}$/MPa and $p_2 = 1.542$. The updated compliance function (7) is plotted in

Figure 1a as the solid curve; it corresponds to the B3 model with modified parameters $q_1^* = p_1 + p_2 q_1 = 7.85$, $q_2^* = p_2 q_2 = 153.78$, $q_3^* = p_2 q_3 = 1.031$ and $q_4^* = p_2 q_4 = 13.454$ (all in 10^{-6}/MPa). The result is somewhat disappointing. The original blind prediction underestimates the compliance after 18.5 years of loading by 14.4% and the updated prediction overestimates it by 14.7%. If the update is performed already after 14 days of loading, the results get even worse, and the extrapolated compliance after 18.5 years of loading is then overestimated by 45.8%; see the dashed curve in Figure 1a.

The reason for the poor accuracy of updated predictions is that the update optimizes the fit of the entire initial period of loading up to time t_m while the main purpose should be an improved accuracy of the extrapolation to longer times. Therefore, it makes sense to reduce the influence of early measurements and emphasize those that are closer to the updating time and thus also to the intended extrapolation. This can be achieved by introducing weight factors that have larger values for measurements at later times. The simplest choice is to take the time elapsed from the first loading up to the given measurement as the weight factor. The definition of the function F to be minimized is then changed from (8) to

$$F(p_1, p_2) = \sum_{i=1}^{m} (t_i - t_0) \left[p_1 + p_2 J_o(t_i, t_0) - J_e(t_i, t_0) \right]^2 \tag{15}$$

and equations (9)–(10) are adjusted accordingly. This modified updating approach leads to a substantial improvement, as shown in Figure 1b. The update after 14 days still does not lead to an improvement (but at least it is not as bad as for the standard updating method), but the update after 143 days gives a very nice prediction of the future evolution of compliance, with the value after 18.5 years overestimated by only 3.6%.

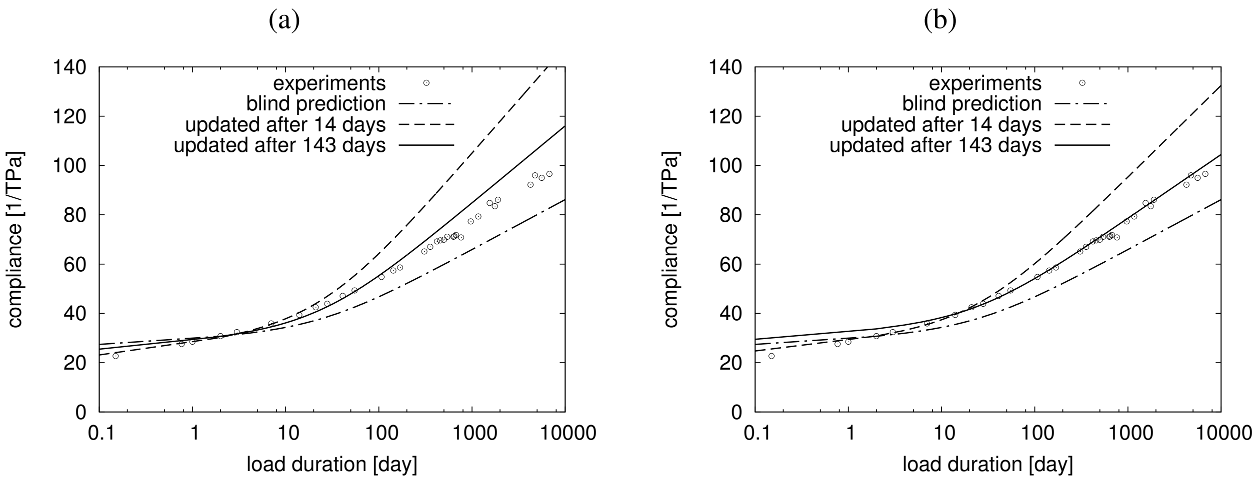

Figure 1. Compliance function of Water Tower Place concrete: (a) standard updating with equal weights of all measured points, (b) modified updating with the weight of each measured point proportional to the duration of loading.

4. Accuracy of Updated Predictions

To compare the accuracy of the original "blind" prediction and the updated predictions with different updating times, we can start from the so-called absolute residual error, which is considered as the root-mean-square (RMS) deviation of the prediction from the experimental data, averaged over the time interval from the updating time t_m up to time $t_{\max}$ at the end of the test. The averaging is done with respect to the logarithmic scale of the load duration, and so the error is defined as

$$e_u^{(m)} = \sqrt{\frac{\sum_{i=m+1}^{n} \ln \frac{t_{i+1}-t_0}{t_{i-1}-t_0} \left[J_u(t_i, t_0) - J_e(t_i, t_0)\right]^2}{\sum_{i=m+1}^{n} \ln \frac{t_{i+1}-t_0}{t_{i-1}-t_0}}} \quad (16)$$

where n is the total number of measurements, and t_{n+1} is set to t_n. The error of the updated prediction can be evaluated for $m \geq 2$, because the updating procedure needs at least the first two measurements, at times t_1 and t_2, for determination of two parameters, p_1 and p_2. Note that the deviations at times preceding or equal to the updating time t_m are not taken into account, and so the interval over which the RMS error is computed diminishes with increasing updating time (this is why the error is called "residual").

The error measure defined in (16) has the dimension of compliance and it can be used for comparison of the relative accuracy of individual models applied to the same test. For evaluation of the average accuracy in the set of 40 tests considered here, it is better to use the normalized error, defined as the absolute error according to (16) divided by the reference compliance value, which is taken as $J(t_0 + 1000, t_0)$ (i.e., as the compliance corresponding to the load duration of 1000 days). The normalized error is dimensionless and its value of 0.1 corresponds to 10% deviation with respect to the reference compliance value.

The dependence of the normalized residual error on the time elapsed from load application to the updating time is graphically presented for individual creep models in Figure 2. All the graphs still refer to one single test of the Water Tower Place concrete. The dashed curves show the error of the prediction based on standard updating and the solid curves refer to the modified updating. The first points of both curves always coincide because they correspond to updating after the second measurement, when two measured values uniquely determine parameters p_1 and p_2, independently on whether weighting is used or not. Later on, both curves in general differ and the modified update typically leads to higher accuracy, with some exceptions in the range from 4 to 80 days for the ACI model and from 1 to 13 days for the GL model.

For comparison, the graphs also contain dash-dotted curves that correspond to the blind prediction, with no updating. For the blind prediction, the model parameters remain fixed and the prediction does not evolve in time. However, to be able to compare directly the error of the blind and updated predictions, the error of the blind prediction is also evaluated over the interval that starts at the current updating time t_m. This residual error is defined by a formula similar to (16), with J_u replaced by J_o. As seen in Figure 2, the residual error of the blind prediction typically increases in time with increasing t_m, which means that the blind prediction is usually more accurate for short load durations than for long ones. Another interesting observation is that the early updates are in some cases less accurate than the blind prediction, which means that updating has an adverse effect on accuracy. This is particularly striking for model B3; see Figure 2a. Here, the blind prediction leads to normalized error (over the entire tested time interval) close to 0.1 while the update based on a few measurements up to load duration of 1 day gives a much higher error, about 0.5. To get improved accuracy as compared to the blind prediction, one needs to take into account measurements from at least 82 days of loading for the standard update and from at least 28 days of loading for the modified

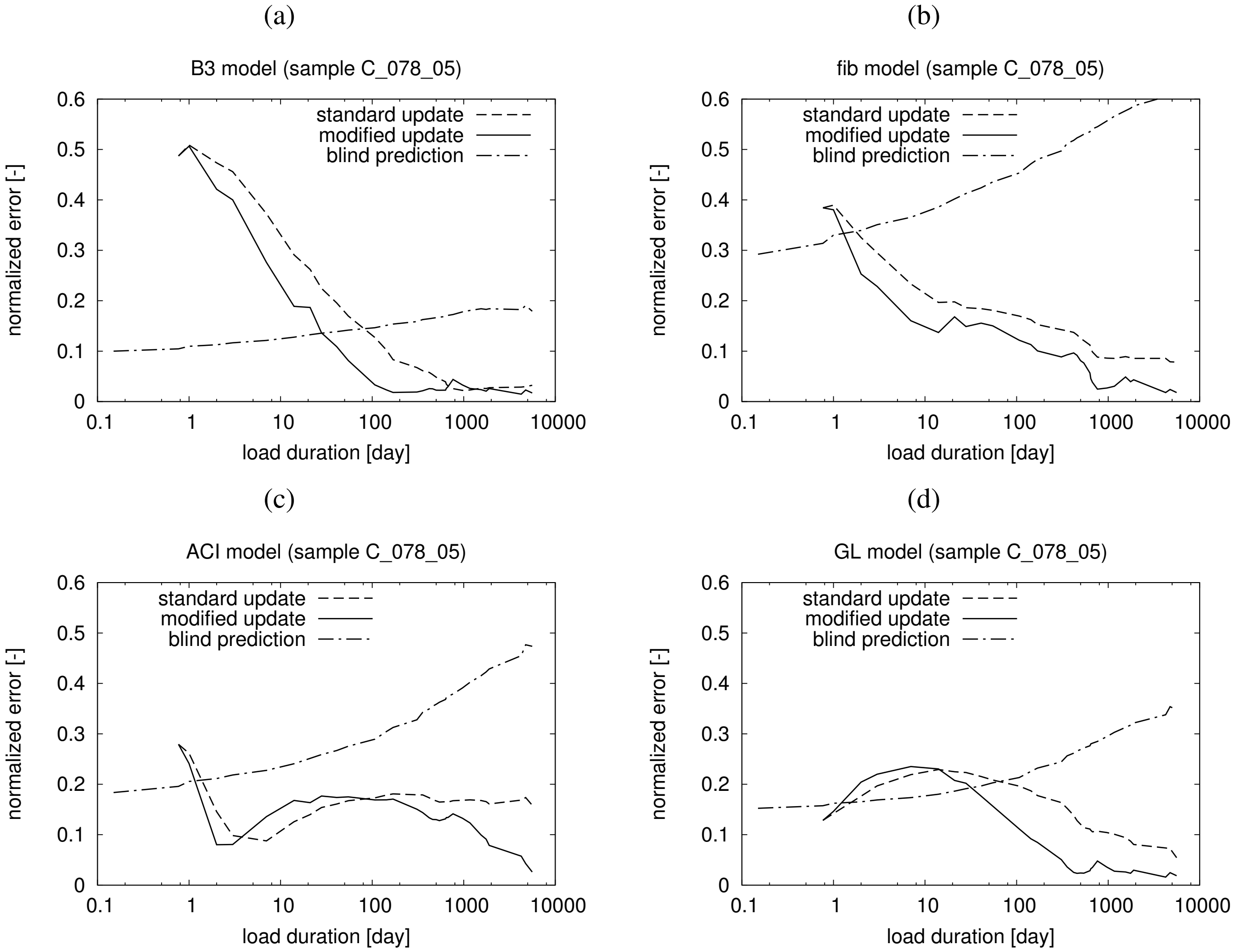

Figure 2. Normalized residual error $e_u^{(m)}/J(t_0+1000,t_0)$ as a function of the load duration up to the updating time, $t_m - t_0$, for Water Tower Place concrete: comparison of the blind prediction with the standard and modified updates for (a) B3 model, (b) *fib* model, (c) ACI model, (d) GL model.

update. A similar phenomenon, albeit less dramatic, can be observed for the other models as well. The initial error of the blind prediction using the *fib* model is much higher than for the B3 model, about 0.3, and the updated prediction becomes more accurate already after less than 2 days; see Figure 2b. For the ACI model, the behavior is similar, with a lower error of the initial blind prediction, about 0.2; see Figure 2c. Finally, for the GL model, the initial accuracy of the blind and updated predictions is comparable, about 0.16, but then the error of the updated predictions grows and remains above the error of the blind prediction up to 31 days for the modified update and up to 64 days for the standard update. Let us emphasize that all these observations refer to one single experimental test and cannot be considered as general statements. For instance, the fact that the blind prediction with the ACI model is more accurate than with the *fib* model is rather an exception. Nevertheless, this specific example illustrates the methodology and brings our attention

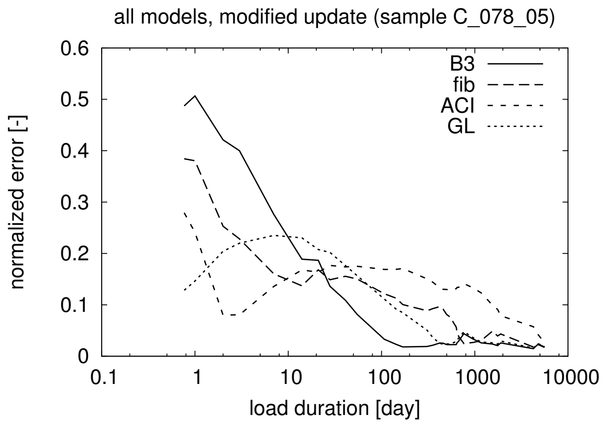

Figure 3. Normalized residual error $e_u^{(m)}/J(t_0+1000,t_0)$ as a function of the load duration up to the updating time, $t_m - t_0$, for Water Tower Place concrete: comparison of modified updates for individual models

to certain unexpected trends. For comparison, the evolution of the residual error based on the modified updating procedure for all creep models is plotted in Figure 3.

As already mentioned, the results in Figures 2 and 3 refer to one single test, and for other tests they can be quite different. To get an idea about the overall performance of individual creep models and updating procedures, it is necessary to take into account all the available tests and perform some averaging. Figure 4 shows the normalized residual error of the updated predictions based on the B3 model for all 40 tests considered in the present study. One can see that the modified updates (Figure 4b) are in general more accurate than standard ones (Figure 4a). After 100 days almost all the individual error points corresponding to the modified updates (perhaps with 2 or 3 exceptions) are below 0.2 and most of them are actually much lower.

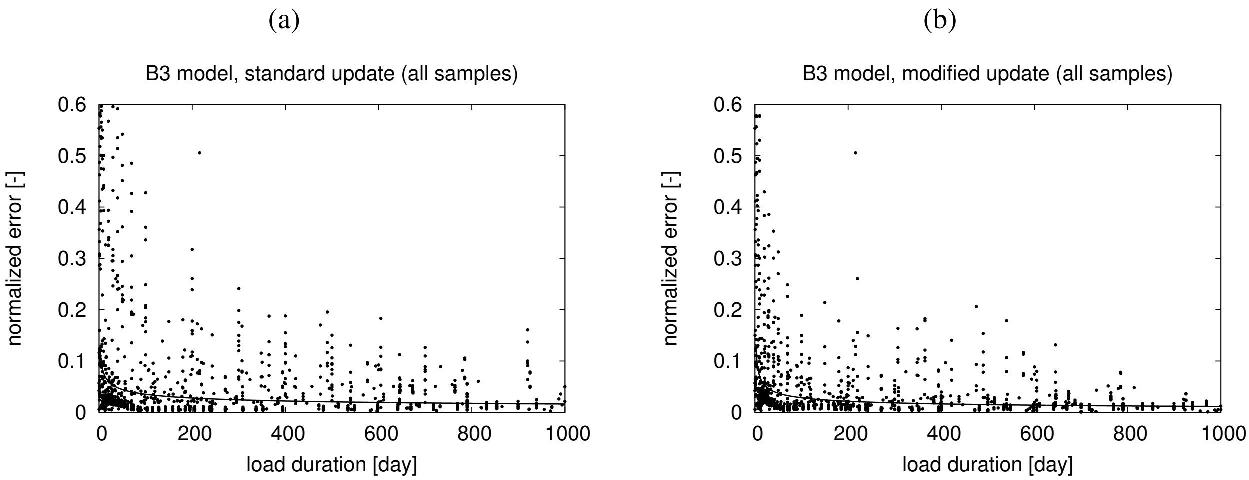

Figure 4. Normalized residual error of updated B3 model for all tests considered: (a) standard updating, (b) modified updating.

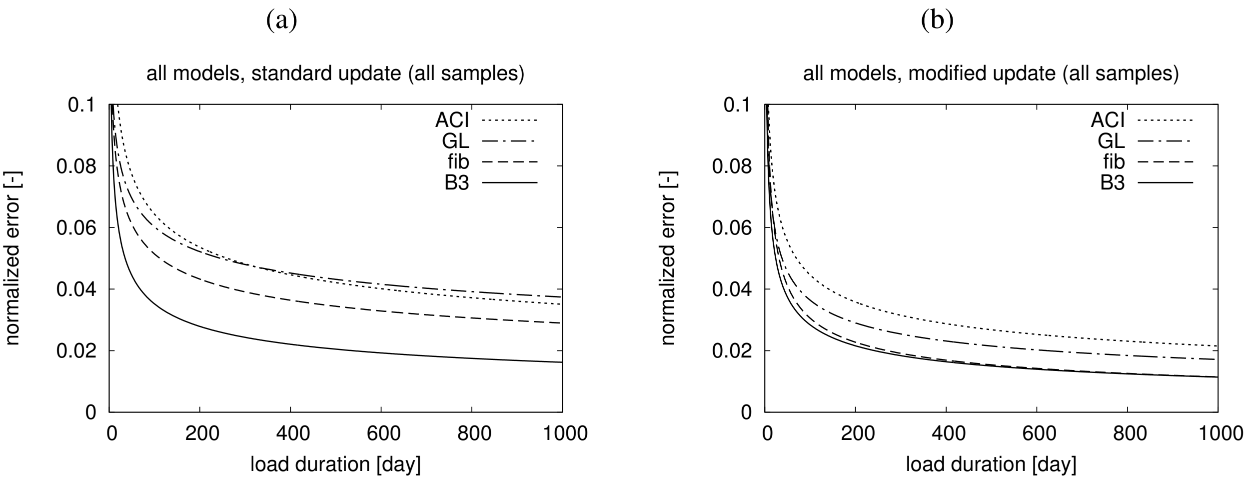

Figure 5. Comparison of power-law curves approximating the average normalized residual error for different models: (a) standard updating, (b) modified updating.

Table I. Average values of load duration after which updating leads to an improved prediction

model	*load duration [day]*
B3	48.7
fib	59.1
ACI	58.0
GL	18.6

For this overall comparison, all the tests have been truncated at load duration of 1000 days, otherwise the meaning of the residual error would be different for tests of different total durations. For easier comparison, the dependence of the normalized residual error on the updating time has been fitted (in the least-square sense) by a power law. The corresponding smooth curves that represent the average errors for individual models are plotted in Figure 5. It is confirmed that the modified updating procedure gives in general better results than the standard one. This effect is particularly strong for the *fib* model and the GL model. With standard updating, model B3 gives by far the highest accuracy, while the *fib* model is the second best. With modified updating, the average performance of the B3 model and the *fib* model is comparable. The GL model gives higher errors of the updated predictions, and the worst results are obtained with the ACI model, which is no surprise because the original version of this model is more than 40 years old.

It is also interesting to compare the typical load durations after which the modified updating procedure leads to an improvement. The average times needed to get at least the same accuracy as with the original blind prediction are summarized in Table I. It turns out that updating is beneficial for the GL model already after 19 days of loading while the other models require between 49 and 59 days. This means that if, for instance, the B3 model is used, it does not seem to be a good idea to adjust the parameters based on measurements that cover only a few weeks of loading, and one should use data covering at least several months.

5. Conclusions

It should be emphasized that the results presented in this short paper are only partial, and a more detailed evaluation remains to be finished. An extension to drying creep represents another important step to be taken before the final conclusions can be drawn. Nevertheless, the preliminary findings lead to the following recommendations:

- Updating of creep predictions based on short-time measured data exhibit higher accuracy if the updating procedure incorporates weight factors that reduce the influence of very early stages of the response.
- The updating procedure can be expected to provide better accuracy than predictions based on concrete mix composition and similar data, provided that the measured response covers a certain minimum period of time, which is in the order of a couple of weeks for the GL model and a couple of months for the other models considered in this study.
- If the updating is based on measurements covering a sufficiently long loading period, the B3 model and the *fib* model seem to provide the most accurate predictions.

Acknowledgements

Financial support of the Czech Science Foundation under project P105/10/2400 is gratefully acknowledged.

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