Random Sets-based estimation of soundings density for geotechnical site investigation

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Abstract: Geotechnical site investigation is a process conducted generally in two steps. One preliminary stage consisting in collecting available information and executing a limited number of soundings on site, and a second step of investigation based on the first stage using more soundings for soil testing. The optimal number of soundings is not known, it depends on number of factors such as geology of the site, soil variability and the type of project to build. Intervals of values are proposed by number of experts (engineers) concerning the soundings density based on preliminary information from site (soil variability, geology, type of project). Each engineer can give an interval of values based on his degree of belief. He will support his judgment by available information. As a first step we used only one parameter (soil variability) to construct the random sets. For certain soil variability degree between [1,10] the expert will give the corresponding number of soundings (with a degree of belief m_{ij}). Using Eurocode7 recommendations for site soundings, we constructed an “objective function” $f(X)$ to rely “soil variability” to the number of soundings. This function permits constructing the random set and obtain the number of soundings by unit area for each expert (engineer). The next step consists in aggregating information from other parameters (Geology, Project type…) and computing the random sets. The construction of upper and lower probabilities permits us optimizing the number of soundings to carry out on site.

Keywords: Geotechnical Investigation; Soundings density; Random Sets; Upper and Lower probabilities.

1. Introduction

Geotechnical site investigation is a process conducted generally in two steps. One preliminary stage consisting in collecting available information and executing a limited number of soundings on site, and a second step of investigation based on the first stage using more soundings for soil testing. The optimal number of soundings is not known, it depends on number of factors such as geology of the site, soil variability and the type of project to build.

The geotechnical engineer in charge of investigation should take into account all those factors and preliminary information to decide on the number of soundings to conduct in the second stage of investigation.

According to Cambefort (1980) there is no specific rule on the number of soundings to execute for geotechnical investigations. If an arbitrary loose mesh of soundings used in the preliminary study shows that the project area is relatively homogeneous then this quantity is satisfactory. However, if the results of the preliminary study show erratic conditions, the site characterization requires more soundings.

5th International Conference on Reliable Engineering Computing (REC 2012)
Edited by M. Vořechovský, V. Sadílek, S. Seitl, V. Veselý, R. L. Muhanna and R. L. Mullen

ISBN 978-80-214-4507-9. Published by Ing. Vladislav Pokorný − LITERA

Previous information on the site is generally given in form of geological and topographic maps and eventual results from adjoining sites. The engineer's judgement is important on the way investigation should be conducted, and especially on the number of soundings to make. The engineer will have a degree of belief about the density of boreholes depending on the preliminary information (soil variability, geologic profiles and type of project). The more important the preliminary information the more significant will be the degree of belief on the density of soundings to carry out. The degree of belief could be expressed as a subjective probability by experts. For a geotechnical investigation to be done, and given preliminary information, experts can formulate a subjective probability (degree of belief) related to the density of soundings to carry out. When variability of soil properties is very important for example experts will propose a "high" degree of belief that the density of boreholes should be "important" to complete an efficient investigation, on the other hand if the given preliminary information indicates a "low" variability of soil this will suggest an "important" degree of belief that the number of soundings to execute out will be "low". The approach will need certain number of experts to be reliable. The more experts the more reliable will be the result.

In a previous study we proposed to use fuzzy sets for this purpose (Boumezerane et al., 2011). In the present work we will use random sets to estimate intervals of soundings to carry out on site.

2. Parameters Affecting Geotechnical Investigations

The distribution of soundings to be made in a project area does not follow particular rules (Magnan, 2000), it depends on preliminary information among which;

- The geologic context of the project area,
- The preliminary results of investigation,
- The project type, and
- The knowledge of the neighbouring areas.

2.1. Geology

The available information about site's geology helps engineers to plan efficient geotechnical investigations. Information is obtained using geological and topographical maps. A visit on site is necessary, it permits having a reliable idea about the visible ground and formations constituting the soil.

The degree of information (knowledge) depends mainly on the scale of geological maps used, on the quality of information available (rough or precise) and on the on-site engineer's judgement. Geological published maps are fundamental tools for any of the analysis; however details have to be revealed by more specific studies. The use of maps is essential to have a first idea on the geologic formations constituting the site, their properties, as well as the possibilities of inadequate or adverse geologic details. Clayton et al (2005) recommend for geotechnical studies to use geologic maps in the scale 1/2500.

The spacing of borings depends on the geology of the area and may vary from a site to another. Boring spacing should be selected to intersect distinct geological characteristics of the project. Borings should be situated to confirm the location of significant changes in subsurface conditions as well as to confirm the continuity of apparently consistent subsurface conditions (US Corps of Engineers, 1994).

2.2. SOIL VARIABILITY

The preliminary step of geotechnical investigation consists in few soundings that permit to have a rough idea about the variability of soil properties. The parameter “Soil Variability” is important for the engineer to decide on how many soundings will be necessary in the second stage of site investigation. The soil variability is related to the number of different soil layers, their orientations and thickness. Average values of soil parameters can be obtained from different points of the site. For important variability of soil properties the density of soundings should be significant.

3. Random Sets Concept and Uncertainty

According to Helton (1997) uncertainty is classified into two distinct groups: aleatory and epistemic. *Aleatory uncertainty* is related to the natural variability of the variables involved. *Epistemic uncertainty* is related to lack of knowledge or data, therefore it can be reduced when new information is available. Possibility, evidence, interval analysis and random set (RS) theories have shown to be appropriate to deal with this type of subjective uncertainty, and here the information is expressed by means of intervals and linguistic terms (Alvarez, 2008).

3.1. RANDOM SETS

A *random set*, sometimes also referred to as a *Dempster-Shafer structure*, is given by finitely any subsets A_i, $i = 1, \ldots, n$ of a given set X, called the *focal sets*, each of which comes with a *probability weight* $m_i = m(A_i)$, $\sum m(A_i) = 1$. An example of a random set is shown in Fig. (Oberguggenberger, 2005).

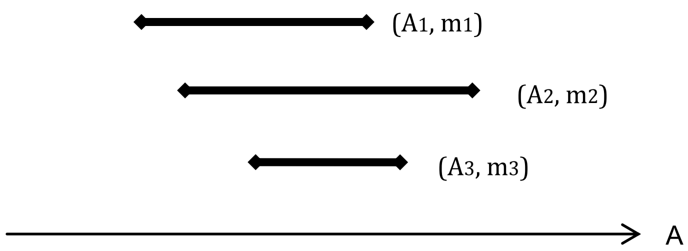

Figure 1. A random set

In the Dempster-Shafer approach (Alvarez, 2008), the random set allows to define a degree of belief $\gamma(S)$ and a degree of plausibility $\eta(S)$, respectively, that the realizations of the parameter A lie in S by;

$$\gamma(S) = \sum_{A_i \subset S} m(A_i)$$

$$\eta(S) = \sum_{A_i \cap S \neq \emptyset} m(A_i)$$

The belief function $\gamma(S)$ or *Bel*, of a subset *S* is a set-valued function obtained through summation of basic probability assignments of subsets Ai included in *S* and the plausibility function $\eta(S)$, or *Pl*, of subset *S* is a set-valued function obtained through summation of basic probability assignments of subsets A_i having a non-zero intersection with *S*. They are envelopes of all possible cumulative distribution functions compatible with the data.

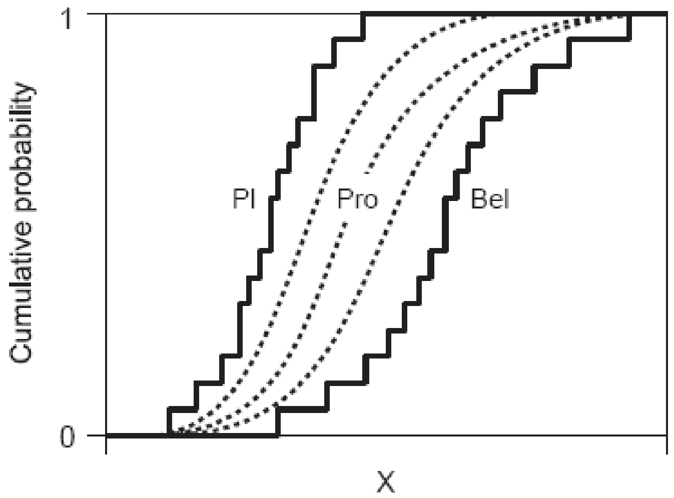

Figure 2. Upper bound (*Pl*) and lower bound (*Bel*) on precise probability (*Pro*)

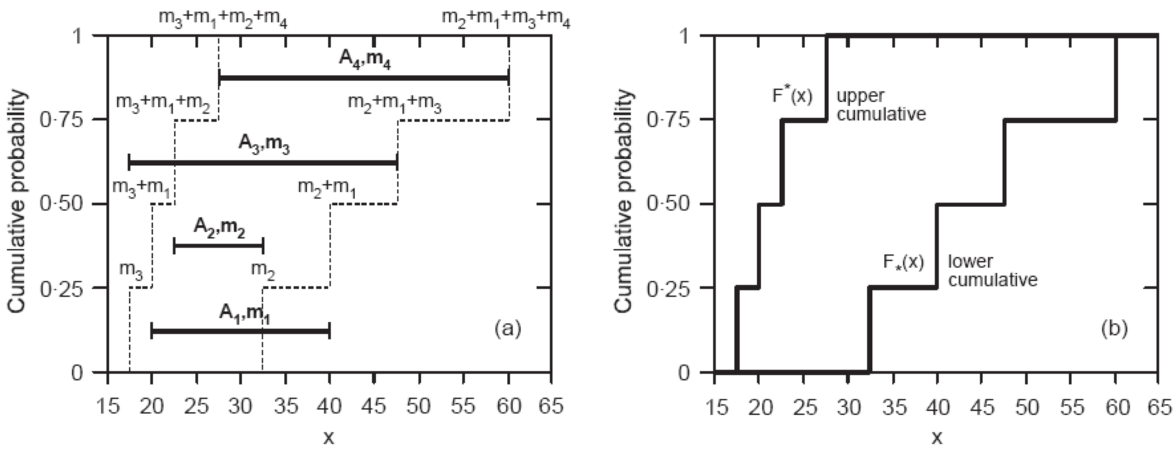

Figure 3. Random set ; (a) construction, (b) upper and lower discrete cumulative distribution function

Let’s consider for instance a Dempster Shafer (D.S) structure which is formed by gathering the information provided by four different sources (e.g. books, experts, previous analysis, etc.) on the friction angle of some soil; each of those opinions will form one element A_i of the focal set A. Suppose that $A = \{A_1 = [20°,22°],\ A_2 = [21°,23°],\ A_3 = [18°,20°],\ A_4 = [20°,25°]\}$. The basic mass assignment given to each of those focal elements will represent the importance of each of those opinions in our assessments. Suppose for example that ($m(A_1) = 0.4$, $m(A_2) = 0.2$, $m(A_3) = 0.1$, $m(A_4) = 0.3$; this means that we are giving to our first source of information the largest relevance (Alvarez, 2008).

4 Situation of the Problem

The idea underlying the use of random sets as a tool to estimate soundings density is supported by their ability to handle vague and uncertain information. The degree of belief an engineer could have given preliminary information is used to construct the upper and lower probabilities to estimate the number of soundings for geotechnical investigation. The calibration is done upon minimal number of soundings per surface recommended by Eurocode7.

Let’s have the opinions of different engineers concerning the soundings density based on preliminary information from site (soil variability, geology, type of project). Each engineer can give an interval of values based on his degree of belief. He will support his judgment by available information. If “Soil Variability” obtained from preliminary soundings is “Very Important” for instance then he will propose an important number of soundings with a strong degree of belief.

How Soil Variability is quantified by experts? A scale between 1 and 10 is proposed representing intervals of “Very Low”, “Low”, “Medium” and “High” Variability. Eurocode 7 recommends 1sounding per 40 × 40 m² as a minimum for an investigation. The degree 1 of soil variability corresponds to a “very low” variability. We consider this degree of variability necessitating the minimal number of soundings recommended by Eurocode7. The maximum number of soundings recommended by codes and some authors (Hunt, 2007) is given by 1 sounding / area of 15 × 15 m². Globally the number of soundings per unit area of 40 × 40 m² varies between 1 as a minimum and 6 as maximum, but it is possible to have more soundings if information is not enough.

Let’s consider the opinions of experts about the density of soundings on site, given soil variability. For certain soil variability degree between [1,10] the expert will give the corresponding number of soundings (with a degree of belief m_{ij}).

5. Point to point approach

We try first with only one expert 1. According to eurocode7 the minimum number of soundings is 1 for an area of 40x40m². This minimum number as explained before could be used for a “Very low” soil variability (which is comprised in the interval [0,2] on the scale). If the maximum number of soundings (6 to 7 / unit area 40x40m²) corresponds to a “High variability” (10 on the scale) we could argue a linear variation and construct an “objective function” to rely “soil variability” to the number of soundings. This function will permit us to construct Upper and Lower probability boxes as a decision aid tool.

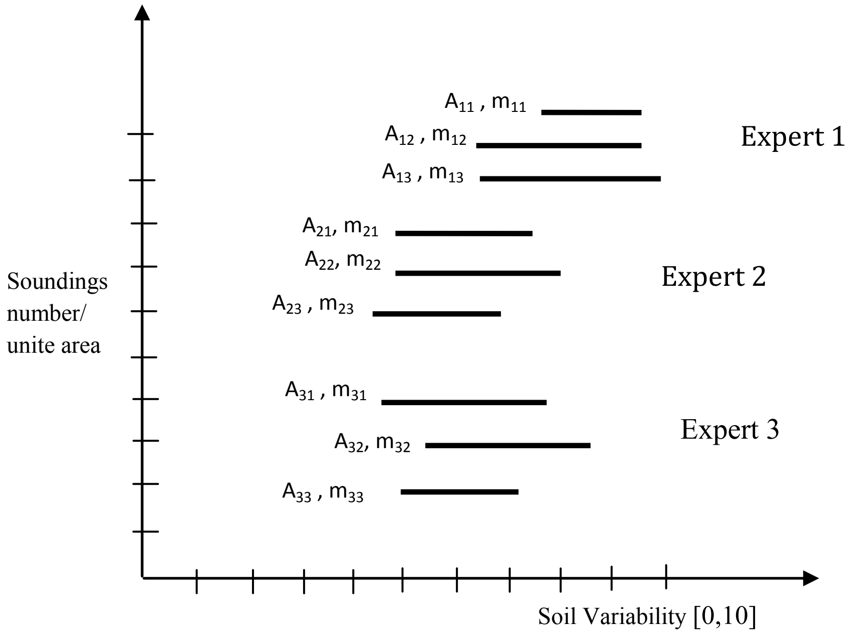

Figure 4. Scheme of random set "Soil Variability" given different experts

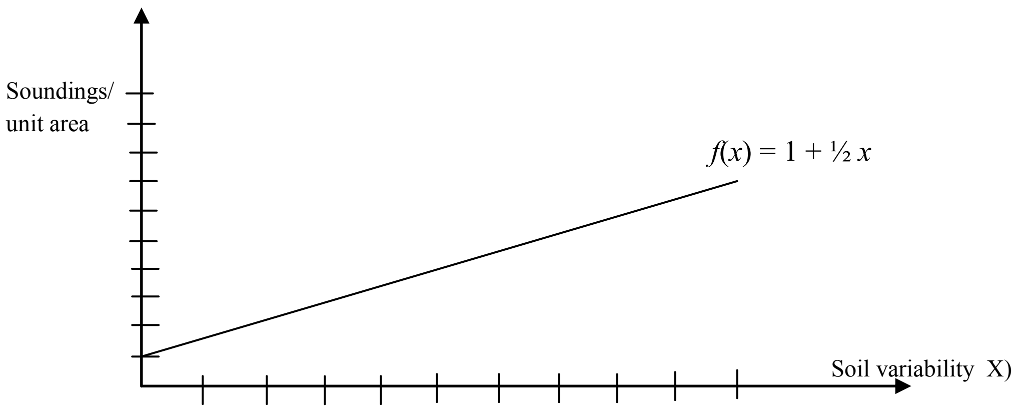

Figure 5. Objective function f(x), point to point approach

The obtained "objective function" is given by $f(x) = (1 + \frac{1}{2}x)$, this function permits to calculate the number of soundings based on the degree of variability.

Example: Expert 1 gave the following number of soundings upon a "low" soil variability, with a degree of belief m_i for each case.

Soil variability = [0,3]; $n_1 = 2$; $m_1 = ¼$
Soil variability = [1,5]; $n_2 = 3$; $m_2 = ¼$
Soil variability = [2,4]; $n_3 = 3$; $m_3 = ½$

Using $f(x)$ we obtain for the number of soundings / unit area ($40 \times 40m^2$) as shown in fig.6 :

Number of soundings [1, 2.5], $m_1 = 1/4$;
[1.5, 3.5], $m_2 = 1/4$,
[2, 3], $m_3 = 1/2$

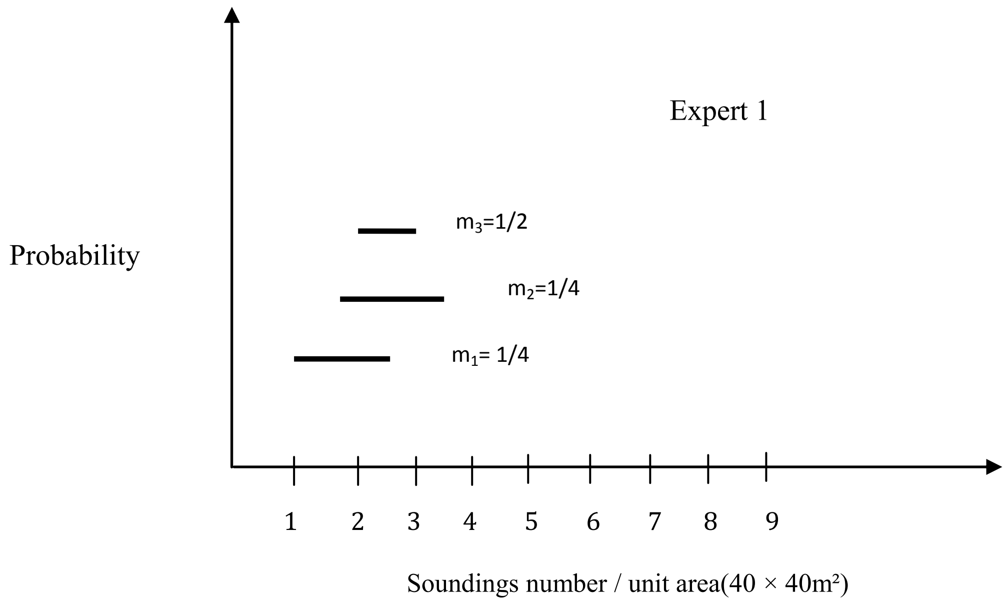

Figure 6. Construction of a random set (Number of soundings) upon $f(x) = (1 + \frac{1}{2}x)$, according to expert 1

The next step is the construction of the upper and lower probabilities (cumulative distribution function) which will give an estimation of the soundings number on site according to one expert.

From the constructed random set we assemble upper and lower probabilities.

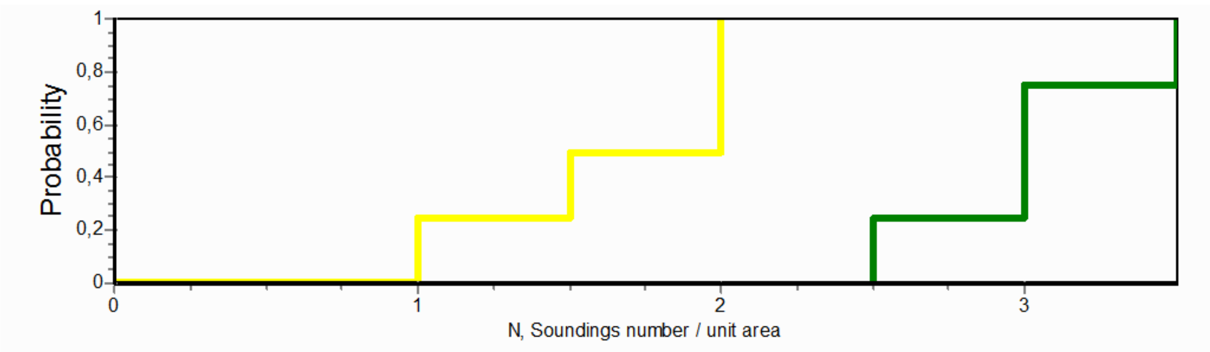

Figure 7. The constructed Upper and Lower probabilities according to one expert.

The suitable number of soundings is around 2 in this case. This example was run to illustrate the way we use the random set-based approach.

When considering more than one expert an aggregation is necessary. According to Hall et al. (2004) when there are "n" alternative random sets describing some variable x, each one corresponding to an independent source of information (expert in this case) for each focal element A,

$$m(A) = \frac{1}{n}\sum m_i(A)$$

In the case when random sets (A_i, m_i) : $i = 1, \ldots, n$ from different sources do not contain the same focal elements a merged random set is obtained using union and $m(A)$ is obtained from the previous equation.

There are other combination rules such as "Dempster rule", Yager's modified Dempster's rule, Inkagi's unified rule of combination, Dubois and Prade's rule and others (Sentz, 2002).

The Dempster's rule combines multiple belief functions through their basic probability assignments (m). The combination (joint) m_{12} is calculated from the aggregation of two pba (probability basic assignment) m_1 and m_2 as it follows:

$$m_{12}(A) = \frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-K} \quad \text{when} \quad A \neq \emptyset$$

$$m_{12}(\emptyset) = 0$$

where

$$K = \sum_{B\cap C=\emptyset} m_1(B).m_2(C).$$

The result of aggregation is still a cumulative function of distribution which could be used as a tool for decision making.

Conclusion

A random set-based approach is introduced to estimate the number of soundings for geotechnical investigations. As a first step we used only one parameter (soil variability) to construct the random sets. For certain soil variability degree between [1,10] the expert will give the corresponding number of soundings (with a degree of belief m_{ij}). Using Eurocode7 recommendations for site soundings, we constructed an "objective function" f(x) to rely "soil variability" to the number of soundings. This function permits constructing the random set and obtain the number of soundings by unit area for each expert (engineer). The construction of upper and lower probabilities permits us optimizing the number of soundings to carry out on site. The proposed system needs to be run on real sites, with different experts opinions and then aggregate them together to obtain a suitable upper and lower probabilities for estimating the number of soundings on site.

References

Alvarez Marín D. A., 2008. *Infinite Random Sets and Applications in Uncertainty Analysis.* Phd Thesis. Unit of Engineering Mathematics. University of Innsbruck, Austria.

Boumezerane D., Belkacemi S. Zlender B. Fuzzy-sets decision-support system for geotechnical site soundings. *Acta Geotechnica Slovenica*, 2011/1: 51-63

Cambefort H. *Géotechnique de l'Ingénieur, reconnaissance des sols*. Eyrolles Editeur - Paris, 1980.

Clayton CRI, Matthews MC, Simons NE. HYPERLINK "http://www.geotechnique.info/" \t "_blank" Site Investigation. Department of Civil Engineering, University of Surrey, 2005.

FHWA (2002a). Sabatini PJ, Bachus RC, Mayne PW, Schneider JA, Zettler TE. *Geotechnical Engineering Circular 5 (GEC5) - Evaluation of Soil and Rock Properties*, Report No FHWA-IF-02-034. Federal Highway Administration, U.S. Department of Transportation, 2002.

Hunt RE. *Geotechnical investigation methods: a field guide for geotechnical engineers*. Taylor & Francis Group LLC, 2007.

Magnan JP. Quelques spécificités du problème des incertitudes en géotechnique. *Revue Française de Géotechnique*, 2000; 93: 3-9.

Oberguggenberger M. Mathematics of uncertainty; Models, Methods and Interpretations; in Felin W., Oberguggenberger M., Vieider R. *Analyzing Uncertainty in Civil Engineering*, Springer Verlag 2005

Sentz K. & S. Ferson, 2002. *Combination of Evidence in Dempster- Shafer Theory*, SANDIA Report, SAND2002-0835, April 2002.

U.S. Corps of Engineers. *Geotechnical Investigation*, Report N° EM 1110-2-2504, 1994.