

Interval Nine-Point Finite Difference Method for Solving the Two-Dimensional Laplace Equation

M. A. Jankowska

Institute of Applied Mechanics, Poznan University of Technology
60-965 Poznan, Poland, malgorzata.jankowska@put.poznan.pl

Keywords: *Laplace equation; nine-point finite difference method; interval methods.*

Abstract

The paper deals with an interval version of the nine-point finite difference method for solving the two-dimensional Laplace equation. The conventional nine-point finite difference method was considered by many authors (see e.g. Orszag and Israeli (1974), Anderson, Tannehill and Pletcher (1984), Boisvert (1981), Duncan (1967)) because of the high order of the local truncation error. The local truncation error is $O(h^4, k^4)$ but it becomes $O(h^6)$ for square mesh.

The interval finite difference method proposed is based on the conventional nine-point method. Furthermore, the local truncation error of the conventional method is bounded by some interval values. Since the endpoints of the error term intervals are approximated, then the interval method considered just verifies the conventional method and we cannot guarantee that the exact solution belongs to the interval solutions obtained. Nevertheless, as the numerical experiments confirm, the exact solution does belong to the interval solutions obtained with the error term approximation proposed.

References

- Anderson, D.A., Tannehill, J.C. and Pletcher, R.H. *Computational fluid mechanics and heat transfer*. Hemisphere Publishing, New York, NY, 1984.
- Boisvert, R.F. Families of High Order Accurate Discretizations of Some Elliptic Problems. *SIAM Journal on Scientific and Statistical Computing*, vol. 2, no. 3, 1981, pp. 268-284.
- Duncan, J.W. The Accuracy of Finite-Difference Solutions of Laplace's Equation. *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-15, no. 10, 1967, pp. 575-582.
- Orszag, S.A. and Israeli, M. Numerical Simulation of Viscous Incompressible Flows. *Annual Review of Fluid Mechanics*, vol. 6, Annual Reviews, Inc., Palo Alto, California, 1974, pp. 281-318.