Inverse Problems with Interval Uncertainties

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Abstract

Inverse problems in science and engineering aim at estimating model parameters of a physical system from available observations (data) of his response (see, for example, Tarantola 1987). From given data, an appropriate 'inverse' algorithm can be formulated for the estimation of an interior property map of the medium (see, for example, Eppstein et al. 2003). Variational least square type approaches are typically adopted with an initial guess, solving the forward model, and then comparing the resulting modeled data with the actual measured data. The initial guess is then corrected by minimizing the data mismatch to yield a better match. The process is iterated until the best match is achieved.

Clearly, data measurements are affected by errors, whose nature depends upon both controllable and uncontrollable factors. Deterministic inverse algorithms hardly provide error bounds on the parameter estimates given uncertainties in the data. A combinatorial approach is computationally unfeasible. On the other hand, a probabilistic approach to solve the inverse problem, as in Kalman Filter estimation (Kalman 1960), propagates uncertainties and provides errors on the parameter estimates. However, such approaches would require a priori assumption on the nature of uncertainties, i.e. data errors are usually assumed as Gaussian. It is desirable to have inverse algorithms that do not rely on the type of uncertainties.

This work addresses this issue, by proposing an interval-based iterative solution for inverse problems that minimizes the overestimation in the target quantities, and uses the containment concept which is intrinsic to intervals as a stopping criterion. Examples will be presented and discussed.

References

Eppstein M. , Fedele F., Laible J. P., Zhang C. , Godavarty A. & Sevick-Muraca E. M. A comparison of exact and approximate adjoint sensitivities in fluorescence tomography. IEEE Transactions on Medical Imaging, vol. 22, No. 10, pp. 1215-1222, 2003.

Kalman, R. E. A New Approach to Linear Filtering and Prediction Problems, *Transaction of the ASME—Journal of Basic Engineering*, pp. 35-45, 1960.

Tarantola, A. *Inverse problem theory: Methods for data fitting and model parameter estimation*. Elsevier, ISBN 0444427651, 1987.