The application of the Probability Transformation Method for the solution of some nonlinear structural relationships

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ABSTRACT

The Probability Transformation Method (PTM) is a powerful tool for finding the relationship between the probability density functions (pdfs) of two random vectors related by an invertible transformation, that is:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}); \quad \mathbf{x} = \mathbf{g}(\mathbf{z}) \quad \Rightarrow \quad p_{\mathbf{z}}(\mathbf{z}) = \frac{1}{\mathrm{Det}\left[\mathbf{J}_{\mathbf{h}}(\mathbf{z})\right]} p_{\mathbf{x}}\left(\mathbf{x}(\mathbf{z})\right)$$

where $\mathbf{J}_{\mathbf{h}}$ is the transformation Jacobian matrix.

The above fundamental relationship can be used in the random static analysis of discretized structural systems subjected to random loads, considering that the input random variable vector $\mathbf{x}$ may represent the vector of the load acting on the structure. In this case $\mathbf{z}$ represents the random response vector. They are related by the transformation $\mathbf{h}(\cdot)$, representing the equilibrium condition of the structural system.

The direct application of the PTM to this kind can show some drawbacks. Usually they belong to three typologies: 1) it cannot be applied if the orders of the input and of the output vectors are different; 2) in some cases the determination of the inverse relationship, that is essential, is practically impossible; 3) when the pdf of only one or a few components of the response vector is required, a very onerous (computationally speaking) saturation of variables is required in order to extract the marginal pdfs from the joint obtained by the above relationship. While the first drawback can be easily overcome by introducing some opportune auxiliary variables, overcoming the other two drawbacks is a very hard task.

In this work a new version of the PTM overcoming the above mentioned problems is introduced. Its goal is the evaluation of the pdf of a generic response component (or the joint pdfs of two components), once that the joint pdf of the input is fixed. The transformation load is assumed to be nonlinear. This may happen when a linear structural system is subjected to a nonlinear combination of loads and/or when the nonlinearity is due to the geometric or mechanical properties of the structural system. An example of the first case is represented by the linear structures subjected to the wind action, when the probabilistic characterization is made on the wind velocity. For the second case a lot of examples could be made.

The proposed approach does not require the determination of the inverse transformation and, obviously, no saturation is necessary. The only computational effort is due to the numerical solution of first order integrals that are usually very simple and, in some particular cases, solved analitically.

The comparison of the results obtained by this new version of the PTM for some structural examples with those obtained by Monte Carlo simulations have always revealed a good level of accuracy.