An Algorithm for Formal Safety Verification of Complex Heterogeneous Systems

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Motivation

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intertwined with digital systems/software
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Finite set often not enough for modeling software.

Plan for Talk

- Modeling formalism for physical/software systems encompassing ODEs and data structures
- Safety verification algorithm

Illustrating Example

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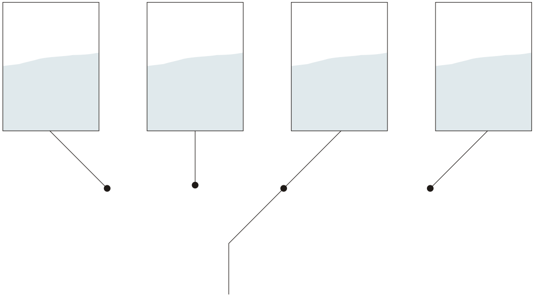
4 tanks T_1, T_2, T_3, T_4 with chemical reactions:



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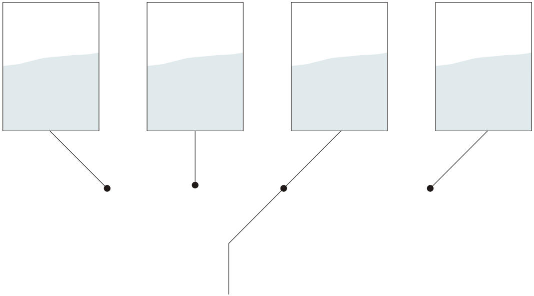

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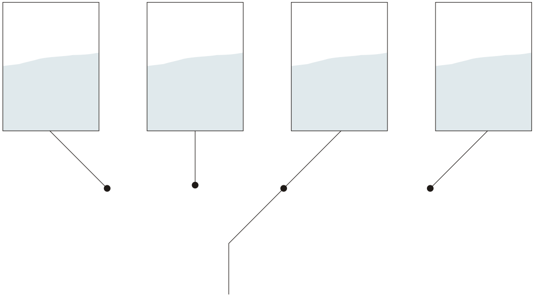

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Central control unit keeps queue of cooling requests,
e.g. $< T_3, T_1 >$

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- k variables ranging over data types $D_1, \ldots, D_k$
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Specification of system behavior:

- Continuous: Extended vector field $\texttt{Flow} \subseteq \Phi \times \mathbb{R}^n$
  - Continuous time change of $\mathbb{R}^n$ according to derivative,
  - $D_1 \times \cdots \times D_k$ stay constant,
- Discrete: $\texttt{Jump} \subseteq \Phi \times \Phi$

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Non-determinism: Relations instead of functions

Formalism: Modeling Language

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For example:

- `Flow` $\subseteq \Phi \times \mathbb{R}^n$:
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$$[\texttt{empty}(Q) \Rightarrow \dot{x} = f(x)] \wedge [\neg\texttt{empty}(Q) \Rightarrow \dot{x} = g(x)]$$

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$$t_1 \geq 100 \Rightarrow Q' = Q + T_1$$

System Evolution

- Continuous: Relation `Flow` $\subseteq \Phi \times \mathbb{R}^n$
  - Results in flows: functions from time to Φ ($\sim$ classical trajectories).
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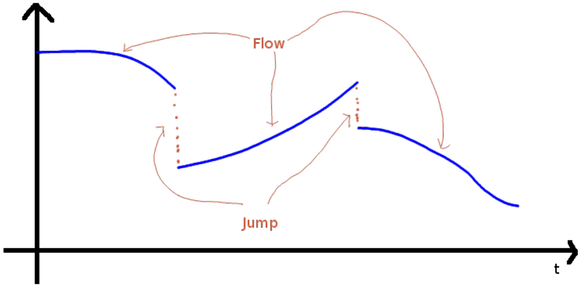
Evolution: Sequence of flows connected by jumps:



Problem: Safety Verification

Given: System +

- set of states `Init` $\subseteq \Phi$ that we consider initial
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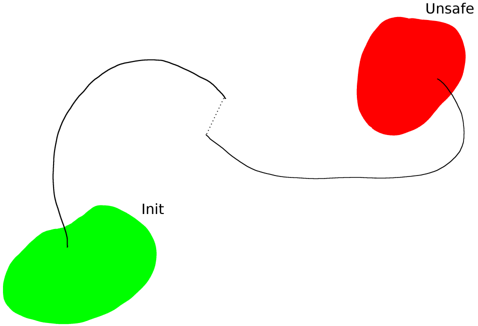

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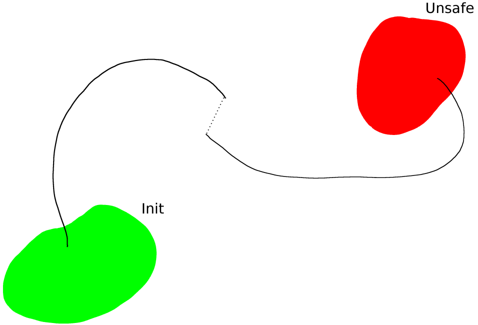

Goal: Algorithm that automatically either

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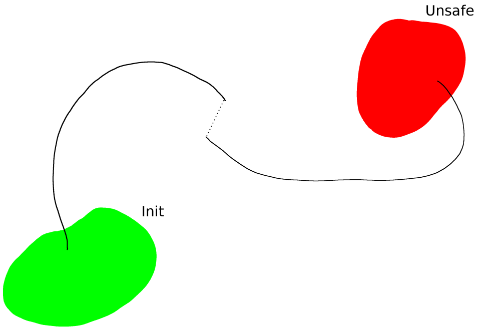

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Rest of talk: Prove non-existence of error

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Approaches to automatically prove non-existence of errors:

- Bounded Model Checking:
  1. Fix upper bound n on time,
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Approaches to automatically prove non-existence of errors:

- Bounded Model Checking:
  1. Fix upper bound n on time,
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- Unbounded Model Checking:
  Prove non-existence of errors over unbounded time
  - Approach 1: Induction
  - Approach 2: Abstraction refinement

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Divide state space Φ into finitely many (non-overlapping) regions

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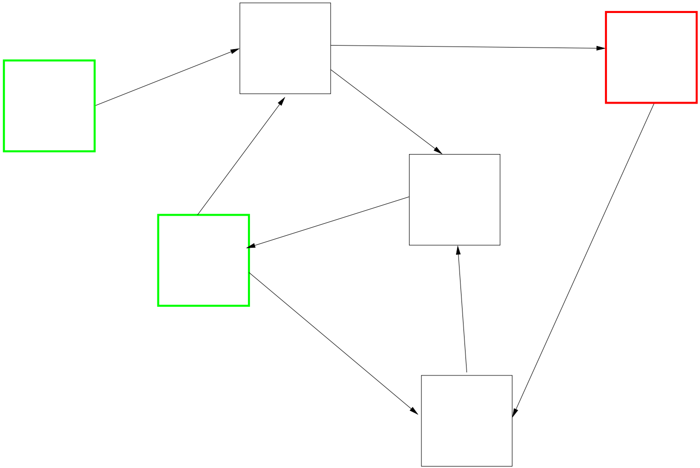
Abstraction:
 Set of finitely many regions
  together with (conservative) transitions

Analysis of Abstraction



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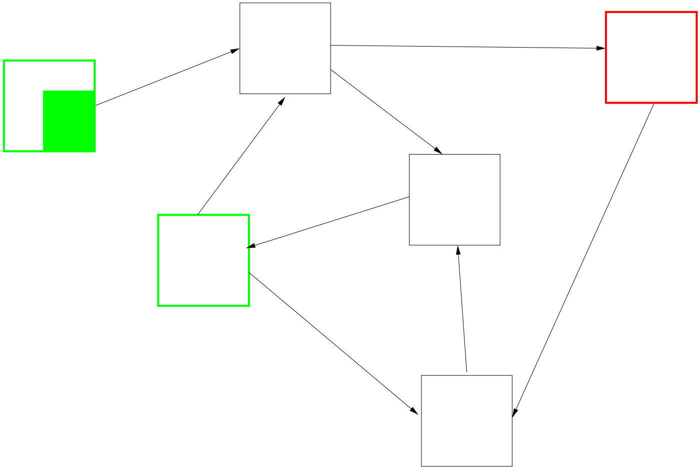

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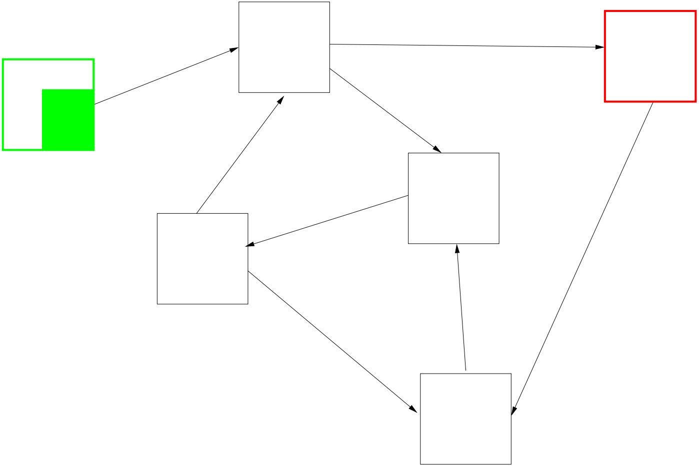

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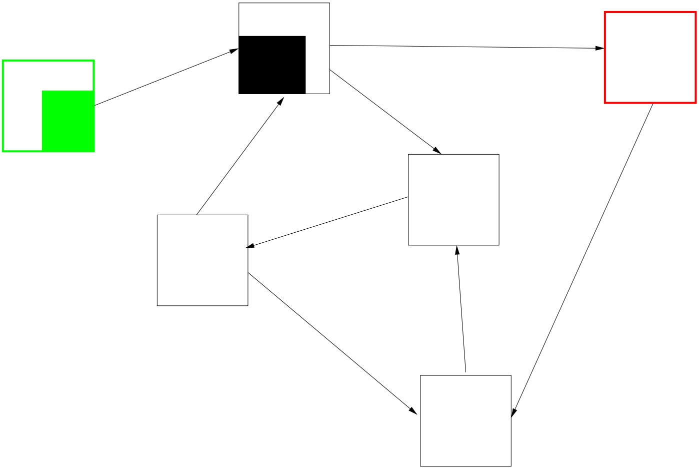

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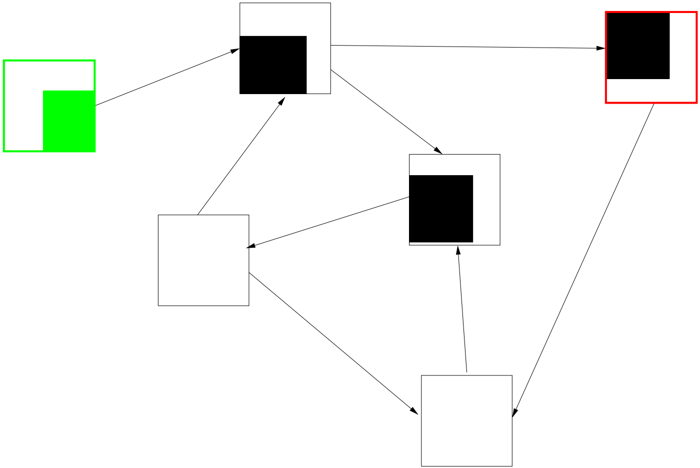

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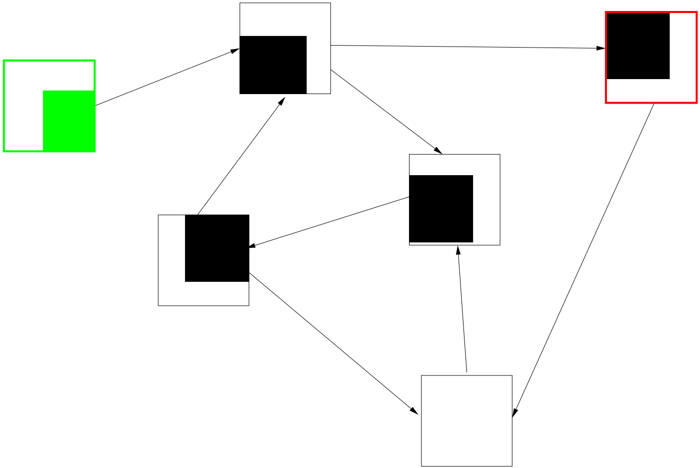

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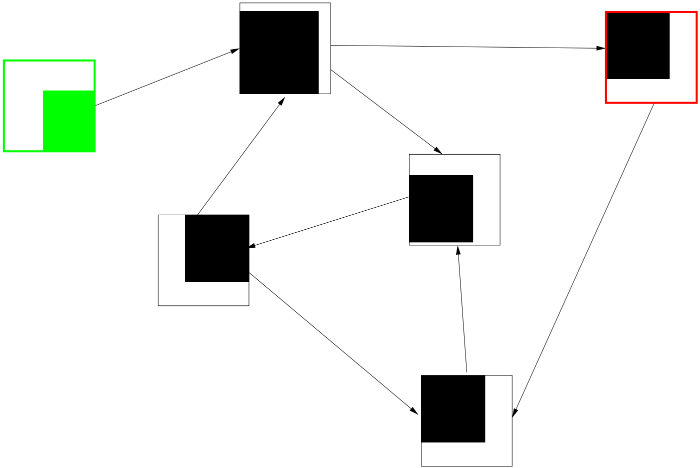

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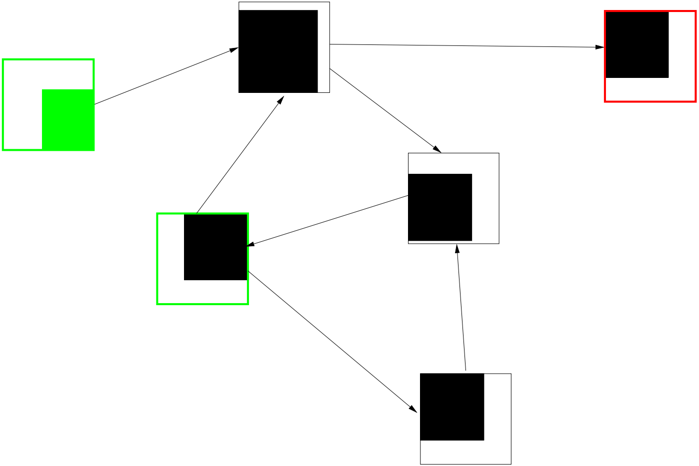

remove unconfirmed transitions (initial/unsafe markings)

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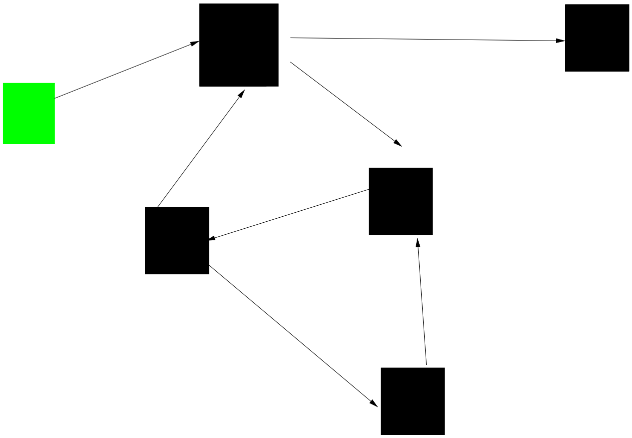

Replace regions by new ones

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Competitive in hybrid systems area

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Next steps:

- Parametric implementation:
  user-provided solvers for data types.
- More efficient handling of continuous evolution