5TH INTERNATIONAL CONFERENCE ON RELIABLE ENGINEERING COMPUTING
PRACTICAL APPLICATIONS AND PRACTICAL CHALLENGES
JUNE 13-15, 2012 | BRNO, CZECH REPUBLIC
REC2012

"Frequency response functions of discretized structural systems with uncertain parameters"

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Introduction

- Standard structural analysis tools are devoted to the numerical evaluation of the system response due to external loads for given geometry and material properties. However, in practical engineering material properties, geometry and boundary conditions often exhibit physical and geometrical uncertainties which may significantly affect the response.

- The uncertainties can be described alternatively by probabilistic or non-probabilistic approaches. The probabilistic approach requires a lot of data, often unavaible, to define the probability density function of the uncertainties.

- Unfortunately, the data about the structural parameters are frequently quite limited. Then non-probabilistic approaches are more suitable, such as convex models, fuzzy set theory or interval models (Ben-Haim and Elishakoff 1990; Elishakoff and Ohsaki 2010), can be alternatively applied.

- In the framework of the non-probabilistic approaches, the **interval model** is today the most used analytical tool. This model is based on the **interval arithmetic** introduced by Moore (1966).

Introduction

- In Structural Dynamics, the ***frequency response function* (*FRF*)**, also called ***transfer function***, is a complex function able to provide information about the behavior of a structure over a range of frequencies.

- The frequency domain approach often gives information useful for structural design purposes that cannot be alternatively caught by the time domain approach.

- In the literature the *FRF*, in presence of uncertain parameters, has been evaluated in the framework of probabilistic approaches (Falsone and Ferro 2005, 2007) and in a non-probabilistic context (Moens and Vandepitte 2004; Manson 2005).

Aim: *to present an approach for the evaluation of the* FRF *of discretized structures with uncertain stiffness properties proposing a novel series expansion of the modal* FRF, *named* ***Rational Series Expansion (RSE)***, *which provides an approximate, but explicit, expression of the* FRF *of the structural systems.*

The proposed series expansion together with the so-called improved interval analysis (Muscolino and Sofi 2012) *is used to obtain the range of the modulus of the* FRFs *of structures with uncertain-but-bounded parameters*.

Problem formulation

- Equations of motion of a quiescent *n*-DOF linear structural system with uncertain stiffness properties subjected to the forcing vector $\mathbf{f}(t)$:

$$\mathbf{M}\ddot{\mathbf{u}}(\alpha,t)+\mathbf{C}\dot{\mathbf{u}}(\alpha,t)+\mathbf{K}(\alpha)\mathbf{u}(\alpha,t)=\mathbf{f}(t)$$

where $\alpha \in \mathbb{R}^r$ is the vector collecting r uncertain real parameters.

- The global stiffness matrix can be written as :

$$\mathbf{K}(\alpha)=\mathbf{K}_0+\sum_{i=1}^{r}\alpha_i\,\mathbf{K}_i$$

where $\mathbf{K}_0$ is the nominal value of the stiffness matrix, which is a positive definite symmetric matrix of order n ,$\mathbf{K}_i$ is a semi-definite positive symmetric matrix of order n and rank p_i and α_i is the symmetric dimensionless fluctuation of the *i*-th uncertain parameters. In structural engineering, it is usually assumed that the uncertainties are not too large, it follows that it can be assumed $|\alpha_i|<1$.

Modal analysis

- Eigenproblem for the nominal stiffness matrix $\mathbf{K}_0 = \mathbf{K}(\alpha_0)$

$$\mathbf{K}_0 \Phi_0 = \mathbf{M}\ \Phi_0 \Omega_0^2 \, ; \quad \Phi_0^{\mathrm{T}} \mathbf{M}\ \Phi_0 = \mathbf{I}_m$$

- Modal decomposition:

$$\mathbf{u}(\alpha, t) = \Phi_0 \mathbf{q}(\alpha, t)$$

- Equations of motion in the modal subspace:

$$\ddot{\mathbf{q}}(\alpha,t) + \Xi\, \dot{\mathbf{q}}(\alpha,t) + \Omega^2(\alpha)\, \mathbf{q}(\alpha,t) = \mathbf{p}(t)$$

- It is worth to note that by virtue of the decomposition of the stiffness matrix, the following relationship holds:

$$\Omega^2(\alpha) = \Omega_0^2 + \sum_{i=1}^{r} \alpha_i \Omega_i^2 \quad \text{where} \quad \Omega_i^2 = \Phi_0^T \mathbf{K}_i \Phi_0$$

Frequency domain response

- In the context of the frequency domain analysis, the set of the algebraic frequency dependent equations governing the response is

$$\left[-\omega^2 \mathbf{I}_m + \mathrm{i}\,\omega\,\Xi + \Omega^2(\alpha)\right]\mathbf{Q}(\alpha,\omega) = \mathbf{P}(\omega)$$

- Modal frequency response vector

$$\mathbf{Q}(\alpha,\omega) = \mathbf{H}(\alpha,\omega)\mathbf{P}(\omega)$$

$$\Omega^2(\alpha) = \Omega_0^2 + \sum_{i=1}^{r} \alpha_i \Omega_i^2$$

where

$$\mathbf{H}(\alpha,\omega) = \left[-\omega^2 \mathbf{I}_m + \mathrm{i}\,\omega\,\Xi + \Omega^2(\alpha)\right]^{-1} = \left[\mathbf{H}_0^{-1}(\omega) + \sum_{i=1}^{r} \alpha_i \Omega_i^2\right]^{-1}$$

is the ***MODAL FREQUENCY RESPONSE FUNCTION (FRF) MATRIX***

and

$$\mathbf{H}_0(\omega) = \left[-\omega^2 \mathbf{I}_m + \mathrm{i}\,\omega\,\Xi + \Omega_0^2\right]^{-1}$$

is the ***FRF matrix of the nominal structural system***

FRF evaluation by Neumann expansion

- To avoid the inversion of the parametric frequency dependent matrix the ***Neumann series expansion*** can be adopted which leads to the following expression:

$$\mathbf{H}(\alpha,\omega)=\left[\mathbf{H}_0^{-1}(\omega)+\sum_{i=1}^{r}\alpha_i\boldsymbol{\Omega}_i^2\right]^{-1}$$

$$\Downarrow$$

$$\mathbf{H}(\alpha,\omega)=\mathbf{H}_0(\omega)+\sum_{k=1}^{\infty}(-1)^k\left[\mathbf{H}_0(\omega)\sum_{i=1}^{r}\alpha_i\boldsymbol{\Omega}_i^2\right]^k\mathbf{H}_0(\omega)$$

- The convergence of this series expansion is guaranteed if and only if the least square norm of the matrix in square brackets is less than one.

- In the following an alternative series expansion of the modal *FRF* matrix for structural systems with uncertain parameters is proposed.

Decomposition of the stiffness matrix

- Starting from $$\mathbf{K}(\alpha) = \mathbf{K}_0 + \sum_{i=1}^{r} \alpha_i \, \mathbf{K}_i$$

- ***Decomposition*** of the *r*-th semi-positive definite symmetric matrix

$$\mathbf{K}_i = \sum_{\ell=1}^{n} \mathbf{k}_i^{(\ell)} \mathbf{i}^{(\ell)T}, \quad (i = 1, 2 \cdots r)$$

$\mathbf{k}_i^{(\ell)}$ $\ell - th$ column of the matrix $\mathbf{K}_i$

$\mathbf{i}^{(\ell)}$ vector of order n containing zeros except 1 in the $\ell - th$ row

- It follows $$\mathbf{K}(\alpha) = \mathbf{K}_0 + \sum_{i=1}^{r} \sum_{\ell=1}^{n} \alpha_i \, \mathbf{k}_i^{(\ell)} \mathbf{i}^{(\ell)T}$$

- By taking into account the previous decomposition of the stiffness matrix, the modal *FRF* matrix can be rewritten in the following form

$$\mathbf{H}(\alpha, \omega) = \left[\mathbf{H}_0^{-1}(\omega) + \sum_{i=1}^{r} \sum_{\ell=1}^{n} \alpha_i \, \mathbf{\Phi}_0^T \mathbf{k}_i^{(\ell)} \mathbf{i}^{(\ell)T} \mathbf{\Phi}_0 \right]^{-1} = \left[\mathbf{H}_0^{-1}(\omega) + \sum_{i=1}^{r} \sum_{\ell=1}^{n} \alpha_i \, \mathbf{v}_i^{\ell} \mathbf{w}^{\ell\, T} \right]^{-1}$$

where $\mathbf{w}^{(\ell)} = \mathbf{\Phi}_0^T \mathbf{i}^{(\ell)}; \; \mathbf{v}_i^{(\ell)} = \mathbf{\Phi}_0^T \mathbf{k}_i^{(\ell)}$

Explicit Frequency Response Function matrix

- After some algebra, by applying the proposed ***Rational Series Expansion (RSE)***, the following relationship holds:

$$\left[\mathbf{H}_0^{-1}(\omega)+\sum_{i=1}^{r}\sum_{\ell=1}^{n}\alpha_i\,\mathbf{v}_i^{(\ell)}\mathbf{w}^{(\ell)^T}\right]^{-1}=\mathbf{H}_0(\omega)-\sum_{i=1}^{r}\sum_{\ell=1}^{n}\frac{\alpha_i}{1+\alpha_i\,b_{i\ell}(\omega)}\mathbf{B}_{i\ell}(\omega)+$$

$$+\sum_{i=1}^{r}\sum_{\ell=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{r}\sum_{m=1}^{n}\frac{\alpha_i\alpha_j}{1+\alpha_j\,b_{j\ell m}(\omega)}b_{j\ell m}(\omega)\,\mathbf{B}_{i\ell m}(\omega)+$$

$$-\sum_{i=1}^{r}\sum_{\ell=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{r}\sum_{m=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{r}\sum_{n=1}^{n}\frac{\alpha_i\alpha_j\alpha_k}{1+\alpha_k\,b_{kmn}}b_{kmn}b_{j\ell m}\mathbf{B}_{i\ell n}+\cdots$$

where

$$b_{i\ell}=\mathbf{w}^{(\ell)T}\mathbf{H}_0\mathbf{v}_i^{(\ell)};\quad \mathbf{B}_{i\ell}=\mathbf{H}_0\mathbf{v}_i^{(\ell)}\mathbf{w}^{(\ell)T}\mathbf{H}_0$$

$$b_{j\ell m}=\mathbf{w}^{(\ell)T}\mathbf{H}_0\mathbf{v}_j^{(m)};\quad \mathbf{B}_{i\ell m}=\mathbf{H}_0\mathbf{v}_i^{(\ell)}\mathbf{w}^{(m)T}\mathbf{H}_0$$

$$b_{kmn}=\mathbf{w}^{(m)T}\mathbf{H}_0\mathbf{v}_k^{(n)};\quad \mathbf{B}_{i\ell n}=\mathbf{H}_0\mathbf{v}_i^{(\ell)}\mathbf{w}^{(n)T}\mathbf{H}_0$$

- The previous ***RSE*** holds if and only if the following condition are satisfied:

$$\left|\alpha_i\,b_{i\ell}\right|<1;\quad \left|\alpha_j\,b_{j\ell m}\right|<1;\quad \left|\alpha_k b_{kmn}\right|<1;\ \ldots$$

Interval Analysis

- The *Rational Series Expansion* before introduced can be also adopted in the framework of the non-probabilistic approaches. Here it is assumed that, according to the interval model, the uncertain variable is bounded between its lower and upper bounds.

- However, the "ordinary" interval analysis suffers from the so-called *dependency phenomenon* (Muhanna and Mullen, 1975; Moens and Vandepitte, 2005) which often leads to an overestimation of the interval width.

- In order to limit the effects of this phenomenon, the so-called "generalized" interval analysis (Hansen, 1975) and "affine arithmetic" (Comba and Stolfi, 1993) have been introduced.

- In the literature, the Frequency response function *FRF* has been evaluated in the framework of probabilistic approaches (Falsone and Ferro, 2005, 2007) and in a non-probabilistic context (Moens and Vandepitte, 2004; Manson 2005).

Uncertain-but-bounded parameters

- The r uncertain structural parameters α_i (i = 1,2,…,r) introduced in the above formulation are assumed independent and are modeled as interval variables.
- According to the *interval analysis*, let $\underline{\alpha} \le \alpha \le \overline{\alpha}$ belong to a bounded set-interval vector of real numbers $\alpha^I \triangleq [\underline{\alpha}, \overline{\alpha}] \in \mathbb{IR}^r$ such that:

$$\alpha_i^I = \alpha_{0,i} + \Delta\alpha_i \, \hat{e}^I, \quad (i = 1,2,\dots,r); \quad \hat{e}^I = [-1,1]$$

where $\alpha_{0,i}$ and $\Delta\alpha_i$ are the mean value or midpoint and deviation amplitude or radius of the i-th real interval variable given, respectively, by:

$$\alpha_{0,i} = \frac{1}{2}(\underline{\alpha}_i + \overline{\alpha}_i); \quad \Delta\alpha_i = \frac{1}{2}(\overline{\alpha}_i - \underline{\alpha}_i)$$

<u>Hp</u>: $\Delta\alpha_i \ll \alpha_{i,0}$

- In structural engineering, the uncertain-but-bounded parameters can be reasonably assumed to posses symmetric deviation amplitude , so that the generic interval variable, according to the improved interval analysis, can be written in affine form as:

$$\alpha_i^I = \Delta\alpha_i \, \hat{e}^I$$

being $\alpha_{0,i}$ and $\Delta\alpha_i > 0$

Dependency phenomenon

- In the "ordinary" interval analysis, an overestimation of the interval frequently occurs when an expression contains multiple instances of one or more interval variables: ***dependency phenomenon***

Real arithmetic:

$$f(x) = x(1-x) \quad \equiv \quad g(x) = x - x^2$$

$$0 \le x \le 1 \quad \Rightarrow \quad 0 \le f(x) = g(x) \le \frac{1}{4}$$

"Ordinary" interval analysis (Moore 1966):

$$f(x^I) = [0,1] \quad \neq \quad g(x^I) = [-1,1]$$

- To limit the catastrophic effects of the dependency phenomenon, the "generalized" interval analysis (Hansen 1975) and the "affine arithmetic" (Comba and Stolfi 1993) have been introduced.

- The extra unitary interval (EUI) has been introduced (Muscolino and Sofi 2012) to treat a single variable with multiple occurrences as a dependent one.

$$\hat{e}_i^I \triangleq [-1,1] \Rightarrow \hat{e}_i^I - \hat{e}_i^I = 0; \quad \hat{e}_i^I \times \hat{e}_i^I = [1,1]; \quad \hat{e}_i^I / \hat{e}_i^I = 1$$

Derived arithmetic rules different from "ordinary interval analysis" rules:

$$x_i \hat{e}_i^I \pm y_i \hat{e}_i^I = (x_i \pm y_i)\hat{e}_i^I; \quad x_i \hat{e}_i^I \times y_i \hat{e}_i^I = x_i y_i \left(\hat{e}_i^I\right)^2 = x_i y_i [1,1]$$

Approximate interval modal FRF matrix

- Based on the ***RSE***, the interval modal *FRF* matrix, in the most general case of discretized structural systems with uncertain-but-bounded stiffness properties, can be expressed in the following approximate explicit form:

$$\mathbf{H}(\alpha,\omega) \approx \mathbf{H}_0(\omega) - \sum_{i=1}^{r}\sum_{\ell=1}^{n} \frac{\Delta\alpha_i\, \hat{e}_i^I}{1+\Delta\alpha_i\, \hat{e}_i^I\, b_{i\ell}(\omega)}\, \mathbf{B}_{i\ell}(\omega), \quad \alpha \in \alpha^I = [\underline{\alpha}, \overline{\alpha}]$$

- Alternatively:

$$\mathbf{H}(\alpha,\omega) = \mathbf{H}_0(\omega) + \sum_{i=1}^{r}\sum_{\ell=1}^{n}\left(a_{0,i\ell}(\omega) + \Delta a_{i\ell}(\omega)\,\hat{e}_i^I\right)\mathbf{B}_{i\ell}(\omega), \quad \alpha \in \alpha^I = [\underline{\alpha}, \overline{\alpha}]$$

where

$$a_{0,i\ell}(\omega) = \frac{(\Delta\alpha_i)^2\, b_{i\ell}(\omega)}{1-\left(\Delta\alpha_i\, b_{i\ell}(\omega)\right)^2}; \quad \Delta a_{i\ell}(\omega) = \frac{\Delta\alpha_i}{1-\left(\Delta\alpha_i\, b_{i\ell}(\omega)\right)^2}$$

or

$$\mathbf{H}(\alpha,\omega) = \mathbf{N}_0(\omega) + \Delta\mathbf{N}(\alpha,\omega), \quad \alpha \in \alpha^I = [\underline{\alpha}, \overline{\alpha}]$$

$$\mathbf{N}_0(\omega) = \mathbf{H}_0(\omega) + \sum_{i=1}^{r}\sum_{\ell=1}^{n} a_{0,i\ell}(\omega)\,\mathbf{B}_{i\ell}(\omega);$$

MIDPOINT

$$\Delta\mathbf{N}(\alpha,\omega) = \sum_{i=1}^{r}\sum_{\ell=1}^{n} \Delta a_{i\ell}(\omega)\,\hat{e}_i^I\, \mathbf{B}_{i\ell}(\omega)$$

DEVIATION

Bounds of the modulus of the nodal FRF

- The aim is to determine the ***range of the modulus of the nodal interval FRFs of linear discretized structures with uncertain-but-bounded parameters***.

- Once the modal F*RF* matrix is known, the square modulus of the *FRF* of the *p-th* DoF of the structural system can be defined as:

$$\left\| H_{N,pp}(\alpha,\omega) \right\|^2 = \phi_{0,p}^{\mathrm{T}} \mathbf{H}^*(\alpha,\omega)\, \phi_{0,p} \phi_{0,p}^{\mathrm{T}} \mathbf{H}(\alpha,\omega)\, \phi_{0,p}, \quad \alpha \in \alpha^I = [\underline{\alpha}, \overline{\alpha}]$$

where $\phi_{0,p}^{\mathrm{T}}$ is the *p-th* row of the modal matrix $\mathbf{\Phi}_0$ solution of the eigenproblem.

- Aiming to evaluate the upper bound and the lower bound of the modulus of the $\left\| H_{N,pp}(\alpha,\omega) \right\|$ previous equation is rewritten as:

$$\left\| H_{N,pp}(\alpha,\omega) \right\|^2 = \mathrm{mid} \left\| H_{N,pp}(\omega) \right\|^2 + \mathrm{dev} \left\| H_{N,pp}(\alpha,\omega) \right\|^2, \quad \alpha \in \alpha^I = [\underline{\alpha}, \overline{\alpha}]$$

where (neglecting the higher-order terms) the ***midpoint*** and the ***deviation*** of the square modulus of the interval nodal FRF can be expressed by:

$$\mathrm{mid} \left\| H_{N,pp}(\omega) \right\|^2 \approx \phi_{0,p}^{\mathrm{T}} \mathbf{N}_0^*(\omega)\, \phi_{0,p} \phi_{0,p}^{\mathrm{T}} \mathbf{N}_0(\omega)\, \phi_{0,p};$$

$$\mathrm{dev} \left\| H_{N,pp}(\alpha,\omega) \right\|^2 \approx \phi_{0,p}^{\mathrm{T}} \mathbf{N}_0^*(\omega) \phi_{0,p} \phi_{0,p}^{\mathrm{T}} \Delta\mathbf{N}(\alpha,\omega)\, \phi_{0,p} + \phi_{0,p}^{\mathrm{T}} \Delta\mathbf{N}^*(\alpha,\omega)\, \phi_{0,p} \phi_{0,p}^{\mathrm{T}} \mathbf{N}_0(\omega)\, \phi_{0,p}$$

Bounds of the modulus of the nodal FRF

- The ***lower bound***, $\left\| \underline{H}_{N,pp}(\omega) \right\|^2$, and the ***upper bound***, $\left\| \overline{H}_{N,pp}(\omega) \right\|^2$, of the square modulus of the nodal FRF of the *p-th* DoF can be evaluated, according to the philosophy of the affine arithmetic, as the minimum and maximum of the various combinations, i.e.:

$$\left\| \underline{H}_{N,pp}(\omega) \right\|^2 = \mathrm{mid}\left\| H_{N,pp}(\omega) \right\|^2 - \Delta\left\| H_{N,pp}(\omega) \right\|^2 ;$$
$$\left\| \overline{H}_{N,pp}(\omega) \right\|^2 = \mathrm{mid}\left\| H_{N,pp}(\omega) \right\|^2 + \Delta\left\| H_{N,pp}(\omega) \right\|^2 .$$

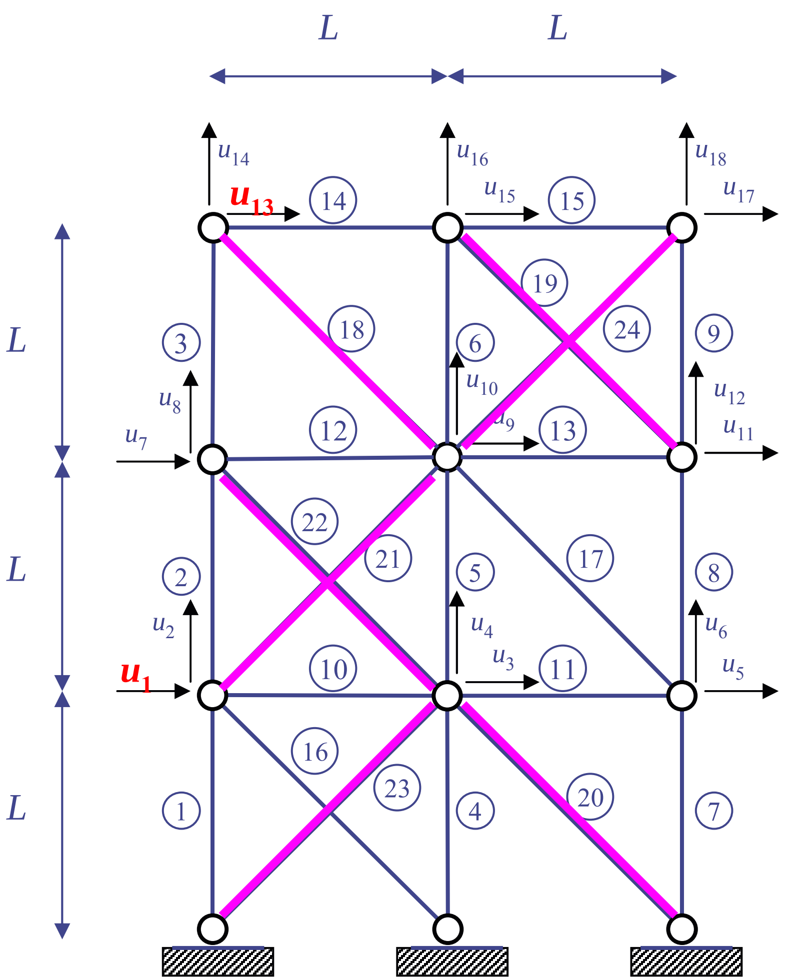
where, according to the main properties of the interval analysis:

$$\Delta\left\| H_{N,pp}(\omega) \right\|^2 = \sum_{i=1}^{r} \left| \sum_{\ell=1}^{n} \phi_{0,p}^{\mathrm{T}} \left[\mathbf{N}_0^*(\omega)\phi_{0,p}\phi_{0,p}^{\mathrm{T}} \Delta a_{i\ell}(\omega)\mathbf{B}_{i\ell}(\omega) + \Delta a_{i\ell}^*(\omega)\mathbf{B}_{i\ell}^*(\omega)\phi_{0,p}\phi_{0,p}^{\mathrm{T}} \mathbf{N}_0(\omega) \right] \phi_{0,p} \right| .$$

- Obviously the lower bound and the upper bounds of the modulus of the nodal FRF of the *p-th* DOF can be obtained straightforwardly by taking the square root of the previous equations.

Numerical Application/1

- **24-bar Truss structure with uncertain Young's moduli:**



- $A_{0,i} = A_0 = 5\times 10^{-4}\ \mathrm{m}^2;\quad L = 3\mathrm{m}$

  $E_{0,i} = 2.1\times 10^{8}\ \mathrm{KN/m}^2\ ;\ i = 1,2,\dots,24$

  $m_{0,j} = m_0 = 500Kg\quad ;\ j = 1,2,\dots,18$

  $\zeta_0 = 0.05$

- 7 uncertain parameters are considered:

  $r = 7\quad (i = 18, 19, \dots 24)$

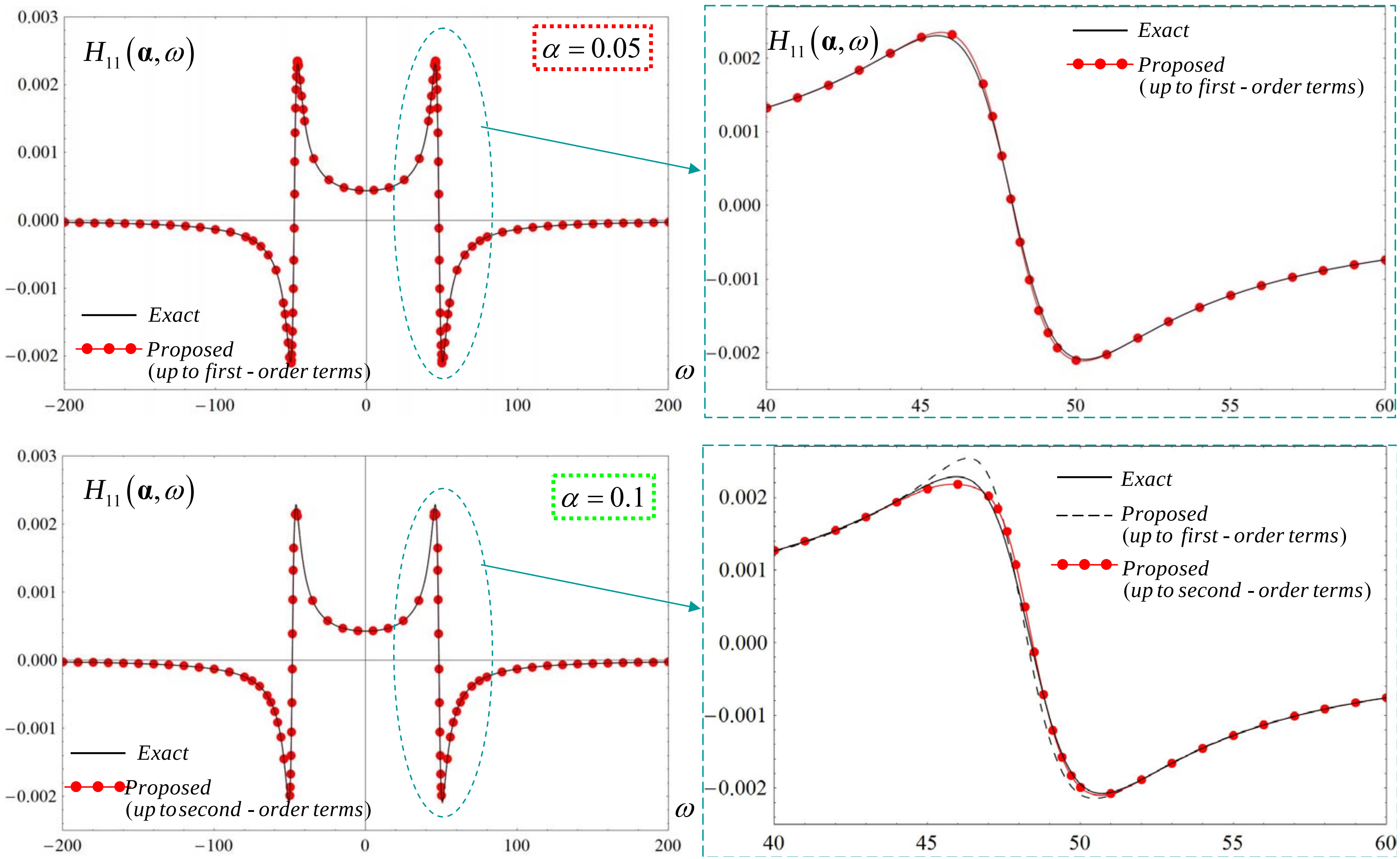
- For truss structures the $\mathbf{K}_i$ matrices are of rank 1 it follows

$$\mathbf{H}(\alpha,\omega) \approx \mathbf{H}_0(\omega) - \sum_{i=1}^{r} \frac{\alpha_i}{1+\alpha_i d_i(\omega)} \mathbf{D}_i(\omega).$$

The results provided by the proposed method are compared with the **exact LB and UB,** which can be evaluated by 2^7 combinations **(vertex method).**

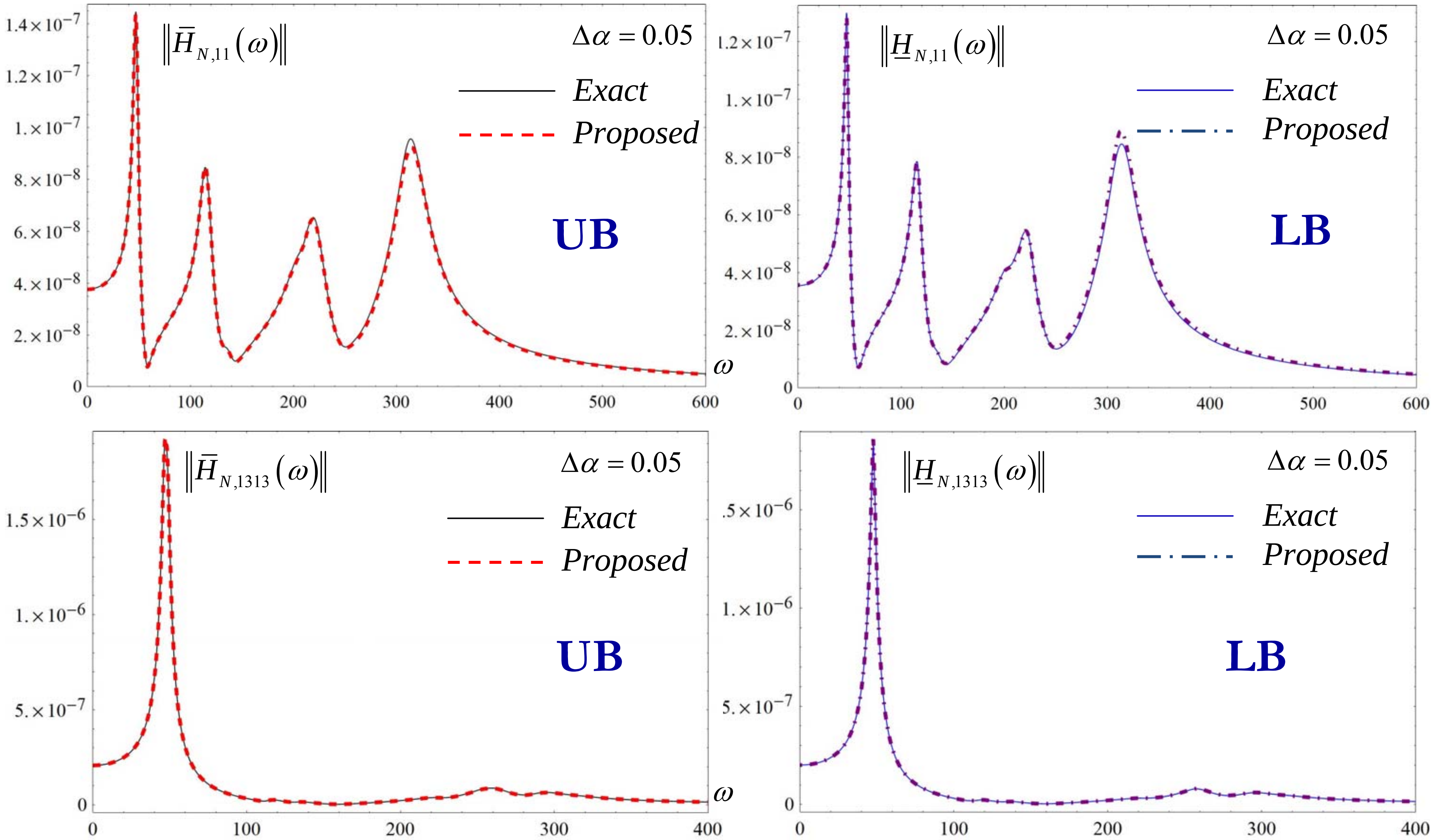
Accuracy of the proposed approach/1

- *FRF* of the first modal coordinate: Comparison between the exact *FRF* and the proposed *RSE*



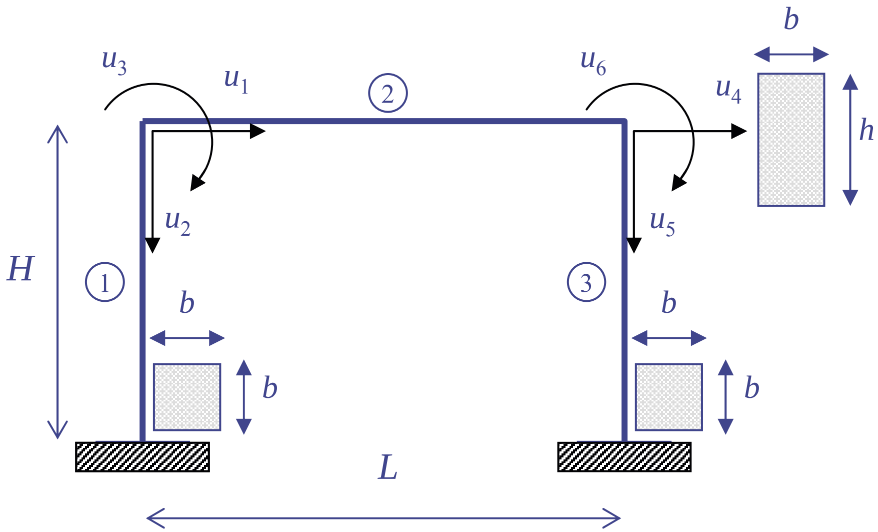
UB and LB of the modulus of the FRF/1

- Comparison between the exact and the proposed upper bound and lower bound of the modulus of the *FRF* of the nodal displacements u_1 and u_{13}: $E_i^I = E_0(1+\Delta\alpha_i\,\hat{e}_i^I)$ (i=18,19,…,24)



Numerical Application/2

- **Flexible frame with uncertain Young's moduli:**



- $b = 0.3\text{m}$; $h = 0.6\text{m}$; L=3m; H=2m

  $E_{0,i} = 2.85 \times 10^7 \text{ KN/m}^2$; $i = 1,2,3$

  $m_{0,j} = m_0 = 5000 Kg$; $j = 1,2,4,5$

  $\zeta_0 = 0.05$

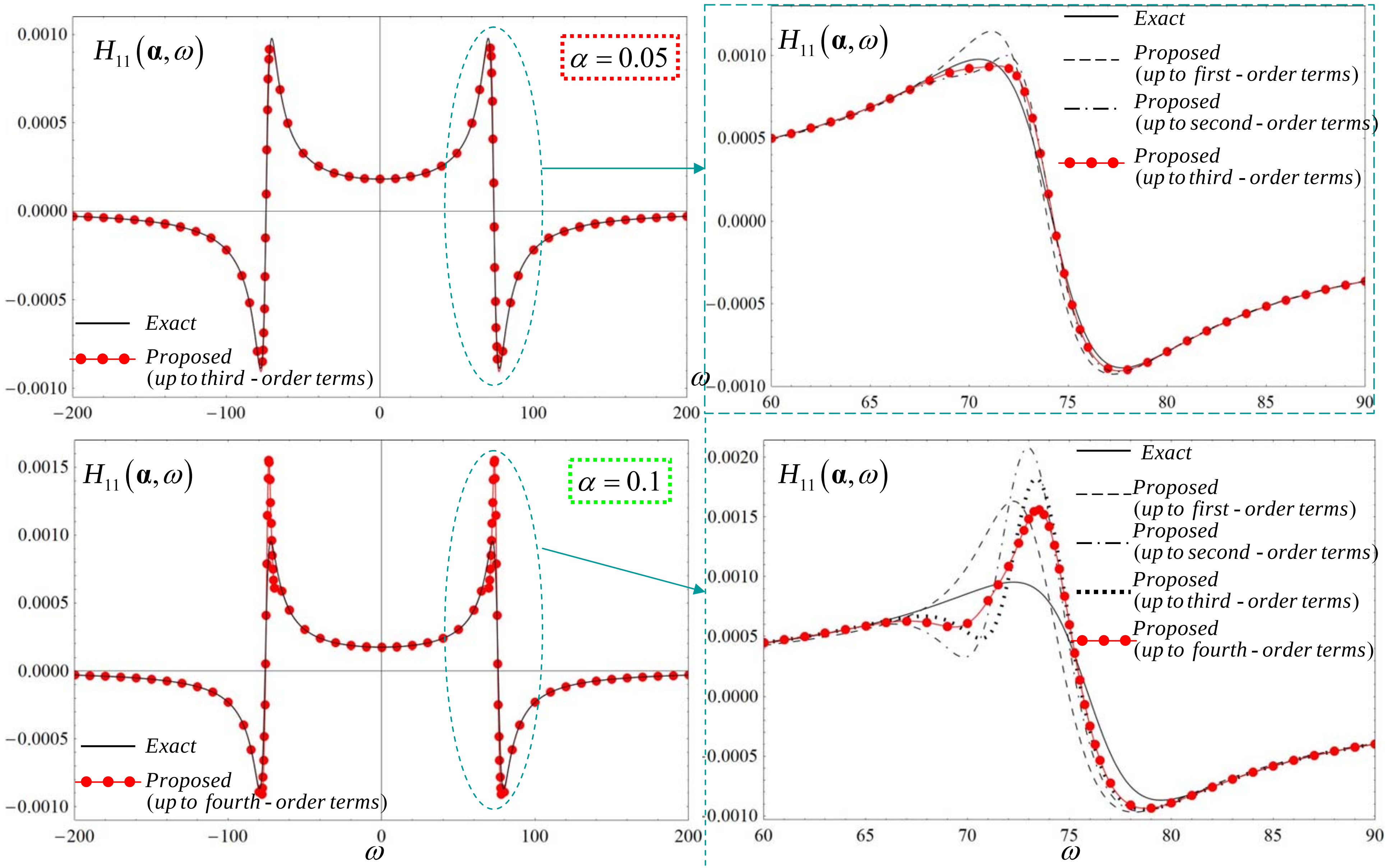
- 3 uncertain parameters are considered: $r = 3$ (i=1,2,3)

- For flexible frames structures the $\mathbf{K}_i$ matrices are of rank 3

The results provided by the proposed method are compared with the **exact LB and UB,** which can be evaluated by 2^3 combinations **(vertex method).**

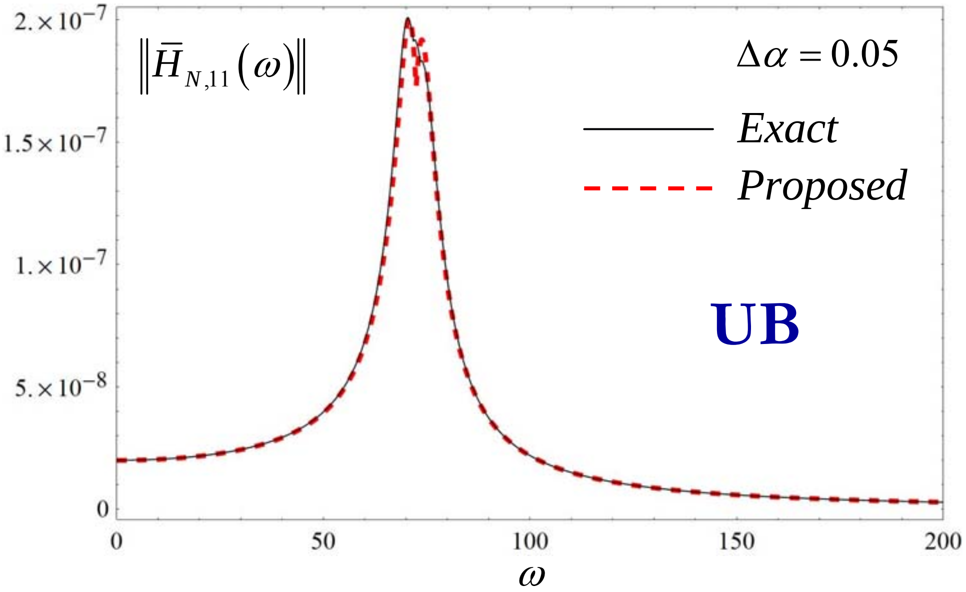
Accuracy of the proposed approach/2

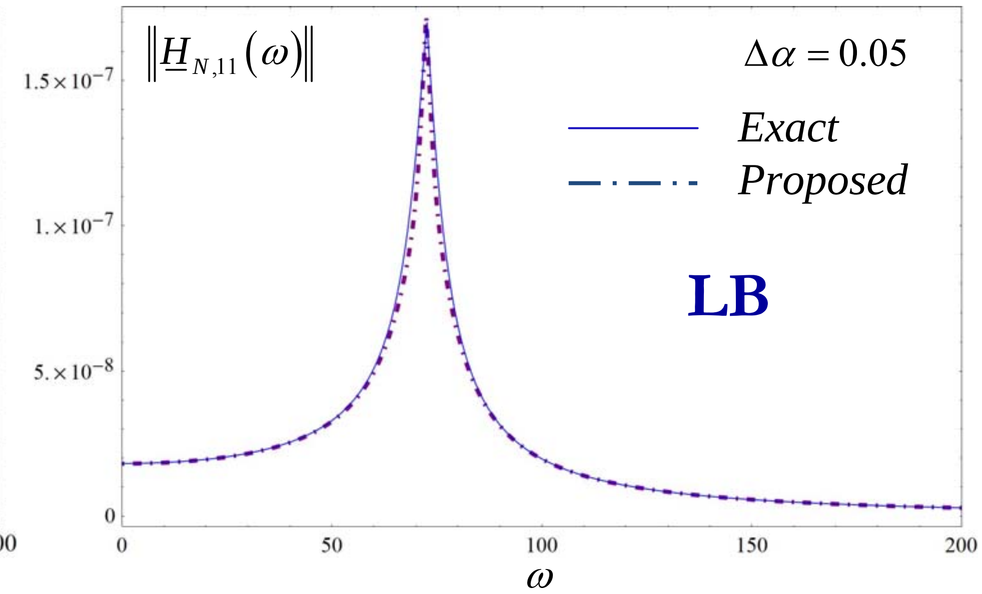
- *FRF* of the first modal coordinate: Comparison between the exact *FRF* and the proposed *RSE*



UB and LB of the modulus of the FRF/2

- Comparison between the exact and the proposed upper bound and lower bound of the modulus of the *FRF* of the nodal displacements u_1: $E_i^I = E_0(1+\Delta\alpha_i\,\hat{e}_i^I)$ (i=1,2,3)





Concluding Remarks

- A procedure for deriving the frequency response function (*FRF*) matrix of linear structures with uncertain stiffness properties has been presented.

- The proposed method relies on the decomposition of the deviation of the stiffness matrix (with respect to its nominal value) which allows to obtain a sum of rank-one matrices, each one associated to a single uncertain parameter.

- A novel series expansion, herein called ***Rational Series Expansion*** (***RSE***), is proposed to approximate in explicit form the *FRF matrix*.

- The accuracy of the proposed *RSE* has been assessed by analyzing a truss structure and a portal frame with uncertain Young's moduli and the versatility of the proposed *RSE* has been demonstrated by modeling the fluctuating Young's moduli as uncertain-but-bounded parameters.

- The estimates of the upper bound and lower bound of the modulus of the *FRF* derived by applying the *RSE* in conjunction with the so-called *improved interval analysis* have been shown to be in good agreement with the exact bounds evaluated following the philosophy of the *vertex method.*