Numerical Approaches to Bayesian Updating of Uncertainty in Model Parameters

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Abstract

This contribution aims to present and compare different numerical approaches to Bayesian updating of uncertainty in computational model parameters. The uncertain parameters stand for material properties and are represented by random variables, described by a non-Gaussian prior distribution. In Bayesian framework, see Tarantola (2005), the goal is to update this prior knowledge using a new information obtained from experimental observations. The most general updating procedure is based on Markov Chain Monte Carlo sampling of the posterior distribution, where the model simulation – often time-consuming – has to be evaluated for every sample, which can make the whole procedure computationaly very exhaustive. To overcome this problem, we employ the stochastic Galerkin method, see Babuška et al. (2004), to construct a polynomial chaos based approximation of the model response, which can be then used within the sampling instead of original model simulations, see Kučerová et al. (2011). Moreover, as an alternative to the sampling procedure, we cast the updating procedure into the linear Bayesian form enabling direct algebraic way of computing the posterior distribution, see Rosić et al. (2011).

References

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